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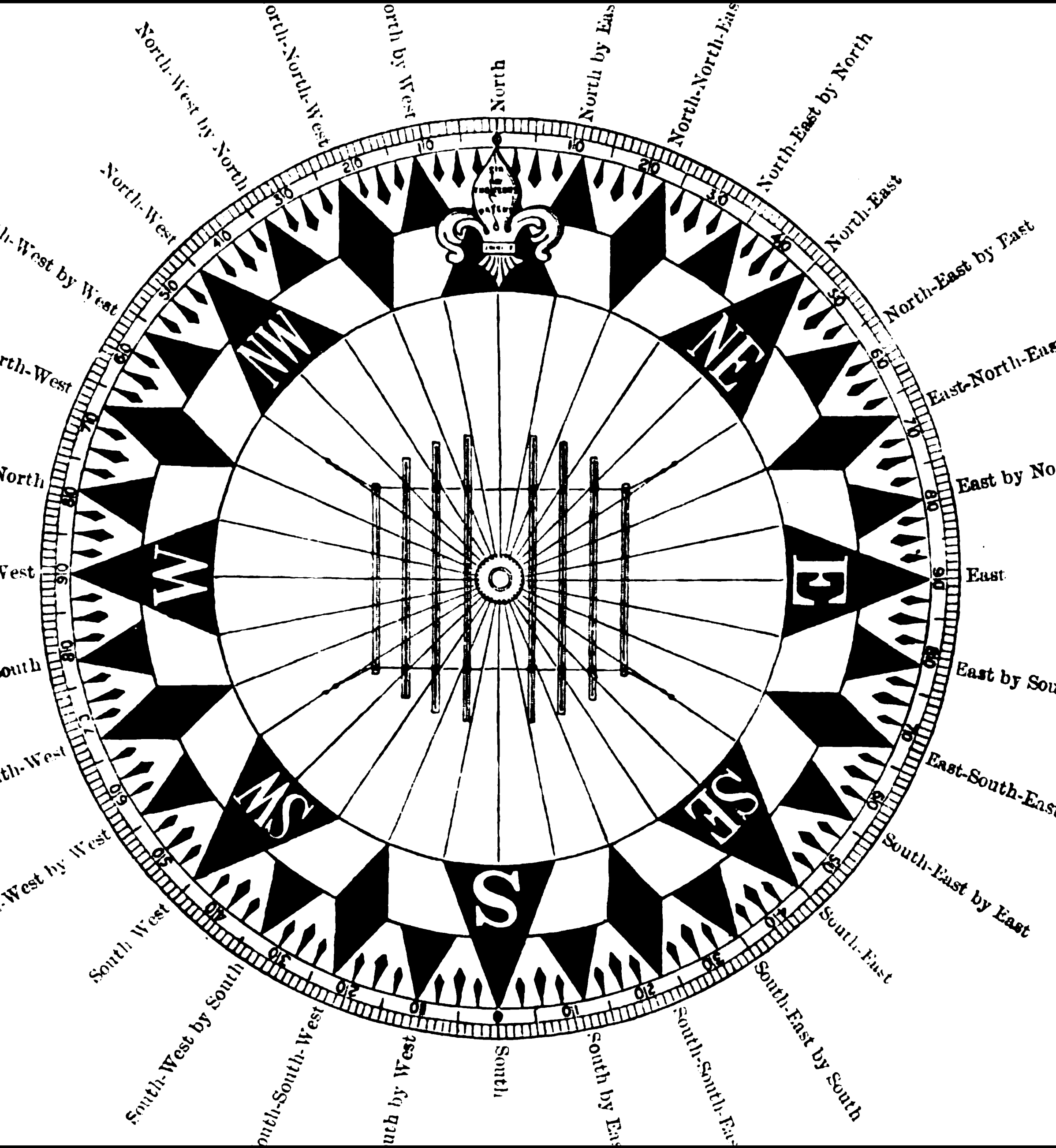

A treatise on navigation and nautical astronomy

William Robert Martin

1837
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A TREATISE ON

NAVIGATION

AND

NAUTICAL ASTRONOMY

COMPILED BY

illiam obert

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PREFACE

TO

THE THIRD EDITION

In preparing the present edition the type has been entirely re-set and the addenda of the last edition embodied in the text.

Numerous important additions and corrections have been made, especially in the chapter devoted to the investigation of Sumner's and other similar methods.

I have, through the courtesy of the executors of the late Admiral Sir Charles F. A. Shadwell, had the very great advantage of reading through and making numerous extracts from his valuable 'Notes on Navigation and Nautical Astronomy,' which embodied the experiences of a long and busy life devoted to both the practical and scientific work of the naval profession, and which it is a matter of regret to all students of the subject were never published.

For the convenience of those among my readers who are more familiar with Norie's nautical tables than with those of Inman, I have drawn up a table (placed so as to face page 1) showing the comparisons between the numbers in each.

At the end of the book will be found specimens of the papers on Navigation worked through by the senior and junior classes at the Royal Naval College.

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Royal Naval College,
Greenwich.

PREFACE

TO

THE FIRST EDITION

IN compiling the following Treatise on Navigation and Nautical Astronomy, which the Lords Commissioners of the Admiralty have honoured by accepting as the text-book for use in the Royal Navy, I have endeavoured to combine in one volume information which should be alike useful to beginners and to practical navigators. The problems dealt with are chiefly those of everyday practice at sea, adapted to the requirements of modern navigation, the introduction of swifter vessels rendering it absolutely necessary for the skilful navigator to have at his command every means of determining a ship's position by day and by night.

Preceding the practical illustrations of each problem I have placed the theory, which is dealt with in such a way as to be intelligible to those conversant with only Plane and Spherical Trigonometry, without a sound knowledge of which it is impossible to understand the art of Navigation.

I have laid great stress on a thorough knowledge of the principles of construction and use of Charts. The latter part of this subject will be found quite within the grasp of beginners, and in every case indicated throughout the book it is highly desirable to use the charts referred to, as impressing on the learner the thoroughly practical nature of the questions dealt with: as only a few charts are supplied for educational purposes, most of the examples on their use are arranged for the Officer's Atlas. Great Circle Sailing I have described from the practical point of view.

the numerous theoretical questions which the mathematical s
tions involve presenting no points of utility to the navigator.

In the chapters on Nautical Astronomy I have endeavou at first to interest the beginner in some of the phenomena general Astronomy, before dealing with the problems relating the navigation of ships.

In the portions relating to Time, I am aware that the planations are unnecessarily diffuse for practical purposes, but so, I think, for the high theoretical standard also aimed at in system of Naval education.

Perhaps the most important chapter in the book is tha which Sumner's and other kindred methods are described: these descriptions I am greatly indebted to my colleague at Royal Naval College, Mr. J. M. Pask, R.N., and to Mr. A Johnson, R.N., late of H.M.S. 'Britannia.'

In the Appendix on Tides will be found a summary of chief facts relating to this very important but seldom stu subject.

I have throughout the book constantly referred to t valuable aids to the practical navigator in the form of var tables, diagrams, &c., placed in the chart boxes, and the us which should, I think, be carefully shown to the learner at e stage of his progress.

I have also frequently referred to works which are eithe the ship's library or generally form part of every officer's pri collection. Among these I have more especially referred 'Wrinkles in Navigation,' by Captain Lecky, R.N.R., a w which I have found most useful in the instruction of my cla in Practical Navigation at the Royal Naval College.

As Inman's Nautical Tables are those most generally use the Navy, I have arranged for the examples to be worked by th and the publishing of this book being considered a suit opportunity for revising and re-arranging both those tables the 'Handbook for the Stars' commonly used in the Navy, references in the text are to both the old and new edition these works.

I have thought it worthy of my subject to spend much t in searching through the nautical records of past years, in o to determine to whom we owe the first publication of the various problems of navigation: the curious information I thus obtained I have placed on record in the foot-notes.

The examples, with some alterations, are chiefly taken from Jeans' 'Navigation.' Although that book has been extensively used in the Royal Navy during the last forty years, there may still exist some errors in the solutions, and for corrections in these I shall always be greatly obliged to my readers.

During the two years I have been engaged in this compilation I have sought and received a vast amount of advice and assistance from my many friends in the service, both present and former members of my classes of senior officers at Greenwich, my present co-workers at the Royal Naval College, my former colleagues at the Hydrographic Office, and members of the staff of H.M.S. 'Britannia,' to all of whom I give my most cordial thanks, and to whom we shall all feel indebted if the present volume answers the purpose for which it is designed, of clearly placing before Naval officers the Theory and Practice of Modern Navigation.

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ROYAL NAVAL COLLEGE,
GREENWICH.

CONTENTS

PART I.—NAVIGATION.

CHAPTER I.

PAGE

General observations—Meridians—Latitudes and longitudes of places—Differences of latitude and longitude—The true course—Rhumb line—Distance—Compass cards and names of points—Log ship—Log line—Patent logs 1

CHAPTER II.

Variation—Changes in variation—Use of Variation chart—Applications of variation—Deviation—Correcting courses for both variation and deviation—Leeway—Correcting compass courses for all sources of error 16

CHAPTER III.

Charts and plans—Mercator's chart—Plane charts—Meridional parts—Construction of Mercator's chart—Laying off courses and measuring distances—Gnomonic projection—Plans—Scales of latitude and longitude—Correction of charts—Details of charts 30

CHAPTER IV.

The protractor—Station pointer—Cross bearings—Bearing and angle—Position determined by angles—Doubling the angle on the bow—Four-point bearings—Danger angle 50

CHAPTER V.

The sailings—Parallel sailing—Plane sailing—Middle latitude sailing—Departure—Mercator sailing—Traverse sailing—The day's work—Soundings 65

APPENDIX.

Swinging ship for deviations of the compass 98

Great circle sailing 101

Calculation of meridional parts 106

Definitions in navigation 110

PART II.—NAUTICAL ASTRONOMY.

CHAPTER VI.

PAGE

Elementary astronomy—Solar system—Fixed stars—Finding the stars—Ecliptic—Equinoctial—Declination—Right ascension—Horizon—Altitudes—Azimuth—The sextant—Correction of altitudes—Dip—Refraction—Semidiameter—Parallax 115

CHAPTER VII.

Time—Sidereal day—Apparent solar day—Mean solar day—Equation of time—Comparisons of sidereal, mean, and apparent intervals of time—Sidereal year—Tropical year—The calendar—Problems on time—*Nautical Almanac*—Greenwich and local times—Correction of declinations, right ascensions, &c.—Correction of altitudes 157

CHAPTER VIII.

Hour angles—Sun's hour angle—Star's hour angle—Times of meridian passages—Moon's meridian passage—Corrections for run—Effect of run on altitude and time 201

CHAPTER IX.

Latitude—Meridian altitudes of the sun, moon, and stars above and below pole—Altitudes near meridian above and below pole—Delambre's method—Pole-star—Double altitudes of sun and stars—Douwes's method—Three altitudes at equal intervals of time apart—Combined altitudes 227

CHAPTER X.

Chronometers—Principle of balance—Longitude by chronometer—Sun chronometer—Star chronometer—Lunar observations—Sun lunars—Star lunars—Lunars by calculated altitudes—Longitude by equal altitudes 274

CHAPTER XI.

Astronomical observations for deviation of compass—Diurnal paths of heavenly bodies—Amplitudes—Altitude azimuths—Time azimuths—True bearing of terrestrial objects—Lengths of days—Time of twilight 315

CHAPTER XII.

Errors of chronometers—The artificial horizon—Absolute altitudes—Equal altitudes—Advantages of equal altitudes—Rates of chronometers—Secondary meridians 335

CHAPTER XIII.

PAGE

Errors of observation—Errors affecting latitude—Errors affecting hour angles—Rates of changes of altitudes—Errors affecting azimuths—Errors caused by spheroidal form of earth—Precession of the equinoxes 360

CHAPTER XIV.

Latitude and longitude from observations at same time—Sumner's method—Marcq Saint-Hilaire's method—Double chronometer rule . . 381

The tides 404

Practical example on navigation 414

Examination for rank of Lieutenant, R.N. 419

Definitions in nautical astronomy 421

INDEX 425

LIST OF PLATES

at end of the book.

PLATE

1. Fixed and Movable Compass Cards for correcting Courses for Variation and Deviation.

2. Chart of Part of the South Coast of England, extending from the Bill of Portland to St. Alban's Head.

3. Plan of the Downs.

4. Napier's Diagram showing the Curve of Deviation drawn from the Deviation Table used in this book (*see* page 21).

5. The above Curve of Deviation drawn on Ruled Foolscap Paper.

6. Archibald Smith's Straight Line Course Table constructed from the Deviation Table on page 21.

7. Lines of Equal Magnetic Variation for 1882.

8. Godfray's Great Circle Diagram, showing a Great Circle Track and a Composite Track.

COMPARISON BETWEEN THE NUMBERS OF INMAN'S NAUTICAL TABLES AND THOSE OF NORIE

Inman	Norie	Inman	Norie	Inman	Norie
3	1 and 2	24	48	44	25
7	3	31	17	46	31
12	20	32	D	47	24
13	5	33	16	48	36
14	8	34	30	49	42
16	4 and 18	36	A	50	55A
17	3*	37	45	52	50
18	6	38	46	54	34
20	18	39	13	55	54
21	9	40	38	56	26
22	10	41	37	62	56
23	14	43	52		

NAVIGATION

AND

NAUTICAL ASTRONOMY.

PART I.–NAVIGATION.

CHAPTER I.

General observations—Meridians—Latitudes and longitudes of places—Differences of latitude and longitude—The true course—Rhumb line—Distance—Compass cards and names of points—Log ship—Log line—Patent logs.

Definition

NAVIGATION has been well defined as 'the art of conducting a ship from port to port, or that science which relates to the determination of the place of a ship on the sea.'

It embraces two distinct modes of procedure, both of which are employed in making a voyage.

Dead reckoning

The first method is practised with the aid of the compass and log line, the former to determine the direction in which the ship is proceeding, the latter to ascertain the distance run in that direction; the result obtained from this record of the direction and distance is termed *dead reckoning*.

Position by observation

The second method is that by which the position is determined astronomically and far more accurately than is possible with the dead reckoning; the principal instruments employed are the sextant and chronometer.

The first of these methods is known as Navigation, the second as Nautical Astronomy.

Application of trigonometry

It cannot too soon be recognised by the reader that both branches of the subject are applications of Trigonometry, the former principally involving the solution of right-angled plane triangles, the latter of spherical triangles; and to confine our attention throughout this part to Navigation alone, we shall find that every practical problem which presents itself can be shown on drawing a suitable figure to be capable of easy solution by those familiar with elementary Trigonometry and the use of logarithms; it will therefore be assumed that the study of those subjects will precede or accompany the perusal of this work, and since Goodwin's *Trigonometry* is the official text-book on that subject in the Royal Navy frequent references will be made to that volume.

Commencing our work with a description of the method of distinguishing the positions of places on the earth by their latitudes and longitudes, we give the following elementary geographical definitions.[1]

Axis The Axis of the earth is that diameter about which it revolves.

Poles The Poles of the earth are the points where the axis meets its surface.

Equator The Equator is a great circle on the earth's surface equidistant from its poles, thus dividing that surface into two equal portions.[2]

Meridian Longitude The positions of places on the earth are determined by their latitudes and longitudes; if we imagine a great circle drawn from pole to pole, such a circle is termed a *meridian*, and the *longitude* of a place is the arc of the equator intercepted between the meridian passing through the place and some arbitrarily chosen meridian (first meridian); that selected in most countries being the meridian passing through Greenwich.

At a conference of delegates from the principal countries of the world (twenty-seven in number), held at Washington, U.S., in October 1884, to fix on a meridian proper to be employed as a common zero of longitude, twenty-two representatives voted in favour of adopting the meridian passing through the centre of the transit instrument at the Observatory of Greenwich as the initial meridian for longitude, and fourteen voted in favour of counting the longitude in two directions up to 180 degrees—east longitude being plus, and west longitude minus.

The equator is supposed to be divided into 360 equal parts, termed degrees, each containing sixty nautical miles or minutes of arc, and longitudes are reckoned both east and west from Greenwich, reckoned numerically in each direction from 0° to

---

[1] For convenience of reference the various definitions in Navigation are brought together and formally stated in the terms sanctioned for use in H.M.S. *Britannia* on pages 110–113. The definitions in Nautical Astronomy are similarly given on pages 421–424.

[2] Great circles are those whose planes pass through the centre of the sphere. Small circles are those whose planes do not pass through the centre of the sphere.

180°, so that the meridian of 180° east coincides with that of 180° west.

Latitude

The *latitude* of a place is the arc of the meridian passing through the place, intercepted between it and the equator, or the angle subtended by that arc at the centre of the earth; latitudes are measured both north and south from the equator, the whole arc of the meridian between it and either pole being supposed to be divided into 90°, reckoned numerically from the equator to either pole, and each consisting of sixty minutes or nautical miles.

The figure of the earth is not that of a sphere, but of an oblate spheroid; hence in different latitudes the arcs subtending angles of one degree at the centre are not of equal lengths; and the sixtieth parts of such degrees vary in length from 6,046 feet at the equator to 6,108 feet at the pole. In navigation the earth is always considered to be a sphere, and the nautical mile 6,080 feet; equivalent nearly to 1⅐ statute miles.

Strictly speaking, while the geographical mile is the length of a minute of arc of the equator, the nautical mile is a minute of arc of a circle passing through the place considered, and having a radius equal to the radius of curvature of the meridian.

Parallels of latitude

Parallels of latitude are small circles on the globe whose planes are parallel to that of the equator.

Difference of longitude

From the foregoing definitions it will be seen that the *difference of longitude* (diff. long.), sometimes called 'easting' or 'westing,' between two places is the arc of the equator intercepted between their meridians, and remembering that we estimate longitudes *both* east and west from Greenwich, it follows that if two places are both east or both west of Greenwich their difference of longitude is found by taking the numerical *difference* of their longitudes; but if one is east and the other west of Greenwich the numerical *sum* of their longitudes, or the defect of that sum from 360°, gives the required difference.

It will be remarked that the diff. long. is the same for all places on the same meridians though the actual distance in miles between them of course varies.

Difference of latitude

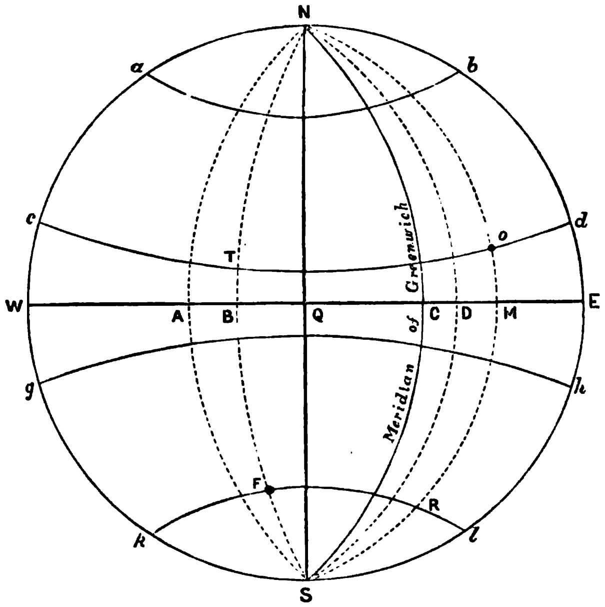
It will also be seen that the *difference of latitude* between two places is the arc of a meridian intercepted between parallels of latitude drawn through them. This is termed TRUE DIFFERENCE OF LATITUDE (T. D. lat.) in distinction to the meridional difference of latitude, which will be defined later on.

True difference of latitude is sometimes called 'northing' or 'southing.'

Since latitudes are measured *both* north and south from the equator, similar distinctions of cases will arise when considering differences of latitude, as we observed before with differences of longitude.

The full meaning of the preceding elementary definitions can be best understood by reference to a globe—a remark which applies equally to all other definitions in navigation—and the reader should accustom himself to the appearance of parallels of latitude and meridians, as seen with the eye in the plane of the equator, and also vertically over either pole. We illustrate this part of the subject by a figure of the earth, drawn on the first of these suppositions (fig. 1).

Fig. 1.



N S represents the earth's axis, N and S being the poles and W Q E the equator; *a b*, *c d*, *g h*, *k l* are portions of parallels of latitude; N A S, N B S, N C S, &c., are portions of meridians, of which N C S is assumed to be the meridian of Greenwich; O M is the latitude of O (north); B F is the latitude of F (south); C M is the longitude of O (east); C B is the longitude of F (west); B M is the difference of longitude between F and O; O R or T F is the difference of latitude between O and F.

The following are computed illustrative examples followed by others for practice:—

Example (*a*). *To find the true difference of latitude in nautical miles or minutes of arc,* having given the latitude from and latitude in.[1]

Examples

1. Find the true difference of latitude, having given latitude from 42° 10′ N., and latitude in 50° 48′ N.

lat. from 42° 10′ N.
lat. in 50 48 N.
8 38
60
T. D. lat. 518′ N.[2]

2. Find the true difference of latitude, having given latitude from 3° 42′ N., and latitude in 2° 50′ S.

lat. from 3° 42′ N.
lat. in 2 50 S.
6 32
60
T. D. lat. 392′ S.

Find the true difference of latitude in each of the following examples:

	Lat. from	Lat. in	Answers.
3.	33° 42′ N.	40° 40′ N.	T. D. lat= 418′ N.
4.	40 40 N.	33 42 N.	„ = 418 S.
5.	3 42 S.	1 40 N.	„ = 322 N.
6.	3 8 S.	14 42 S.	„ = 694 S.
7.	68 48 N.	38 30 N.	„ =1818 S.
8.	14 14 N.	0 0	„ = 854 S.

Example (*b*). *To find the latitude in,* having given the latitude from and true difference of latitude.

Examples

9. Find the latitude in, having given the latitude from 42° 30′ N., and true diff. lat. 342′ N.

60)342′ N.
T. D. lat. 5° 42′ N.
lat. from 42 30 N.
lat. in 48 12 N.

10. Find the latitude in, having given the latitude from 42° 30′ S., and true diff. lat. 342′ N.

60)342′ N.
T. D. lat. 5° 42′ N.
lat. from 42 30 S.
lat. in 36 48 S.

Find the latitude in, in each of the following examples:

	Lat. from	T. D. lat.	Answers.
11.	30° 10′ N.	182′ N.	Lat. in 33° 12′ N.
12.	3 2 S.	190 N.	„ 0 8 N.
13.	2 48 S.	368 N.	„ 3 20 N.
14.	2 48 S.	288 N.	„ 2 0 N.
15.	4 48 N.	288 S.	„ 0 0
16.	0 10 N.	228 N.	„ 3 58 N.

---

[1] The latitude and longitude of the place sailed from is called that *from*, the latitude and longitude of the place arrived at is called that *in*.

[2] The letters N. and S. attached to the diff. lat. denote the *direction* in which the change of position has taken place, and similarly in subsequent cases with regard to the letters E. and W. attached to the diff. long.

Example (*c*). *To find the difference of longitude in minutes of arc*, having given the longitude from and longitude in.

Examples

17. Find the difference of longitude, having given the longitude from = 110° 42′ W., and longitude in = 100° 42′ W.

long. from 110° 42′ W.
long. in 100 42 W.
10 0
60
diff. long. 600′ E.

18. Find the difference of longitude, having given long. from 12° 10′ E., and long. in 2° 45′ W.

long. from 12° 10′ E.
long. in 2 45 W.
14 55
60
diff. long. 895′ W.

Find the difference of longitude in each of the following examples:

	Long. from	Long. in	Answers.
19.	33° 40′ E.	40° 10′ E.	Diff. long. 390′ E.
20.	104 0 W.	110 30 W.	„ 390 W.
21.	2 45 W.	3 30 E.	„ 375 E.
22.	0 0	4 10 W.	„ 250 W.
23.	3 10 W.	3 10 E.	„ 380 E.
24.	179 0 E.	179 0 W.	„ 120 E.[1]

Example (*d*). *To find the longitude in*, having given the longitude from and the difference of longitude.

Examples

25. Find the long. in, having given long. from 38° 42′ W., and diff. long. 384·5′ W.

60)384·5
6° 24·5′ W.

long. from 38° 42′ W.
diff. long. 6 24·5 W.
long. in 45 6·5 W.

Find the longitude in, in each of the following examples:

	Long. from	Diff. long.	Answers.
26.	62° 32′ E.	1000·5′ W.	Long. in 45° 51·5′ E.
27.	2 30 E.	126·6 E.	„ 4 36·6 E.
28.	3 40 W.	220·0 E.	„ 0 0
29.	0 0	100·4 W.	„ 1 40·4 W.
30.	179 59 W.	2·0 W.	„ 179 59·0 E.[2]

---

[1] If the difference of longitude found exceed 180°, it must be subtracted from 360°, and the remainder brought into minutes of arc must be considered the difference of longitude, with the contrary name attached to it; this is seen to be correct since in the example given long. 179° E. = long. (360° − 179°) W.

[2] If the long. in exceed 180°, subtract it from 360°, and attach to the remainder the contrary name.

TRUE COURSE.

True course.

We proceed next to give a clear idea of the form of the track a vessel makes when proceeding from one port to another; unless prevented by adverse winds and currents such a voyage is usually made by steering in one definite direction all the time, which is equivalent to saying that the angle between *every* meridian crossed and the ship's track is the same in magnitude. From our definition of meridians on page 2 we see that they are not parallel to one another except at the equator, but converge to points at the poles; if, then, the track of a vessel, steering a direct course from B to L, be represented in fig. 2 as seen from the plane of the equator, and also in fig. 3 as seen when looking down from a point vertically over the north pole, and if N *a* S, N *d* S in fig. 2, and N *a*, N *d* in fig. 3 represent portions of meridians passing through B and L, while N *b* S, N *c* S and N *b*, N *c* are two among any number of parts of meridians which may be supposed to be drawn between B and L, the actual track will be represented by the lines B G, G K, K L, drawn so that the angles N B G, N G K, N K L are all equal; when, however, we draw similar lines between a *great number* of intervening meridians, the resulting form of the track will be a *curve* making equal angles with *every* meridian.

FIG. 2.

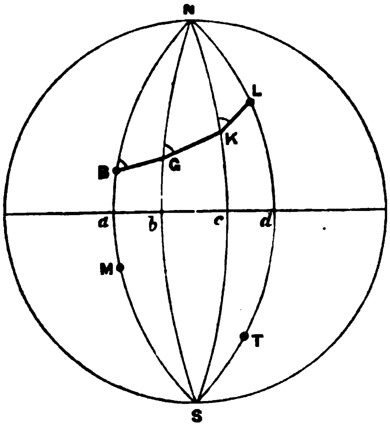
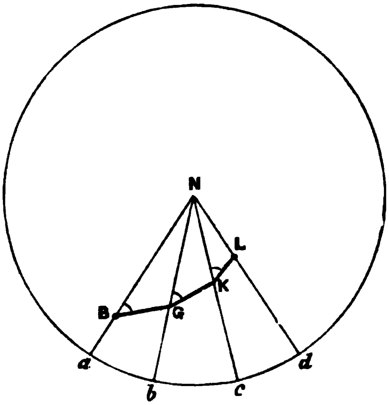


FIG. 3.



Rhumb line

This curve is termed the RHUMB LINE,[1] and the common angle it makes with every meridian is known as the TRUE COURSE, while the length in nautical miles of the curve B L is the DISTANCE between B and L.

---

[1] It is also known as the Loxodromic curve or Equiangular spiral, and is of such a form that if a vessel could maintain *one* course while sailing round the world, she

Distance

On similarly drawing a curve, making equal angles with every meridian, between two places, M and T, in south latitude, it will be seen that, when steering apparently a direct and unchanging course from one place to another, the track of the ship is *really* a curve which is always concave to the nearer pole, except when the places are situated north and south of each other, or on the equator or the same parallel of latitude when they lie east and west of one another.

Compass

From the definitions of course and distance we proceed to a description of the instruments by which these elements are practically measured, observing that, since no written description of *any* instrument can adequately take the place of personal inspection, such descriptions will only be given in general terms.

Since the meridians themselves are imaginary circles, the course of a vessel evidently cannot be directed by them; but since, by applying certain corrections to the indications given by a compass, the positions of these meridians can always be ascertained, the course is directed by that instrument.[1]

The mariner's compass is constructed on the principle of attaching a graduated circular card to a magnet or combination of magnets (usually called 'needles') placed parallel to one another underneath it, so that, when freely suspended, a certain point on the graduation points to the northward. The details of construction greatly vary, but the form in general use at sea is that patented by Sir W. Thomson (now Lord Kelvin) in 1876, and known by his name. This is now used as the 'standard' compass in the Royal Navy.

Compass card

All compasses agree in the card attached to the magnets being graduated on the same plan. The whole circumference is divided into thirty-two equal parts, the divisions between them being called points, to each of which a distinctive name is given; the intervening spaces are divided into half and quarter points, also

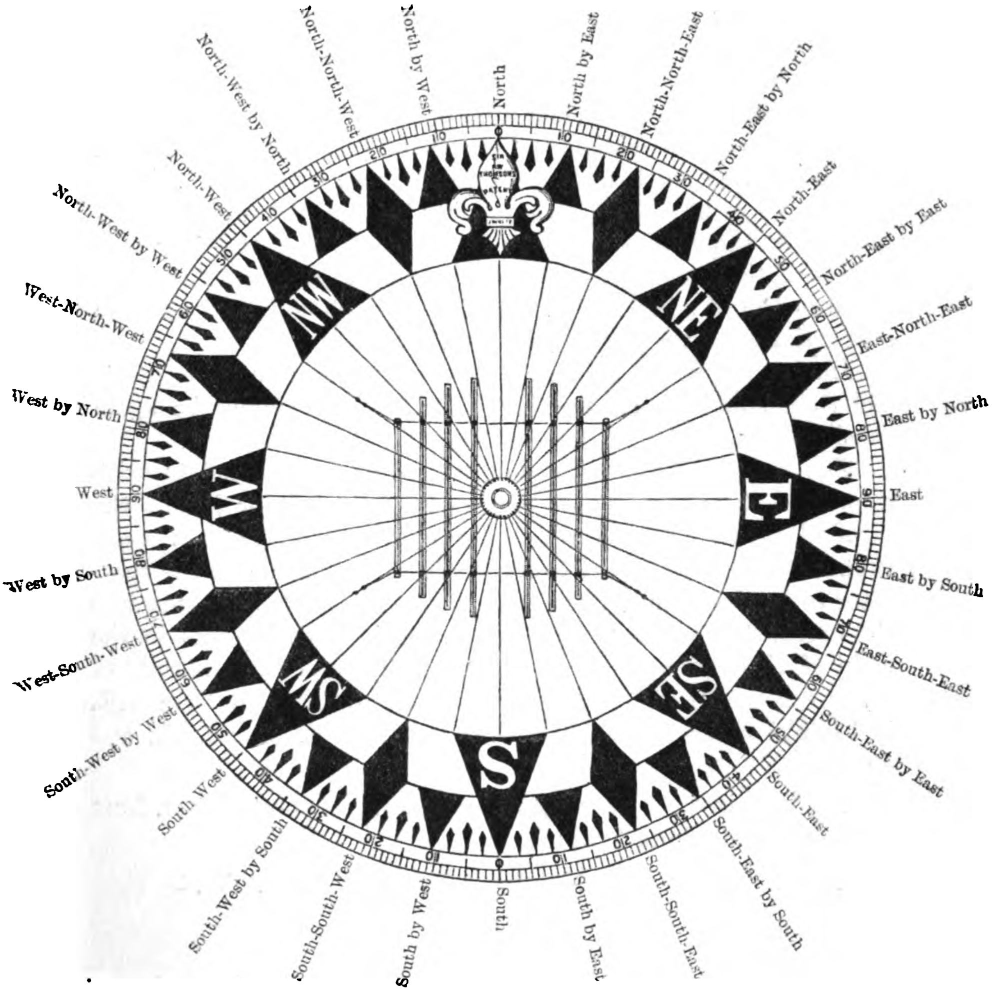
---

would only return to the point of departure in the cases of that course being N, S, E or W; in every other case the rhumb line tends constantly to the nearer pole, but never reaches it.

[1] The course actually steered in a ship is directed by a compass placed conveniently near the wheel, but constant comparison is made between its indication and that of a compass placed in a carefully selected position, and known as the Standard compass.

distinguished by names. The point over the north-seeking end of the attached magnets is indicated by a fleur-de-lys, supposed to be a modified form of a cross.[1]

Fig. 4.



Good compass cards have also another graduation in degrees, the card being divided into four quadrants; the zero points are at both north and south, and the degrees are marked on the edge

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[1] In ancient treatises on Navigation the *east* point is treated as the principal point on a compass card. The mariner's compass is said to have been known to the Chinese in B.C. 2630.

of the card, numerically increasing in both directions to 90° at east and also at west.

Names of points

The names of the points of the compass must be thoroughly impressed on the memory, and the reader should accustom himself to remember the order of names repeated either directly or in reverse order (familiarly known as 'boxing the compass'), to facilitate which a representation of a Thomson compass card is here given.

The north, east, south, and west points are known as *cardinal* points, and those equidistant from them (north-east, south-east, south-west, and north-west) are termed *quadrantal* points.

The points having been thoroughly learned, the reader should then learn the names of the subdivisions, which, together with the connection between the two systems of graduation, will be found in Inman's tables, No. 1, which table should be studied in connection with the preceding figure.

Having acquired familiarity with the names of both the points and their subdivisions, this important subject should be again considered in the following way:

The points of the compass are frequently referred to with respect to their position or bearing to the *right* or *left* of the cardinal points towards which the spectator is looking: thus, suppose the compass card to represent the horizon, and the spectator to be placed in the centre of the card and looking towards the north, then a point such as N.E. is said to be 4 points to the right of N. (written thus—4 r. N.): E. by N. is 7 points right of N. or 7 r. N. If the spectator is looking towards east, then N.E. is 4 left of E. or 4 l. E., E.N.E. is 2 left of E. or 2 l. E., and so on.

Examples

31. Required the bearings of the following points—first, from North; second, from East:

N.N.E. N.E. by N. N. by E. N.N.W. N.W. W. by N.

Answers.

Bearings from	N.N.E. or	N.E. by N. or	N. by E. or	N.N.W. or	N.W. or	W. by N. or
North...	2 r. N.	3 r. N.	1 r. N.	2 l. N.	4 l. N.	7 l. N.
East.....	6 l. E.	5 l. E.	7 l. E.	10 l. E.	12 l. E.	15 l. E.

32. Required the bearings of the following points from North, East, South, and West respectively:

S. by E. S.E. by S. S.E. by E. S.S.W. W. by S. E.S.E.

Answers.

Bearings from	S. by E. or	S.E by S. or	S.E. by E. or	S.S.W. or	W. by S. or	E.S.E. or
North ...	15 r. N.	13 r. N.	11 r. N.	14 l. N.	9 l. N.	10 r. N.
East ...	7 r. E.	5 r. E.	3 r. E.	10 r. E.	15 r. E.	2 r. E.
South ...	1 l. S.	3 l. S.	5 l. S.	2 r. S.	7 r. S.	6 l. S.
West ...	9 l. W.	11 l. W.	13 l. W.	6 l. W.	1 l. W.	14 l. W.

Examples

33. Required the compass names of the following points :

2 r. N. 5 l. N. 3 r. S. 12 r. S. 5 r. E. 4 l. W.

Answers.

2 r. N. or N.N.E.
5 l. N. or N.W. by W.
3 r. S. or S.W. by S.
12 r. S. or N.W.
5 r. E. or S.E. by S.
4 l. W. or S.W.

Each point of the compass being subdivided into *quarter points*, and named from the adjacent points, 2½ points to the right of north is N.N.E.½E.; 7¾ points to the left of north is W.¼N.

34. Required the bearings of the following points—first, from North; second, from East:

N.N.E.¼E. N.¾E. E. by N.½N. N.E.½N. N.W.½W. N.¾W.

Answers.

Bearings from	N.N.E.¼E. or	N.¾E. or	E. by N.½N. or	N.E.½N. or	N.W.½W. or	N.¾W. or
North ...	2¼ r. N.	¾ r. N.	6½ r. N.	3½ r. N.	4½ l. N.	¾ l. N.
East ...	5¾ l. E.	7¼ l. E.	1½ l. E.	4½ l. E.	12½ l. E.	8¾ l. E.

35. Required the bearings of the following points—first, from South; second, from West :

S. by E.½E. S.S.E.¾E. S.¾W. S.W. by W.¾W. W. by S.¾S.

Answers.

Bearings from	S. by E.½E. or	S.S.E.¾E. or	S.¾W. or	S.W. by W.¾W. or	W. by S.¾S. or
South...	1½ l. S.	2¾ l. S.	¾ r. S.	5¾ r. S.	6¼ r. S.
West ...	9½ l. W.	10¾ l. W.	7¼ l. W.	2¼ l. W.	1¾ l. W.

36. Required the compass names of the following points:

2¼ r. N. 1¾ l. N. 10¼ r. S. 7¾ l. N. 3½ r. S. 3½ l. S.
6½ r. S. 10½ l. S. 14¾ r. N. 8 r. N. 8 l. S. 15½ r. N.

Answers.

2¼ r. N. or N.N.E.¼E.
6½ r. S. or W. by S.½S.
1¾ l. N. or N. by W.¾W.
10½ l. S. or N.E. by E.½E.
10¼ r. S. or N.W. by W.¾W.
14¾ r. N. or S. by E.¼E.
7¾ l. N. or W.¼N.
8 r. N. or East.
3½ r. S. or S.W.½S.
8 l. S. or East.
3½ l. S. or S.E.½S.
15½ r. N. or S.½E.

The compass card with attached magnets is mounted on a pivot, balanced so as to lie parallel to the plane of the horizon, and enclosed in a brass bowl with a glass cover, on the inside of which towards the ship's head is drawn a narrow vertical line called the *lubber line*, carefully painted so that a line joining it with the centre of the compass card shall be parallel to a line from the stem of the ship to the middle of her stern (technically known as a *fore and aft* line).

Lubber line

The binnacle, in which the compass bowl is suspended, being part of the ship, the lubber line moves with every movement of the ship's head; the freely suspended compass points steadily to the northward, so that the graduation of the card under the lubber line indicates the direction of the ship's head, or its angular distance in points or degrees from the north point of the compass: this is known as the COMPASS COURSE; it would also indicate the *true* course if the north point of the compass card always pointed to the true or geographical north; this, however, is seldom the case, but, as the amount of such error is always known, we see that with this instrument, by applying a known correction, we can always ascertain the angle the ship's track makes with the meridian.

Compass course

MEASUREMENT OF DISTANCE RUN.

Log ship

The earliest attempts to measure the distance run by a ship were made by attaching a line to a log of wood,[1] which on being thrown overboard remained stationary in the water, while the onward progress of the ship carried over a certain length of line in a certain measured interval of time, from which the speed could be calculated. A slight modification of the original method was afterwards introduced, by what is known as a log *ship* being substituted for the original log of wood. This method of measuring distances, though rough, is still used.

The log ship is usually made of thin wood in the form of a sector of a circle of about 5 inches radius, and weighted with lead on the curved side so as to float upright; the log line attached to this is divided into equal spaces called knots.[2]

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[1] First mentioned in Bourne's *Regiment for the Sea*, published in 1577. A very ancient mode of measuring a ship's speed through the water was by what was termed the Dutchman's log.

A log of wood was thrown from the bows of a ship and the time noted when it came abreast of each of two marks on the deck at a carefully measured distance apart.

[2] The divisions of the log line do not commence at the log ship, but at some distance from it; the intervening length is termed *stray line*.

Log line

In order to ascertain the speed of the ship per *hour*, the length of each knot on the log line must bear the same ratio to the length of a nautical mile that the observed interval of time bears to an hour, or

$$\frac{\text{Number of feet in one knot}}{\text{number of feet in one mile}}=\frac{\text{interval of time in seconds}}{\text{number of seconds in one hour}}$$

so that if the interval be 28 seconds and the nautical mile be considered to contain 6,080 feet,

$$\text{Length of each knot}=\frac{28}{3600}\times 6080=47 \text{ feet } 3 \text{ inches.}$$[1]

It is usual to permanently mark these lengths on the deck, frequently comparing the *wetted* log line with them.

The interval of time is generally measured by a sand glass, consisting of two small glass bulbs connected by a narrow neck, which on being inverted allows a carefully adjusted quantity of sand to run from one bulb to the other in 28 seconds; when the speed of the vessel is greater than 5 knots a 14 second glass is used, and each half knot reckoned as a knot.

A preferable way is to take the time by a watch.

Heaving the log

This method of ascertaining the speed is technically known as *heaving the log*, and is practised every hour, the actual distance run *during* the hour being estimated; though only giving an approximation, especially when steaming at high speeds, the log, as affording independent evidence, is always used even when more perfect instruments are employed for the same purpose.

Errors of log line

The effect of using a log line incorrectly marked in conjunction with a correct glass would be:

(1) If the length of the knot is *too short* a greater number would run out in the same interval of time, and the estimated speed of the ship would be *too great*.

(2) If the length of the knot is *too great*, the estimated speed will be *less* than the true speed.

Errors of glass

The effect of using an incorrect glass in conjunction with a correct line would be:

(1) If the interval of time is *too great*, more than the true number of knots will run out, and the ship will appear to be sailing faster than the true speed.

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[1] Before 1635, when Norwood approximately measured the length of a degree, it was the custom to use a 30-second glass with a knot of the log line 42 feet in length. The true length, as he showed, should be (for a 30-second glass) 51 feet nearly.

It is curious to note that hour and half-hour sand glasses were used for keeping time on board vessels of the Royal Navy until 1839.

(2) If the interval is *too short*, the apparent speed will be less than the true speed.

The above theoretical considerations may be combined by observing that the estimated speed varies *directly* as the time, and *inversely* as the length of the knot.[1]

Example 37. If the length of each knot on the log line is 45 feet and the sand glass runs out in 31 seconds, what is the *true* distance corresponding to a *recorded* distance of 150 miles?

$$\text{True distance} = \frac{150 \times 28 \times 45}{31 \times 47} = 129{\cdot}7 \text{ miles.}$$

Example 38. If the sand glass runs out in 26 seconds and the knot on the log line is 49 feet long, what would be the *recorded* distance for a true run of 165 miles?

$$\text{Recorded distance} = \frac{165 \times 26 \times 47}{28 \times 49} = 147 \text{ miles.}$$

Patent logs To obtain a *continuous* record of speed and distance run the PATENT log is used; there are numerous forms of this instrument, but they are mostly based on the indications given by the revolution of a small fan or screw, connected by a line with a case enclosing clockwork, which indicates on a dial plate the revolutions made.

The instrument is towed overboard and occasionally hauled in and read off. The onward progress of the ship causes the screw to revolve, and, since from its form a certain number of revolutions corresponds to a nautical mile, the dial plate, having small rotating hands, somewhat similar to those of an ordinary gas meter, can be graduated so that the distance run can be read off with considerable accuracy.[2]

Patent logs generally have a small index error, which can be tested only when in sight of land by comparing a distance *indicated* with that *measured* on a chart between two known positions of the ship; this is only applicable when there is no current or tide, or when a sufficiently long run can be made to eliminate the effect of the latter by experiencing the influence of two tides in *opposite* directions; the index error varies with the speed of the ship, and at very high speeds no form of patent log gives satisfactory results.

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[1] For examples on these theoretical sources of error, *see* Goodwin's *Problems in Navigation and Nautical Astronomy*.

[2] A form of patent log known as Walker's taffrail log is now supplied to H.M. Navy. The principle of this is that of a rotating fan or screw, but the indications can be read off without hauling in the instrument. It may be interesting to note that in Bourne's *Inventions and Devices*, 1578, there is a description of a proposed patent log regulated by a revolving *wheel* enclosed in a case to be towed astern of a ship, and in which the recording apparatus appears to be precisely similar to that used at present.

No further mention of such a valuable instrument appears until 1773, when Captain Phipps took two on trial in his Arctic voyage. These were of very similar construction to Walker's harpoon log, formerly supplied to H.M. ships.

Under ordinary circumstances (especially in paddle-wheel steam vessels) the number of revolutions of the engines affords an excellent guide both as to present speed and distance run.

Ground log

In localities where the currents are uncertain the course and distance made good is sometimes found in shoal water by attaching a lead to the log line. When thrown overboard this remains stationary on the bottom, and the compass course made good will be obtained by reversing the bearing of the direction of the line, since the vessel will be set away from the position of the lead partly by her own progress and partly by the current.

The speed is obtained in the same way as with the ordinary log line.

Note.—The course (true) *steered* is the angle between the meridian and the direction of the ship's head; the course (true) *made good* is the angle between the meridian and the ship's real track over the ground.

CHAPTER II.

Variation—Changes in variation—Use of Variation chart—Applications of variation—Deviation—Correcting courses for both variation and deviation—Leeway—Correcting compass courses for all sources of error.

Corrections to compass courses

THE *true* course made good, or angle between the track of a ship and the meridian passing through the ship, is practically obtained by applying certain corrections to the indication by compass of the direction of the ship's head. These corrections are known as the Variation, Deviation, and Leeway; the combined effect of the two first is termed the Total Error of the compass.

VARIATION OF COMPASS.

Apart from any magnetic influence of the ship (which causes what is termed Deviation), the north point of the compass card, when freely suspended, is in most parts of the world drawn either to the eastward or westward of the true or geographical north. The amount by which it is thus in error is known as the Variation, and it is termed easterly when the north end of the compass is drawn eastward of the true north, and westerly when drawn in the opposite direction.

Changes in variation

From a great number of observations, extending over many years, and made both on land and on board ships over every part of the navigable seas, charts have been prepared showing the amount and direction of this variation. The same observations have revealed the fact that everywhere changes are taking place, as an illustration of which the following table is given of the changes in the variation which have occurred at London since 1580:[1]—

1580......	11° 17′ E.	1700......	9° 40′ W.	1800......	24° 36′ W.
1622......	6 12	1720......	13 00	1820......	24 11
1634......	4 5	1740......	16 10	1831......	24 00
1657......	0 0	1760......	19 30	1850......	22 59
1672......	2 30 W.	1790......	23 39	1898......	16 20

An illustration taken from the southern hemisphere is afforded by the fact that the south point of Africa, on its discovery in 1497, received the Portuguese name of L'Agulhas, or the Needle, from the fact of the compass in its vicinity pointing to the true north; at present in the same position the variation is 31° W.

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[1] Taken from Lloyd's *Treatise on Magnetism.*

Variation chart

Our knowledge on these two important points is embodied in a *Variation chart*,[1] of which a reduced copy is shown on Plate 7. For facility of reference, places where the same amount of variation exists are joined by curved lines, those indicating easterly error being distinguished in character from the curves of westerly variation, and the amount in degrees shown on each fifth curve.

In order to know the variation at any position of a ship when far from land, that position should be placed on this chart by its latitude and longitude, when the required amount may be read off; this will be the variation corresponding to the year in which the chart was published. A further correction for the changes which have occurred since then will be taken from the small diagram on the upper part of the Variation chart.[2] For the convenience of navigation when near the land, on the coasting charts then used, compass cards are depicted representing the north point drawn eastward or westward of the meridian by an amount equal to the variation at the place.

Application of variation

To illustrate the method of applying the variation we will suppose a vessel to be in lat. 10° 00′ S., long. 20° 00′ W., and wishing to steer N. 45° E. true. On placing this position on the Variation chart the variation is seen to be 21° 40′ W., corresponding to the year 1895, in which the chart was published. On referring to the upper small diagram in the given position the annual change is seen to be + 3′, or increasing 3′ annually; hence the variation at this position in 1898 is 21° 49′ W. (approximately 22°).[3] Now if N S (fig. 5, page 18) represents the meridian passing through the ship, and N_1 S_1 the direction of the north and south line on the compass card (making an angle of 22° with N S), it is evident that when steering a *true* course of N. 45° E. the direction of ship's head by the compass[4] will be N. 67° E. Similarly if wishing to steer north (true) the compass course must be N. 22° E.

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[1] The first Variation chart was drawn by Halley in 1700, when it was hoped that a comparison of the variation *observed* in a ship with that shown on the chart would be a valuable means of ascertaining the longitude.

[2] These annual changes are shown on Plate 7.

[3] Courses and the corrections to be applied to them are practically only required to be known to the nearest *degree*.

[4] Throughout these remarks on the effect of *Variation* we assume that the compass is not affected by *Deviation*. As these errors do in reality occur together the subject will be again considered from that point of view.

To further illustrate this important point, suppose a vessel in lat. 40° 00′ S., long. 150° E., to be steering by *compass* N. 55° W., and we require to know the true course. On reference to the Variation chart, as before, it is found that in 1895 the variation is 9° 48′ E. (approximately 10° E.).

Similar letters being used as before, on referring to fig. 6, it will be evident that, since the north point of the compass card is drawn 10° to the eastward, a *compass* course N. 55° W. corresponds to a *true* course N. 45° W.; also if steering North by compass the true course will be N. 10° E.

From these considerations the practical rules follow, that when the variation is easterly the correction to a given compass course in order to find the true course must be *applied* to the *right*, by an observer supposed to be at the centre of the card and looking at the point under the lubber line, and *vice versâ* when the variation is westerly.

FIG. 5.

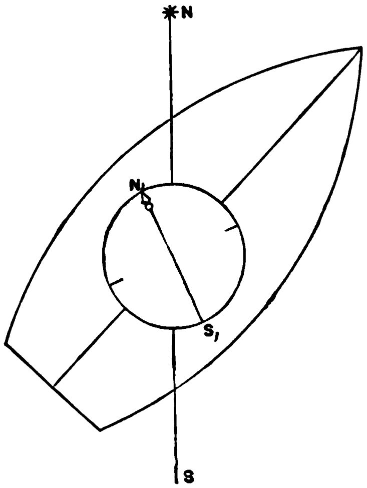
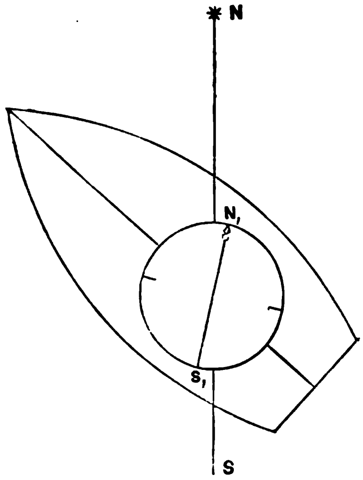


FIG. 6.



Also when desiring to steer a certain *true* course, the corresponding *compass* course will be found by reversing the above process.

Magnetic course

The direction towards which the north point of a compass points (if unaffected by deviation) is termed Magnetic North, and the course indicated the Magnetic course, sometimes called the 'Correct Magnetic course,' or the magnetic direction of the ship's head.

The variation is always represented in *degrees*, while the course is sometimes estimated in *points* and sometimes in *degrees*. Examples of correction in both cases will be given.

Practical rules

Rule (*a*). *To find the magnetic course*, the true course being given.

Easterly variation allow to the left.
Westerly „ „ right.

Rule (*b*). *To find the true course*, the magnetic course being given.

Easterly variation allow to the right.
Westerly „ „ left.

Examples

1. Find the magnetic course, having given the true course N.E.½E. or N. 51° E. and variation 11° easterly.

	pts.	qrs.		
True course	4	2	right of north	or N. 51° E.
Variation	1	0	left	or 11 E.
Magnetic course ...	3	2	right of north	or N. 40° E. or N.E.½N.

2. Find the true course, having given the magnetic course N.W.½W. or N. 51° W. and variation 36° westerly.

	pts.	qrs.		
Magnetic course ...	4	2	left of north	or N. 51° W.
Variation	3	1	left	or 36 W.
True course	7	3	left of north	or N. 87° W. or W.¼N.

Find the magnetic course in each of the following examples :

	True Course.	Var.	Answers.	
3.	N.N.E.½E.	3° W.	N.N.E.¾E.	or N. 31° E.
4.	N.	17 E.	N. by W.½W.	„ N. 17 W.
5.	S.S.W.	22 W.	S.W.	„ S. 45 W.
6.	S.W.	0	S.W.	„ S. 45 W.
7.	N. by W.¼W.	14 W.	N.	

Find the true course in each of the following examples :

	Magnetic Course.	Var.	Answers.	
8.	N.N.E.	25° W.	N.¼W.	or N. 2° W.
9.	N.W.	20 E.	N.N.W.¼W.	„ N. 25 W.
10.	S.W.¾W.	17 E.	W. by S.¾S.	„ S. 70 W.
11.	S.	22 W.	S.S.E.	„ S. 22 E.
12.	W.	28 E.	N.W. by W.½W.	„ N. 62 W.

Fixed and movable compass cards

It is of the utmost importance to be able with facility to express compass courses as true courses or as magnetic courses, and *vice versâ*. As the same rules apply to the method of correction both for variation and deviation, the reader is advised in each of the above examples to draw a figure similar to figs. 5 and 6, or to use the fixed and movable compass cards on Plate 1,[1] when the movable card, representing the compass card of the ship, should be turned until the north point is separated from the north point of the fixed card (representing *true* north) by an arc equal to the given variation (East or West), when the connection between *any* magnetic course on the inner card and the corresponding true course as shown by the outer fixed card will be at once seen.

DEVIATION OF COMPASS.

Deviation

Deviation is an effect of the magnetic attraction on the compass of the iron used in constructing a ship; this attraction manifests itself as the ship swings round by drawing the north point of the compass card eastward or westward of the direction in which it would otherwise point, which direction we have defined as the Magnetic North.

Magnetic meridian

Hence we see that the variation is the angle between the true meridian and the magnetic meridian, the plane of the latter being that of the vertical circle in which the compass needle would rest if undisturbed by the magnetism of the ship, while the deviation is the angle between the magnetic meridian and the actual north and south line shown by the compass card as disturbed by the iron in the ship.

The variation is of course unaffected by the direction of the ship's head, its amount being dependent only on geographical position, but the deviation (being caused *by* the ship) changes in amount as she turns round. Usually there are two directions of the ship's head on which there is *no* deviation, or where that direction as shown by compass differs from the *true* direction only by the amount of the *variation*; similarly there are usually two directions of the ship's head where the deviation is large, in many ironclads amounting before correction to from 40° to 60°.

As those causes which produce such large deviations on some points decrease the directive force by which the compass points to the northward when the ship's head is on other points, it would not only be inconvenient but dangerous to attempt to navigate a ship under such circumstances; means, however, exist by which these deviations can be greatly reduced in amount, and in some cases almost completely corrected.

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[1] The same practical mode of illustrating the effect of variation is found in Norwood's *Seaman's Practice*, 1659.

This important part of the art of navigation is fully described in the works mentioned in the foot-note,[1] while the means by which the deviation itself is obtained will be found in the Appendix;[2] having been determined, it is tabulated as follows.

DEVIATION TABLE.[3]

I. Ship's Head, or Course by the Standard Compass	II. Deviation of the Standard Compass	I. Ship's Head, or Course by the Standard Compass	II. Deviation of the Standard Compass
North	3° 10′ W.	South	3° 10′ E.
N. by E.	2 35 E.	S. by W.	0 5 E.
N.N.E.	8 10 E.	S.S.W.	3 0 W.
N.E. by N.	13 10 E.	S.W. by S.	6 30 W.
N.E.	16 50 E.	S.W.	9 40 W.
N.E. by E.	19 30 E.	S.W. by W.	13 0 W.
E.N.E.	20 30 E.	W.S.W.	16 10 W.
E. by N.	21 5 E.	W. by S.	19 15 W.
East	20 20 E.	West	21 10 W.
E. by S.	19 15 E.	W. by N.	23 20 W.
E.S.E.	18 5 E.	W.N.W.	24 0 W.
S.E. by E.	16 30 E.	N.W. by W.	23 35 W.
S.E.	14 40 E.	N.W.	22 0 W.
S.E. by S.	12 5 E.	N.W. by N.	19 0 W.
S.S.E.	9 40 E.	N.N.W.	14 50 W.
S. by E.	6 0 E.	N. by W.	9 15 W.

Since those causes which produce the deviation are liable to change from the effects of time, change of geographical position of the ship, firing heavy guns, redistribution of stores or cargo, &c., the deviation table requires constant correction, and every available means should be employed to constantly examine and rectify it (*see* Chap. XI.).

We now give rules and examples on the mode of applying the correction for deviation, advising the reader both to draw figures similar to figs. 5, 6 on page 18, and to test the results by the fixed and movable compass cards on Plate 1.[4] As the course is sometimes given in points and sometimes in degrees, specimens of correction for both cases will be given.

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[1] Evans's *Elementary Manual* and Towson's *Practical Information*; there is also a valuable chapter in Lecky's *Wrinkles in Navigation.* The mathematical theory will be found in the Admiralty *Manual of Deviation of the Compass*, and an elementary manual for use of sub-lieutenants is supplied by the Hydrographic office.

[2] In an article contributed by Captain E. W. Creak, R.N., F.R.S., Superintendent of Compasses, Admiralty.

[3] The deviations in this table are greater than those which H.M. ships generally have, but it is to be noted that frequently vessels which leave England with small deviations have, unless the compass is frequently corrected, large deviations in high southern latitudes.

[4] In using these cards for correction of *deviation* the outer fixed card must be considered as pointing to the *magnetic* north, and the north points of the cards separated by an arc equal to the given deviation.

Practical rule

Rule (*a*). *To find the magnetic course*, having given the compass course and the deviation.

Easterly deviation allow to the right.
Westerly „ „ left.

Examples

13. Correct the compass course W. by S. for deviation 19° W. (known from table on page 21).

	pts.	qrs.	
Compass course......	7	0 right of S. or S. 79° W.	
Deviation	1	3 left	19° W.
Magnetic course......	5	1 right of S. or S. 60	W. or S.W. by W¼W.

14. Find the magnetic courses in the following examples, taking the deviation from the table on page 21:

Compass Courses.

N.N.E.; N. 62° E.; E. by N.; S. 30° E.; S.W.; N. 80° W.; N.N.W.

Answers:

N.N.E.¾E.; N. 82° E.; E.¾S.; S. 19° E.; S.W.¾S.; S. 77° W.; N.W.¾N.

Practical rule

Rule (*b*). *To find the compass course*, having given the magnetic course and deviation.

Easterly deviation allow to the left.
Westerly „ „ right.

Examples

15. Required the compass course, the magnetic course being N.E. by E. or N. 56° E. As a first approximation the deviation is looked out from the table on page 21 for the *magnetic* course and applied thus:

	pts.	qrs.	
Magnetic course	5	0 right of north or N. 56° E.	
Deviation for N.E. by E.	1	3 left	19° E.
Compass course nearly	3 N.E.¾N.	1 right of north or N. 37° E.	

That this is only an approximation follows from the consideration that if we steer by compass N.E.¾N., on applying the deviation due to that point (15° E.) the resulting magnetic course is N. 52° E., differing from that required, the reason of this difference being that the original deviation applied was in error; if, however, we now apply to the given magnetic course the deviation due to N.E.¾N. (the approximate compass course) the result will be

N.E.¼N. or N. 41° E., which will be found on correction to agree with the given magnetic course.

Where the *change* of deviation on adjacent points is small this recorrection may be neglected.[1]

Examples

16. Required the compass courses in the following examples, the magnetic courses being—

North; N. 41° E.; E.N.E.; S. 19° E.; S.S.W.½W.; S.62° W.; N. 70° W.; N. by W.

Answers:

N.¼E.; N. 31° E.; N.E.½E.; S. 30° E.; S.W. by S.; S. 81° W.; N. 48° W.; N.½W.

Combined effect of variation and deviation

Having now illustrated the modes of application of variation and deviation *separately*, two figures will be given showing their *united* acticn, since in practice in almost all cases they have thus to be considered.

FIG. 7.

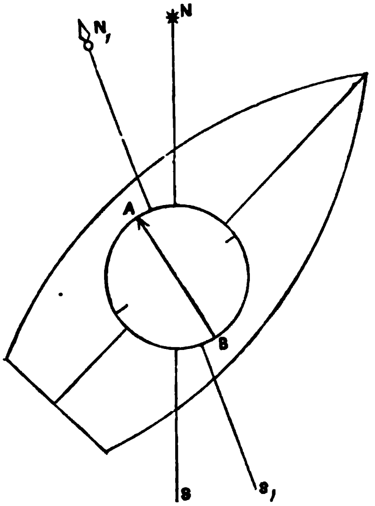
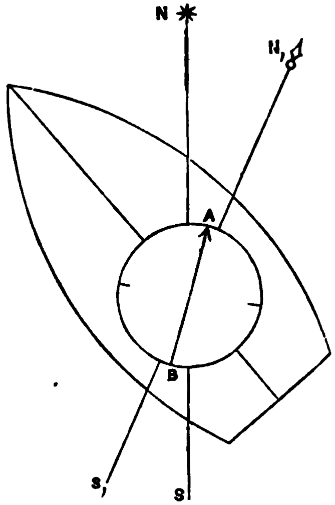


FIG. 8.



In both these figures N S represents the true meridian, and $N_1 S_1$ the magnetic meridian or direction in which, but for the influence of the *ship*, the compass would point, the variation in fig. 7 being 15° W. and in fig. 8, 25° E.

A B in both figures represents the north and south line on the compass card, the deviation in fig. 7 being 5° W., and 10° W. in fig. 8.

In converting a compass course into a true course it is immaterial which correction is applied first, or whether they are applied together; but in converting a true course into a compass course the variation should be first applied and then the deviation.[1]

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[1] The method of converting compass courses into corresponding magnetic courses and *vice versâ*, by means of Napier's curve and the Straight line method, is described on pages 24 and 100.

The following numerical examples are appended, for which rules have already been given; the deviation to be taken from the table on page 21.

Examples

Find the true course in each of the following examples:

	Compass Course.	Var.	Answers.
17.	N.N.E.	25° W.	N.½E.
18.	N. 45° W.	19 E.	N. 48° W.
19.	S.W.¾W.	17 E.	S.W. by W.¼W.
20.	S.	22 W.	S. 19° E.
21.	W.	28 E.	W.½N.

Find the compass course in each of the following examples:

	True Course.	Var.	Answers.
22.	N. 28° E.	3° W.	N. 22° E.
23.	N.	17 E.	N. by W.
24.	S.S.W.	22 W.	S.W. by W.¼W.
25.	S.W.	0	S.W. by W.¼W.
26.	N. 14° W.	14° W.	N. 3° E.

NOTE.[2]—Experience having proved the great value of the free use of Napier's diagram when dealing with the large deviations sometimes found in iron ships, the following description might prove useful.

1. The deviations of the Standard Compass on a certain number of points having

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[1] In practice, on commencing an ocean voyage, the *true* course to the port of destination is taken from the chart and corrected first for the variation at the place of departure, and then for deviation in order to obtain the first *compass* course by which to steer. As the voyage progresses and the variation changes, corresponding alterations are made in the course steered.

When near the land, the course from a ship's position to the port of destination is taken from a *magnetic* compass drawn on the charts then used; this only requires correction for *deviation* to give the *compass* course by which to steer.

In all cases of *calculating* a ship's position (see Chapter V.) the courses steered must be converted into *true* courses.

As an aid to the memory in applying variation and deviation (in doing which it requires much practice to avoid making errors), it is convenient to embody the precepts in the following letters:

1. If the true course is given to find the compass course, the letters E.T.W. indicate that easterly variation and deviation are applied to the *left*, and westerly variation and deviation to the *right*.

2. If the compass course is given to find the true course, the letters W.C.E. may assist in remembering that westerly variation and deviation are applied to the *left*, and easterly variation and deviation to the *right*.

[2] This note is taken from the *Admiralty Manual of Deviations of the Compass*.

An illustration of the diagram is appended at end of this volume representing the curve of deviations used in this book.

been observed in the manner described in the Appendix, two problems remain to be solved in order to derive the greatest advantage from the observations: 1st. From observations made on a small number of points to find the most probable value of the deviations on the intermediate points. 2nd. From observations made on a large number of points to find the most probable value of the deviations on each point.

2. A graphic solution (which is due to the late J. R. Napier, Esq., F.R.S.) has the advantage of being equally applicable, whether the points on which observations have been made are or are not precisely equi-distant. It requires no calculation, and only a moderate degree of neat-handedness.

The method consists of two parts, the diagram and the curve. The diagram is the same for all vessels; it is therefore engraved.

Description of the Diagram.

3. The diagram consists of a vertical line of convenient length—say 18 inches—divided into 32 equal parts, the divisions representing the 32 points of the compass in the following order, beginning at the top: North, N. by E., N.N.E., &c. The line is also divided into 360 equal parts, representing degrees, and these divisions are numbered from 0° at the top to 360° at the bottom. They are also numbered, according to the usual mode of dividing the circumference of the compass card, from 0° at North and South, up to 90° at East and West. The line may in fact be considered as the margin of a compass card cut at the North point and straightened. The vertical line is intersected at each of the 32 points by two straight lines, inclined to it at angles of 60°. One, a *plain* line, inclined to the right; the other, a *dotted* line, inclined to the left.

Requisite Observations to be made.

4. The least number of observed deviations available for obtaining a complete curve are the deviations on 4 points distributed equally, or nearly so, round the compass; but, if possible, the deviations should be observed on 8 or more points. If the deviations are observed on 4 points only, these should be at or near N.E., S.E., S.W., and N.W. The points next in importance are North, East, South, West.

Cases may also occur in which by the ship swinging round at her anchor in a tide way, or to the wind, or by the aid of a steam-tug, the deviations may be observed on various directions of the ship's head, not being necessarily exact points of the compass; or similarly whilst under steam or sail at sea, a number of azimuths of the sun may be observed, and hence the deviation obtained.

In these cases the graphic method here described furnishes a ready and effectual mode of obtaining a result in which the errors of individual observations are as far as possible compensated, and any egregious errors eliminated. The following examples for constructing the curve of deviation will render intelligible this method.

Construction of the Curve of Deviations.

5. The observed deviations are to be laid down on the diagram in the following manner. Take, on the vertical line, one of the COMPASS courses on which an observation has been made, and lay off the amount of the observed deviation on the dotted line which passes through it, or if no dotted line passes through it, then in a direction parallel to the dotted lines, to the right if the observed deviation be easterly, to the left if westerly, and mark the point so determined with a cross, or dot encircled in ink. Observe that the scale on which the deviations are to be so laid off is the same as the scale of the vertical line, viz., $\frac{1}{20}$ of an inch to the degree. Perform the same operation for each observed deviation. Then with a pencil and a light hand, draw a flowing curve, passing as nearly as possible through all the courses or dots encircled; and when satisfied with the curve in pencil, draw it in ink. This is the curve of deviations.

6. The following rules will serve to facilitate the use of the diagram.

Rule I.

From a given Compass Course, to find the corresponding Magnetic Course.

On the vertical line take the given compass course. Move in a direction parallel to the *dotted* lines till you arrive at the curve, and then move in a direction parallel to the *plain* lines till you get back to the vertical line. The point in the vertical line at which you arrive is the magnetic course required.

Example.—If the Standard Compass course be N.N.E., follow the *dotted* line of that rhumb till it meets the curve, and then return to the vertical line in a direction parallel to the *plain* line; it will be found to intersect it at N. 30° 40′ E., the required magnetic course.

Rule II.

From a given Magnetic Course, to find the corresponding Compass Course.

On the vertical line take the given magnetic course. Move in a direction parallel to the *plain* lines till you arrive at the curve, and then move in a direction parallel to the *dotted* lines till you get back to the vertical line. The point on the vertical line at which you arrive is the compass course required.

Example.—If the magnetic course required be N.N.E., follow the *plain* lines of that rhumb till it meets the curve, and then returning to the vertical line in a direction parallel to the *dotted* line, it will be found to intersect it at N. 16° 50′ E. or N. by E. ½ E., the required course by Standard Compass.

Leeway.

In a steamship when making a voyage in *calm* weather, or in a sailing vessel with a fair wind, the direction of the ship's head by compass, when corrected for variation and deviation, correctly indicates also the true direction of her track through the water; but when the wind blows on her side, or in any direction except from right ahead or astern, it tends to drive the vessel before it, and the direction of her head no longer corresponds with that of her track; the angular distance between them, usually estimated in points, is termed leeway.

This is very variable in amount in different ships, and even in the same ship under different circumstances, depending upon the form of the hull under water and the force and direction of the wind; it may generally be disregarded in steamers, except when blowing hard.

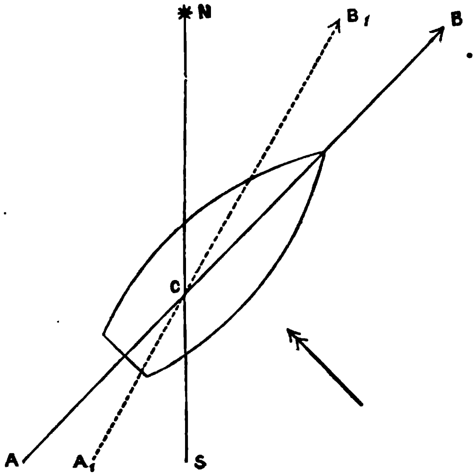
As there is no trustworthy method of measuring leeway it is estimated by eye, the observer being guided by the visible track of the ship in the water astern. Fig. 9 on page 27 will more clearly show the nature of this correction.

In this figure N S represents the true meridian, N C B the *apparent* true course or true course steered (found by correcting the compass course for variation and deviation), N C B_1, the *really* true course or that made good, which, owing to the action of the wind (shown by a double arrow), is in this case 1½ points to leeward of N C B, the angle $A\,C\,A_1$ ($=B\,C\,B_1$) being estimated as 1½ points. If then the apparent true course be N.E. the effect of leeway in this case will be that the really true course made good becomes N.N.E.½E.

Practical rule

Leeway is allowed away from the direction of the wind: thus when on the port tack, or with the wind on the port side of the ship, it is allowed to the right, and with the wind on the starboard side to the left, right and left being estimated on the supposition that the observer is at the centre of the compass and looking in the direction in which the vessel is proceeding.

Fig. 9.



Examples

Correct the following courses for leeway, so as to find the true courses:

	Apparent Course.	Wind.	Leeway.	Answers.
27.	N.N.E.	W.N.W.	1½ pts.	N.E.½N.
28.	N.W.	N.N.E.	2	W.N.W.
29.	E.S.E.	S.	2½	E.½N.
30.	E.	N. by E.	¾	E.¾S.

Correct the following compass courses for deviation, variation, and leeway, so as to find the true courses. The deviation is found in the table on page 21, and the variation is supposed to be in each example 28° W.

	Course.	Wind.	Leeway.	Answers.
31.	N.W.¼W.	W.S.W.	2½ pts.	W. by N.¾N.
32.	S.E.½E.	E.N.E.	2¼	S.E.¾S.
33.	W.¼S.	S.S.W.	2	S.W. by W.½W.
34.	N.¾W.	W. by N.	1½	N.N.W.½W.

These examples may be worked out in the following manner:

				pts.	qrs.	
Ex. 31.	Compass course			4	1. l. N.	
	Deviation.........	2	0 l.			
	Variation.........	2	2 l.			
				4	2 l.	
				8	3 l. N.	
	Leeway.........................			2	2 r.	
	True course..................			6	1 l. N.	W. by N.¾N.

NOTE.—Since leeway can only be *roughly* estimated, it is sufficiently accurate when the circumstances are such that this correction has to be applied to work to points and quarter-points, *not* to degrees.

35. Find the true courses in each of the following examples:

Compass Course	Variation	Deviation (from table, page 21, or from Napier's curve, Plate 4)	Leeway	Wind	Answers
N.¾E.	12° E.	0	¾ pt.	E. by N.	N. by E.
N. 47° E.	9 W.	18° E.	0	...	N. 56° E.
E. by N.¼N.	21 E.	2 pt. E.	1¼ pt.	North	S.E.
S. 81° E.	19 E.	20° E.	0	...	S. 42° E.
E. by S.¾S.	12 W.	1¾ pt. E.	¾ pt.	N.E. by N.	S.E.¾E.
S. 51° E.	18 E.	16° E.	0	...	S. 17° E.
S.S.E.¼E.	24 W.	1 pt. E.	1¼ pt.	S.W.	S'.S.E.¼E.
S. 5° E.	21 E.	4° E.	0	...	S. 20° W.
S.¾W.	21 W.	0	1 pt.	E.S.E.	S.¼E.
S. 36° W.	18 E.	7° W.	0	...	S. 47° W.
S.W.¼W.	24 W.	1 pt. W.	¾ pt.	W.N.W.	S.½W.
S. 62° W.	11 E.	14° W.	0	...	S. 59° W.
W. by S.¾S.	11 W.	1½ pt. W.	1 pt.	South	S.W.¾W.
West	20 E.	21° W.	0	...	S. 89° W.
W.¼N.	2 W.	2 pt. W.	½ pt.	N.N.W.	S.W.¼S.
N. 78° W.	5 E.	23° W.	0	...	S. 84° W.
N.W. by W.¾W.	11 W.	2 pt. W.	1¾ pt.	S.W.	W. by N.
N. 52° W.	12 E.	23° W.	0	...	N. 63° W.
N.W.¼W.	19 W.	2 pt. W.	1 pt.	N.N.E.	W. by S.
N. 36° W.	17 E.	20° W.	0	...	N. 39° W.
N.N.W.¼W.	30 W.	1¼ pt. W.	¼ pt.	West	W.N.W.
N. 19° W.	35 E.	13° W.	0	...	N. 3° E.
N. by W.¼W.	32 W.	1 pt. W.	1¼ pt.	N.E. by E.	W. by N.½N.

36. Find the compass course in each of the following examples, allowing deviation from the table on page 21, or from Napier's curve on Plate 4:

True Course	Variation	Answers	True Course	Variation	Answers
North	12° E.	N. 6° W.	S.E. by S.	12 W.	S. 35° E.
N. 12° E.	12 W.	N. 18° E.	S. 27° E.	25 E.	S. 70° E.
N.N.E.¼E.	21 E.	N. 5° E.	South	30 W.	S. 37° W.
N. 41° E.	19 W.	N. 43° E.	S. by W.¼W.	19 E.	S. 10° E.
N.E.¼E.	20 E.	N. 21° E.	S. 41° W.	7 W.	S. 62° W.
N. 62° E.	11 W.	N. 54° E.	S.W. by W.¾W.	21 E.	S. 55° W.
E. by N.¼N.	5 E.	N. 52° E.	S. 80° W.	12 W.	N. 64° W.
N. 81° E.	4 W.	N. 65° E.	West	9 E.	N. 76° W.
East	20 E.	N. 51° E.	W.N.W.	7 W.	N. 40° W.
E.¾S.	12 W.	East	N. 42.° W.	28 E.	N. 48° W.
S. 79° E.	12 E.	N. 68° E.	N.W.¼W.	84 W.	N. 7° W.
E. by S.¼S.	21 W.	S. 74° E.	N. 34° W.	35 E.	N. 47° W.
S. 60° E.	25 E.	N. 74° E.	N.N.W.¼W.	27 W.	N. 4° E.
S.E.¾E.	4 W.	S. 67° E.	N. 9° W.	4 E.	N. 7° W.
S. 40° E.	8 E.	S. 65° E.			

CHAPTER III.

Charts and plans—Mercator's chart—Plane charts—Meridional parts—Construction of Mercator's chart—Laying off courses and measuring distances—Gnomonic projection—Plans—Scales of latitude and longitude—Correction of charts—Details of charts.

Charts

CHARTS are representations of portions of the earth drawn on a flat surface. Since it is impossible by any method to *accurately* represent such a curved surface on a plane, some distortion must necessarily be introduced;[1] it depends upon the object in view where this unavoidable distortion is made. There are various modes of making such representations (termed projections), but for nautical purposes only two are employed, viz. Mercator's and the Gnomonic projection.[2]

Mercator's chart

The distinctive character of a Mercator's chart is that the meridians, instead of being represented by curves meeting in a point at either pole, are depicted as parallel straight lines, drawn at the same distance apart in all latitudes as they are at the equator, the degrees of latitude shown on the chart being lengthened in the same proportion that the meridians are separated, so that each small portion of the chart is *similar* to that portion of the globe which it represents.[3]

The advantage of adopting this device is that the real track of a ship, proceeding from port to port and steering one continuous course, instead of being represented as a *curve* (previously defined as the rhumb line), is correctly shown in direction as a *straight line* joining the two places, the common angle made by the intersection of this line with every intervening meridian being the true course steered.

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[1] A sphere or spheroid has not what is termed a *developable* surface—that is (unlike a conical or cylindrical surface), when represented as a plane, portions of its surface must be distorted either in area or in relative angular position.

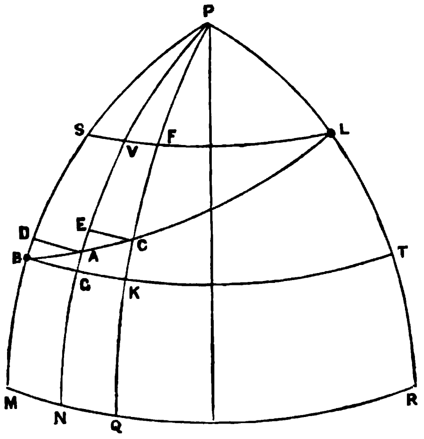
[2] A description of the principal projections employed in drawing *maps* will be found in Harbord's *Glossary of Navigation*. See also Plates 2 and 3.

[3] Before the adoption of Mercator's projection the degrees of lat. and long. on sea charts were everywhere shown *equal* to one another; such distortions were known as Plane charts.

Mercator's charts were first drawn by Gerard Mercator in 1569, and the mathematical principles of their construction explained by Mr. E. Wright in 1599.

To illustrate this important projection by two figures, we will represent the rhumb line between two well known places, such as Bermuda and the Lizard head, and show the successive steps by which it is drawn as a straight line on the corresponding Mercator's chart.

Fig. 10.

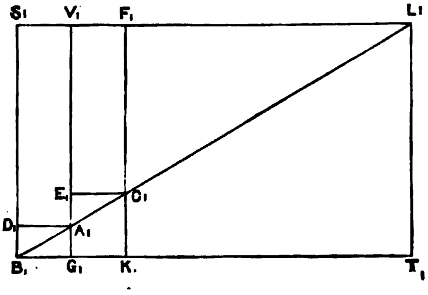


On fig. 10, representing a portion of the globe between the pole P and equator M N Q R, Bermuda is represented at B and the Lizard at L, while P M, P N, P Q are meridians supposed to be drawn very near to each other.

S L and B T are parallels of latitude drawn through L and B, and B A C L represents the rhumb line between those places, intersecting the meridians P N, P Q in A and C, through which points portions of parallels of latitude D A, E C are drawn.

To represent this rhumb line on a Mercator's chart in fig. 11, a straight line $B_1 T_1$ is drawn to represent B T, but taken of the same length as M R, and hence equal to the difference of longitude between B and L.

Fig. 11.



Distances $B_1 G_1$, $G_1 K_1$, $K_1 T_1$ are set off on this line equal to M N, N Q, Q R, and through $B_1 G_1 K_1 T_1$ perpendiculars $B_1 S_1$, $G_1 V_1$, $K_1 F_1$, and $T_1 L_1$ are drawn to represent the portions of meridians, S B, V G, F K, L T.

Now in considering how to represent a small portion of the earth's surface D B G A by a *similar* figure on the chart, since already distortion in one way has been introduced by making $B_1 G_1$ greater than B G, similarity can only be obtained by representing B D by $B_1 D_1$, which is increased in the same proportion.[1]

If then $D_1 A_1$ be drawn parallel to $B_1 G_1$, and $B_1 A_1$ joined, the latter will represent the portion B A of the actual track, and will make an angle $D_1 B_1 A_1$ with the meridian equal to the true course D B A.

---

[1] M N or $B_1 G_1 = B G \times$ sec. lat. of B. *See* page 66.

On considering the next small portion of the track A C, for the same reason as before the arc E A must be shown as a *longer* line $E_1 A_1$ on the chart, but the increase of length will be greater than in the former case, since the distortion caused by making the meridians parallel to one another is greater at C than at A, these meridians on the globe being more inclined to one another at C than at A.

Then on drawing $E_1 C_1$ parallel to $G_1 K_1$ and joining $A_1 C_1$ the portion A C of the rhumb line will be correctly represented on the chart by $A_1 C_1$; and since the angle $E_1 A_1 C_1$ will be equal to the angle E A C, and therefore also to the angle $D_1 B_1 A_1$, it follows that $A_1 C_1$ will be in the same straight line with $B_1 A_1$.

By thus similarly treating each successive *small* portion of the rhumb line we can finally represent it in direction on the chart by the *straight* line $B_1 L_1$, which will make with every meridian the common angle $S_1 B_1 L_1$ equal to the true course; $B_1 L_1$ will, however, obviously exceed B L (the distance) in length.

Meridional difference of latitude

The line $S_1 B_1$ into which S B, the *true* difference of latitude, has been extended is termed the MERIDIONAL DIFFERENCE OF LATITUDE between B and L.

Meridional parts

Tables known as Tables of Meridional Parts have been constructed, showing the lengths in nautical miles into which the true latitude must be expanded when constructing a Mercator's chart; on reference to this in Inman's Tables, No. 7, it will be seen that at the equator the length of one mile of latitude on the globe is represented by the same length on a Mercator's chart, while in lat. 30° the length of the true latitude or distance from the equator in miles (30° × 60 = 1800) must be expanded to 1888·38, and in lat. 60° each mile of true latitude is expanded to *twice* its natural length, since in that latitude the meridians passing through two places are only half the distance apart that they are at the equator.

It will also be noticed that the meridional diff. lat. between the equator and lat. 83° 52′ is twice the length of the true diff. lat., and that, since in such high latitudes the meridians rapidly approach each other, the meridional differences of latitude also rapidly increase, and the mile of latitude between 89° 59′ and 90° becomes infinite in length.

In high latitudes, therefore, charts are not constructed on Mercator's but on the Gnomonic projection, which will presently be described.

NOTE.—The terms *Meridional parts* and *Meridional difference of latitude* presenting some difficulties to beginners, the following additional definitions are given:

By Meridional parts is meant the ratio which the line representing the latitude bears to the line representing one minute of longitude on a Mercator's chart. Meridional difference of latitude is the ratio which the line representing the difference of latitude bears to the line representing one minute of longitude on a Mercator's chart. Thus when we see from Inman's table that the meridional parts for 10° are 603·07, it means that on a Mercator's chart the line representing the distance from the equator to the parallel of 10° is 603·07 times as long as the line on that chart representing one minute of longitude.

Similarly, on taking out the meridional difference of latitude between 59° and 60° (= 118·28) it means that on a Mercator's chart the line representing the difference of latitude between 59° and 60° is 118·28 times as long as the line on that chart representing one minute of longitude.

Examples

1. Find the meridional difference of latitude, having given latitude from 42° 10′ N., and latitude in 50° 48′ N.

lat. from 42° 10′ N.
lat. in 50 48 N.
mer. parts...2795·2 N.
mer. parts...3549·8 N.
mer. diff. lat. ...754·6 N.

2. Find the meridional difference of latitude, having given latitude from 3° 42′ N., and latitude in 7° 32′ S.

lat. from 3° 42′ N.
lat. in 7 32 S.
mer. parts...222·2 N.
mer. parts...453·3 S.
mer. diff. lat....675·5 S.

Find the meridional difference of latitude in each of the following examples:

	Lat. from	Lat. in	Answers.
3.	34° 42′ N.	33° 15′ N.	M. D. lat. = 104·9 S.
4.	14 14 N.	30 14 N.	„ = 1041·6 N.
5.	84 10 N.	80 30 N.	„ = 1681·5 S.
6.	2 8 S.	3 10 N.	„ = 318·1 N.
7.	4 5 N.	4 5 S.	„ = 490·4 S.
8.	0 0	2 45 N.	„ = 165·1 N.

Construction of Mercator's chart

A Mercator's chart is constructed as follows:

Draw at the bottom of a sheet of paper a straight line to represent the most southern parallel of latitude required for the chart; divide it into equal parts, as degrees, &c., regulating the length of each degree according to the scale required: if the chart is to be drawn to a scale of one inch or 0·7 inch equal to a degree of longitude, make the length of each degree on the line drawn one inch or 0·7 inch accordingly. The line so drawn at the bottom of the paper we may call the *graduated parallel;* at each extremity of this line erect a perpendicular;[1] these perpendiculars are the

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[1] The accuracy with which these perpendiculars have been erected will be finally

graduated meridians, on which must be marked the lengths of each degree of latitude.

To obtain the linear measure of the degrees of latitude.

Write down on a slip of paper, in a vertical column, the degrees of latitude which the chart is to contain, beginning with the highest degree: take out from Inman's tables the meridional parts for each degree, and write them down opposite their corresponding latitudes; take the successive differences between the first and second, second and third, &c., of these meridional parts, and thus make a second vertical column. Then, to find the points on the graduated meridians through which each parallel of latitude is to be drawn, transfer these meridional differences of latitude to the graduated meridians, by measuring with dividers on a scale the length in inches and decimals of an inch corresponding to the number of minutes, &c., contained in each meridional difference of latitude taken in order, making a dot on the graduated meridians at the extremity of each measure; connect these dots by straight lines: these will be the parallels of latitude required.

The intermediate meridian lines are then to be filled in, by drawing lines parallel to the graduated meridians through the divisions of the graduated parallel, or through every fifth degree, or through as many as may be considered sufficient; a compass should then be drawn on the chart (or more than one, if the chart is large); this will be useful to determine the bearings of different points, or for finding the latitude and longitude of the ship when her course and distance run are given. To construct the compass, take some convenient intersection of a meridian and parallel as a centre, and describe a circle with any suitable radius; mark the points of the circumference cut by the meridian with the letters N. and S., and complete the compass by inserting the other points and half and quarter points.[1]

To lay down upon the chart a point whose latitude and longitude are given.

Laying down positions

Lay the edge of a ruler[2] along the given parallel of latitude; measure on the graduated parallel the degrees, &c., between the given longitude and the longitude of the nearest meridian line

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tested by ascertaining that the diagonals of the parallelograms formed by the outer parallels and meridians are of equal length.

[1] Compasses graduated thus are given on Ocean charts; those shown on Coasting charts are divided from the *Magnetic* meridian, and usually into degrees as well as points.

[2] Parallel rulers were invented in 1584 by Gaspar Mordente.

drawn on the chart; apply this difference to the edge of the ruler east or west of that meridian as the case may be, and the point on the chart whose latitude and longitude are given will be found.

Example

Let it be required to lay down on chart No. 2615 a point whose latitude is 50° 31′ 30″ N., and longitude 2° 12′ 10″ W.

Place the edge of the ruler over latitude 50° 31′ 30″ N. on the chart, and with a pair of compasses take 2′ 10″ (the difference between 2° 10′ and 2° 12′ 10″, assuming that the meridian line of 2° 10′ is the nearest on the chart), and lay it along the ruler towards the left from the meridian of 2° 10′: the position of the required point will then be determined.

In this manner a ship's daily track is usually laid down; for the latitude and longitude being known at noon, her position is given at that time, and the entire track during the voyage can be seen by connecting, by straight lines, the points on the chart thus found.

To find the course and distance between two given places on the chart.

Laying off courses

(1) *To find the course.*—Place the edge of a parallel ruler over the two places on the chart, and keeping one part of the ruler firm, move the other till the edge passes through the centre of the nearest compass described on the chart: the edge thus lying on the compass will point out the course between the two places. It also may be found by means of the small semicircular protractor contained in most cases of mathematical instruments in the following manner:

Place the plain edge of the protractor against the edge of the ruler as it lies upon the two places, and slide it along till the centre of the protractor is on one of the meridian lines; then the course in degrees will be seen on that point in the circumference of the protractor through which the meridian line passes. A rectangular protractor will determine this with equal facility.

Measuring distances approximately

(2) *To find the distance.*—The distance is approximately found on transferring by dividers the space between the two places on the chart to the side line, or graduated meridian, *as nearly opposite the two places as possible:* the degrees, &c. (turned into minutes), which this space measures on the graduated meridian will be the distance required in nautical miles.

If the places have the same or nearly the same latitude, the distance is found more accurately as follows: Take half the space between them; apply it to the graduated meridian above and below the parallel on which the places are situated: the difference between the degrees of the extreme points (turned into minutes) will be the distance required (nearly) in miles. If the places have the same longitude, it is evident that the difference of their latitudes (or the sum, if they are on different sides of the equator) will be the distance between the two places.

Measuring distances accurately

If two places A and B are in the same latitude their distance apart is *accurately* measured as follows: Join the two places on the chart and lay off with a protractor from either extremity of this line an angle equal to the colatitude (90° – lat.). Measure from that extremity in this latter direction a length B C equal to the diff. long. of the places; the perpendicular distance C D from the end of this line to that originally drawn between the places *measured on the graduated parallel* gives the required distance accurately.

Fig. 12.

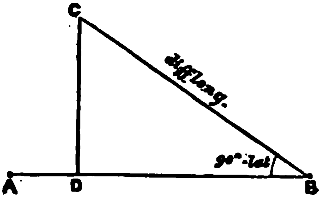
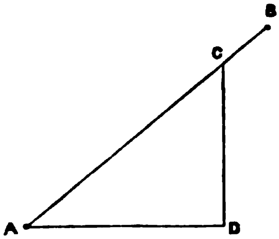


Fig. 13.



If the two places A and B differ both in lat. and long. join them and find their diff. lat. in degrees and minutes; extend the compasses to the same number of degrees and minutes on the graduated *parallel* (this is the true diff. lat.). Find the point on the line joining the places which is at this perpendicular distance (C D) from a parallel drawn through one of them; the distance of this point from that place (A C) *measured on the graduated parallel* is the true distance required.[1]

Fig 13

To find the latitude and longitude in by the chart, having given the course and distance from a given place.

Fixing positions on chart

Lay down the course on the chart in the manner pointed out on page 35. To the line (or edge of the ruler) thus lying in the direction of the course, apply the distance run (turned into degrees and minutes if necessary), measured with dividers from that part of the graduated meridian which is adjacent to the given place, and to that towards which the ship is sailing: this distance so taken along the line or edge of the ruler from the place sailed from will determine the position of the ship on the chart.

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[1] Given in Wright's *Errors in Navigation*, 1599; see also fig. 45, page 78. Although this method is *theoretically* accurate, it is to be observed that, owing to a certain amount of distortion inevitably introduced in printing charts, it is more accurate to obtain *exact* distances by calculation, as explained further on.

Construction of chart

The following example of constructing a Mercator's chart and of tracing the ship's track thereon is similar to that given at the Royal Naval College at the examination of sub-lieutenants.

9. Construct a Mercator's chart on a scale of one inch = one degree of longitude, extending from lat. 54° N. to 58° N., and from long. 178° E. to 178° W., and lay down thereon the ship's track from a position in lat. 56° 50′ N., long. 178° 50′ E.—namely, the following several true courses and distances:

True courses and distances	S. by W.	E.S.E.	N.N.E.	N.½E.	N.W. by N.	W.S.W.
	90′	100′	65′	60′	80′	75′

First, to construct the chart within the given limits.

This is done by following the directions previously given.

(1) At the bottom of the paper draw the graduated parallel (fig. 14), and divide it into four equal parts each one inch long, to represent the four degrees of longitude—namely, from 178° E. to 178° W.—and erect the graduated meridians at each extremity of the graduated parallel.

(2) Subdivide each degree of the graduated parallel into six or any other convenient number of equal parts:[1] if into six, as in the diagram, then each subdivision will be 10′. This may now be used as a scale of equal parts from which to set off any longitude distances on the chart.

(3) Write down the meridional parts for each degree of latitude so as to get the M.D. lat. for each degree, beginning with the highest; thus:

Lat. 58°...merid. parts...4294
,, 57 ... ,, ...4183...M.D.lat. = 111′ between 57° and 58°
,, 56 ... ,, ...4074... ,, = 109 ,, 56 ,, 57
,, 55 ... ,, .. 3968... ,, = 106 ,, 55 ,, 56
,, 54 ... ,, ...3865... ,, = 103 ,, 54 ,, 55

(4) Transfer the lengths in inches corresponding to 103′, 106′, 109′, 111′, taken off a scale of inches, to the graduated meridians;[2] draw the parallels of latitude and intermediate meridians and mark them with their proper degrees, and if it is found that the

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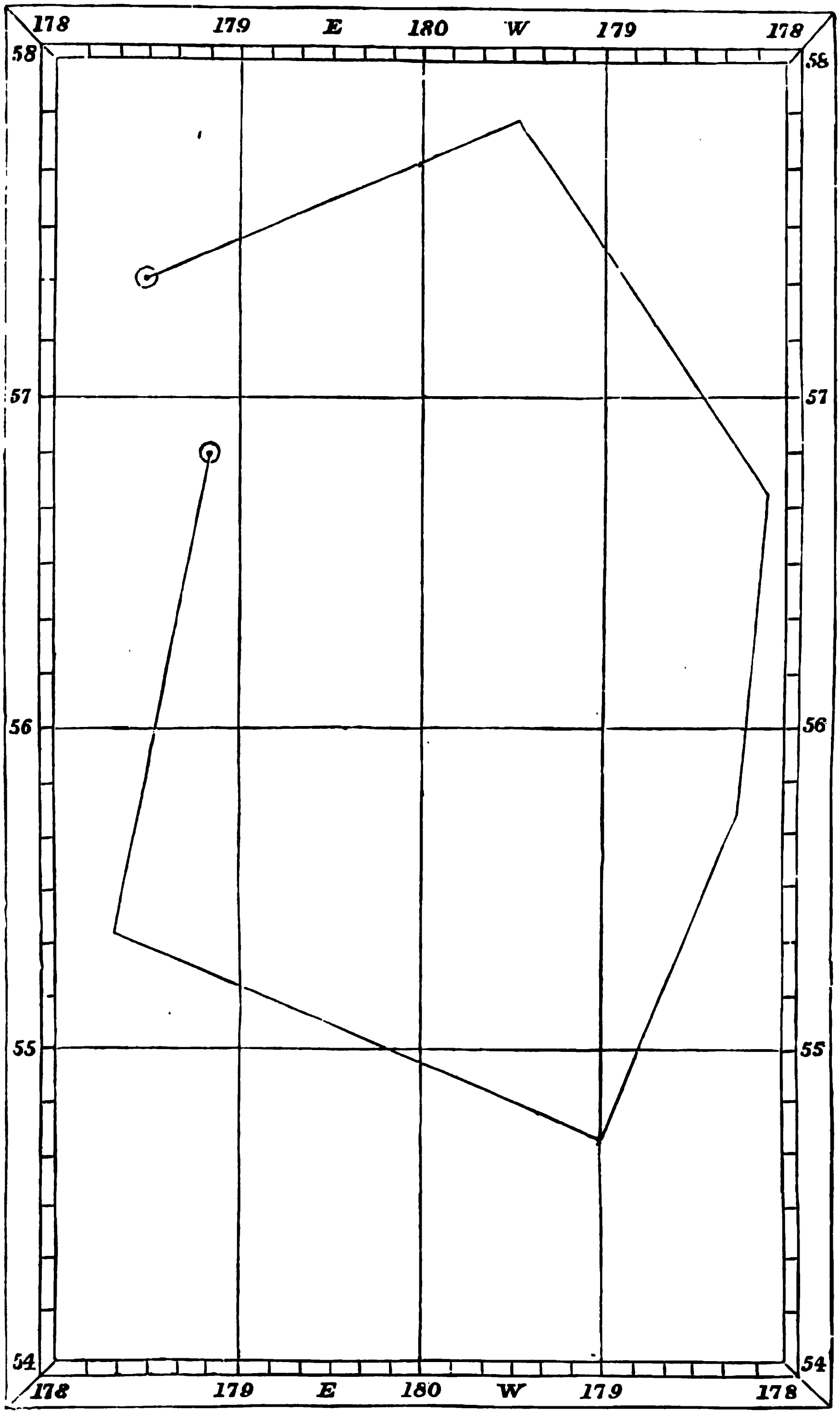
[1] On Admiralty charts the degrees are usually divided into ten equal parts, and similarly the degrees of latitude.

[2] These lengths are found by dividing each of the quantities by 60 and multiplying the results by the inches of the scale (in this case by one inch).

FIG. 14.



diagonals of the parallelogram formed by the outer parallels and meridians are equal, the chart will be ready to lay down on it the track of the ship—that is, the several true courses and distances made during the day. This is done as follows:

(1) Find the point on the chart corresponding to the place the ship sailed from; thus:

Lay the edge of a ruler over lat. 56° 50′ N.; and since the longitude is 178° 50′ E.—that is, 10′ to the left of 179° E.—with a pair of dividers take a distance of 10′ from the graduated parallel and lay it along the ruler to the left from meridian 179° E.; make a small dot at the point, and the position of the ship—namely, lat. 56° 50′ N. and long. 178° 50′ E.—is determined.

(2) To draw the several lines to represent the ship's track:

Laying down a ship's track

From the point thus found draw a line in the direction of S. by W. or 1 point west of meridian (179° E.)[1] and equal in length to 90′ remembering to take all the *distances from those parts of the graduated meridians adjacent to the respective courses.* From the southern extremity of this line draw a line E.S.E. 100′, and proceed in the same manner to lay down the several other courses given in the question; when it will be found that the ship has arrived at a place in lat. 57° 21′ N. and long. 178° 35′ E.

NOTE.—In using a Mercator's chart it will be observed that comparisons cannot be readily made by eye of the relative *areas* of places situated in different latitudes, since from the nature of the projection those in high latitudes are drawn on a larger scale than those in low latitudes.

In all latitudes, however, *small* portions of the earth are correctly represented in *shape*.

Mercator's charts show accurately the relative *positions* of places as regards the rhumb line, but not their relative *distances* from each other.

Objects which to the eye appear in line will not be so represented on a Mercator's chart unless they are on the same meridian or on the equator.

Gnomonic projection

Plans of small portions of the earth's surface and generally also maps of the polar regions are drawn on the Gnomonic projection.

The appearance and properties of such charts are well illustrated by a representation of the north polar regions, to draw which, as we have already seen, Mercator's projection is very unsuitable, the lengths of successive degrees of latitude increasing so rapidly.

If we suppose a flat sheet of paper A B C D laid on a transparent globe (fig. 15), in such a way that its middle part touches the north pole P, and an observer to be situated at O, the centre of the globe, then the parallels of latitude K L, M N (supposed to be visible) will appear on the paper as concentric circles, described round P as a centre, while the portions of meridians PM, PQ, PT will appear on the paper as straight lines PM_1, PQ_1, PT_1, radiating from P, since they are the edges of planes all of which pass through O and P. Such a sheet of paper, when seen from *outside* the globe, will appear as in fig. 16, and it will be observed that while the distortion in the appearance of regions very near the pole is slight, this rapidly becomes greater as the distance of the paper from the globe increases; also that, for the same reason that the meridians appear as straight lines, *all* arcs of great circles are similarly represented.[1]

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[1] Either by drawing a compass, as recommended on page 34, or with a protractor.

On a plan of any place not at the pole the meridians and all other great circles will still be represented by straight lines, but the parallels of latitude become curves or portions of ellipses, the paper now being supposed to touch the earth at the middle of the place represented.

Fig. 15.

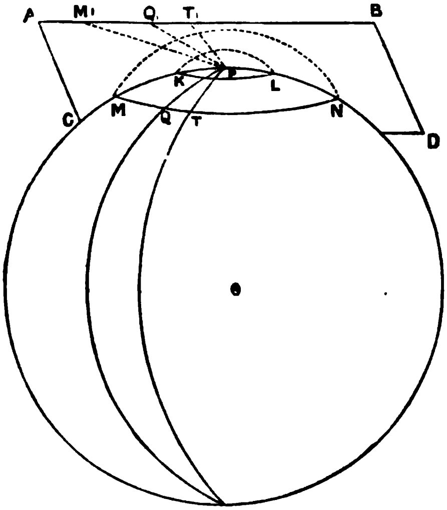
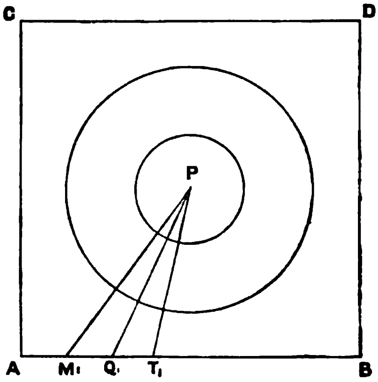


Fig. 16.



Plans are not *graduated*[2]—that is to say, meridians and parallels are not drawn on them, but measurements of distance, latitude, and longitude are obtained from scales.

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[1] For the purposes of general navigation polar charts are of course but seldom required, but plans of harbours and small islands are also published on this projection. In nautical surveying *all* charts are *necessarily* at first drawn on the gnomonic projection, since the directions of points of land from each other must be laid down on the paper by straight lines, and this is the only projection on which such directions are truly so represented.

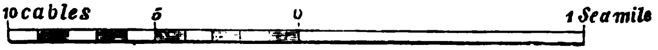
[2] *See* Plate 3.

Scale of latitude and distance

By the nautical mile we understand a mile of *latitude*, or, as already defined on page 3, one-sixtieth part of a degree of latitude; this increases slightly in length between the equator and poles, owing to the form of the earth not being that of a perfect sphere.

A scale of latitude and distance represents the number of inches on the plan corresponding to the length of a mile of latitude on the earth, the proportion between these lengths being chosen when drawing the plan of the place; usually the length of one or more *whole* miles is shown, together with one mile subdivided into ten equal parts or cables.

FIG. 17.—Scale of Latitude and Distance.



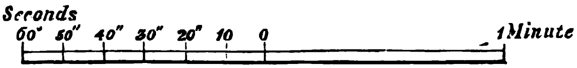
Scale of longitude

Differences of longitude on a plan cannot be measured by the above scale, since the same actual *length* which at the equator represents a mile of longitude in any other latitude must represent *more*, since meridians are not parallel but inclined to one another.

For convenience in measuring longitudes a scale similar to fig. 18 is usually drawn on plans.

This scale is formed from the scale of latitude by multiplying it by the cosine of the latitude. If the scale of latitude is 4 ins. = one mile in lat. 50°, the scale of longitude or length corresponding to one minute of longitude = 4 ins. × cos 50°. In some cases, such as laying down positions given in Notices to Mariners, on plans having no scale of longitude, the following method is useful:

FIG. 18.—Scale of Longitude.



Required $1\frac{1}{2}$ min. of longitude having only a scale of latitude (AB), lat. 30° (Fig. 19, page 42).

At A make angle B A C = the latitude, or 30°. From lat. scale take *b* B = $1\frac{1}{2}$ miles of lat., and make A C = *b* B. From C drop perpendicular C D. Then A D = $1\frac{1}{2}$ min. of longitude in lat. 30°.

Natural Scale

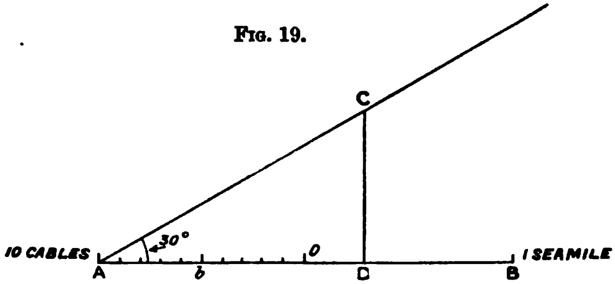
The natural scale (given on plans as a vulgar fraction) expresses the proportion the plan bears lineally to the portion of the globe it represents.

Thus, on a plan drawn on a scale of 3 inches = one mile, if the number of feet in the nautical mile at the latitude of the plan is 6076, then the natural scale $= \frac{3}{6076 \times 12} = \frac{1}{24304}$.

To illustrate the use of the above scales, it must be remarked that the lat. and long. of *one* point on a plan is always given in the title, and to obtain the position of any other point, if a meridian line is drawn through the given point, and a perpendicular on this drawn from the second, the intercepted part of the meridian, measured on the scale of latitude, will give the difference of latitude; and the distance east or west, transferred to the scale of *longitude*, will give the difference of longitude between the points. To facilitate drawing these lines, the marginal lines on a plan are drawn North and South and East and West true.

Even when places are east and west of one another, the *distance* between them must be measured on the scale of latitude, as by a mile of distance we of course mean a mile of latitude.

FIG. 19.



We proceed now to give some remarks equally applicable to charts and plans.[1]

Dates of correction

It is important in using an Admiralty chart or plan to know up to what date it has been corrected; the copper plate from which the impressions are taken is in many cases as old as the original survey, and the dates on which alterations and corrections have been made will be found on the printed chart in either or all of the following places:

If the alteration is of such magnitude that those in already possession of the chart could not themselves correct it by hand, the copper plate is first corrected and new copies issued, with instructions to destroy all old copies, the date of such an alteration being inserted either in the title or under the lower margin of the chart in the middle.

If the alteration is a *small* one the existing charts are not destroyed, but notices (Notices to Mariners or Hydrographic Notices) are sent to all H.M. ships for such correction to be made by hand; the copper plate is at the same time corrected, so that future copies will not be in error, and the month and year when such correction was made is engraved in the left-hand lower corner of the chart.

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[1] An interesting account of Admiralty charts will be found in the *Journal of the Royal Geographical Society* for 1881, written by a former Hydrographer, the late Captain Sir F. J. Evans, R.N., K.C.B., F.R.S. *See* also "Notes bearing on the navigation of H.M. ships."

Number

All charts and plans placed in the chart boxes are numbered, the number being found in the right-hand lower corner, and also on the back.

Title

Abbreviations

The title of a chart contains a brief extract of all information relating to it, together with a full explanation of the abbreviations employed on it. A list of *all* the abbreviations used on *all* charts is placed in the chart box.

Variation on coasting charts

To facilitate laying down courses on coasting charts, or those used when near the land, magnetic compasses are shown having the north point drawn east or west of the true meridian, by an angle equal to the variation at the place; these compasses are graduated from the *magnetic* north, and are divided into points and quarter-points, and usually into degrees also.

The date on which these compasses were made to point correctly is given in the title, together with the annual change of variation, which should be allowed for when great accuracy is desirable.

In laying down a compass course on such a chart it is only requisite to correct it for *deviation*, the resulting magnetic course being taken directly from the nearest compass on the chart.

High water line

On looking in from the sea, the first *firm* black line on the coast represents the high water line; all to seaward of this covers and uncovers.

Soundings

All soundings on Admiralty charts are given *either* in fathoms or feet, as explained in the title, but *both* are never shown on the same chart; they show in almost all cases the depths at the time of low water at ordinary spring tides, so that except at such times and at particular seasons of the year[1] there is always a greater depth than that indicated.

Heights

All heights are expressed in feet, and measured above the level of high water at ordinary spring tides, so that, except at such times and the particular seasons previously alluded to, all objects are higher out of the water than the figures written against them would indicate.

Lights

The distances at which lights are visible are expressed in nautical miles,[2] the observer being supposed at a height of 15 feet above high water. If higher above the water the light should be seen farther, and *vice versâ*.

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[1] *See* page 407.

[2] Reference should be made to the Admiralty Light Lists, on the first pages of which will be found a full description of the various characteristics of lights.

Bearings All bearings given on Admiralty charts are magnetic, and are such as would be observed from a ship, or in other words from seaward.[1]

Many items of information relating to practical navigation are frequently given on each of three different forms, viz., the charts, sailing directions, and light lists; where discrepancies exist between these, the chart must always be taken as the guide.

EXAMPLES.

Positions by Latitude and Longitude.

10. Upon chart No. 2615, plot the following positions and take off the *compass* courses and distances between each pair:

1.	Lat..........50° 28′ 36″ N.	and	Lat..........50° 31′ 22″ N.
	Long....... 2 20 00 W.		Long....... 2 17 00 W.

Ans. Comp. co. N. 40° E.; Dist. 3·4.′

2.	Lat..........50° 30′ 5″ N.	and	Lat..........50° 32′ 22″ N.
	Long....... 2 20 00 W.		Long.......2 20 00 W.

Ans. Comp. co. N. 15° E.; Dist. 2·3′.

3.	Lat..........50° 29′ 00″ N.	and	Lat..........50° 29′ 00″ N.
	Long....... 2 15 18 W.		Long....... 2 20 30 W.

Ans. Comp. co. N. 47° W.; Dist. 3·3′.

4.	Lat..........50° 30′ 10″ N.	and	Lat..........50° 31′ 56″ N.
	Long....... 2 9 8 W.		Long....... 2 3 28 W.

Ans. Comp. co. N. 64° E.; Dist. 4·1′.

5.	Lat..........50° 28′ 10″ N.	and	Lat..........50° 32′ 5″ N.
	Long....... 2 10 30 W.		Long....... 2 4 20 W.

Ans. Comp. co. N. 46° E.; Dist. 5·6′.

6.	Lat..........50° 27′ 20″ N.	and	Lat..........50° 34′ 35″ N.
	Long........ 2 8 5 W.		Long....... 2 21 45 W.

Ans. Comp. co. N. 19° W.; Dist. 11·4′.

NOTE.—In this and examples on following pages the deviation table on page 21 is to be used.

Positions by Bearing and Distance.

11. Upon chart No. 2615, place the following positions and take off their latitudes and longitudes:

1. 2 miles, 4 cables S. by W.¼W.(magnetic) from Shambles light vessel.

Ans. Lat. 50° 28′ 28″ N. Long. 2° 19′ 36″ W.

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[1] A complete 'Catalogue of Charts' is published annually by the Admiralty, together with a supplementary list every two months.

2. 3 miles, 2 cables E.¼S. (true) from Breakwater light.

Ans. Lat. 50° 34′ 48″ N. Long. 2° 19′ 52″ W.

3. 9 cables S. 47° E. (magnetic) from Portland High Light.

Ans. Lat. 50° 30′ 56″ N. Long. 2° 26′ 0″ W.

4. St. Alban's flagstaff bearing N. 42° E. (compass), distant 3¼ miles. Ship's head N.W.

Ans. Lat. 50° 31′ 28″ N. Long. 2° 3′ 27″ W.

5. Bindon hill summit bearing N. by W.¼W. (compass), distant 2¾ miles. Ship's head N.E.½E.

Ans. Lat. 50° 34′ 32″ N. Long. 2° 12′ 26″ W.

6. Shambles light bearing S. 86° W. (compass), distant 4½ miles. Ship's head East.

Ans. Lat. 50° 31′ 8″ N. Long. 2° 12′ 55″ W.

NOTE.—The *balls* on light vessels and the *foot* of St. Alban's flagstaff are the fixed positions.

Upon the plan of Valetta harbour, Malta (in Officers' Atlas), place the following positions:

7. 8½ cables East (magnetic) from Sliema point.

8. 7 cables N. 47° E. (true) from Dragut point.

9. 6¼ cables N.N.E. (compass) from St. Elmo light. Ship's head S. by E.

10. 1 mile, 8 cables S. 64° E. (magnetic) from St. Julian's tower.

11. 4¾ cables S. 80° E. (true) from St. Julian's tower.

12. 7 cables N. 5° E. (compass) from St. Elmo light. Ship's head South.

Mercator's Charts.

1. Construct a Mercator's chart upon a scale of 2 inches = a degree of longitude, extending from lat. 50° 20′ S. to 53° 10′ S., and long. 3° 10′ W. to 1° 40′ E.

A vessel from a position in lat. 51° 20′ S., long. 00° 40′ W., steers as follows:

Comp. Course.	Deviation.	Distance.	
W.¾S.	14° W.	60 miles	variation 19° W.
E.S.E.	13 E.	85 ,,	
S.S.W.¼W	5 W.	15 ,,	

Required the lat. and long. of position arrived at.

Ans. Lat. 52° 39′ S. Long. 00° 18′ E.

2. Construct a Mercator's chart on a scale of 3·7 inches = a degree of longitude, extending from lat. 57° to 58° N., and from long. 1° to 4° E.

A ship sailed from lat. 57° 10′ N., long. 1° 30′ E., as follows:

Magnetic.	Distance.	
N. by E.	35 miles	variation 20° W.
S.E. by S.	50 „	
N. by E.¾E.	25 „	
S.S.E.½E.	40 „	

Required the lat. and long. of position arrived at.

Ans. Lat. 57° 13′ N. Long. 3° 29′ E.

3. Construct a Mercator's chart upon a scale of 1·5 inch = a degree of longitude, extending from lat. 40° to 45° N., and long. 178° E. to 178° W.

A ship sailed from lat. 40° N., long. 180°, as follows:

Magnetic Course.	Distance.	
N. 38° W.	70 miles	variation 10° E.
N. 30 E.	145 „	

The current during the whole period set East (true) 35 miles.

Required the lat. and long. of position arrived at.

Ans. Lat. 42° 53′ N. Long. 177° 53′ W.

4. Construct a Mercator's chart on a scale of 0·75 inch = a degree of longitude, extending from lat. 49° N. to 53° N., and long. 8° W. to 15° W.

A ship sailed from lat. 49° 20′ N., long. 14° 15′ W., as follows:

Magnetic Course.	Distance.	
East	102 miles	variation 25° W.
E. by N.	68 „	
E. by N.¾N.	40 „	

Upon the completion of the last course what is the magnetic bearing of a point of land situated in lat. 51° 40′ N., long. 9° 24′ W.?

Ans. N. 20° E.

5. Construct a Mercator's chart upon a scale of 2·5 inches = a degree of longitude, extending from lat. 57° 00′ N. to 59° 00′ N. and long. 1° 30′ W. to 1° 30′ E.

A ship is 12 miles S.¼W. (true) of an islet lying in lat. 58° 20′ N., long. 0° 50′ W., and then steers as follows:

Comp. Course.	Deviation.	Distance.	
S.S.E.	3° E.	30 miles	variation 22° W.
N.E. by E.¼E.	2 E.	50 „	
N.W. by W.¼W.	11 W.	30 „	

How does the islet bear (true) on completion of the last course?

Ans. N. 88° W.

6. Construct a Mercator's chart upon a scale of 1·5 inch=a degree of longitude, extending from lat. 2° 30′ N. to 2° 30′ S., and long. 0° to 4° W.

A ship sailed from lat. 1° 45′ S., long. 0° 30′ W., as follows:

True Course.	Distance.
N. 47° W.	80 miles
N. 30 W.	85 „
N. 14 E.	60 „

A second ship sailed from lat. 1° 50′ N., long. 3° 12′ W., as follows:

True Course.	Distance.
S. 12° E.	90 miles
East.	95 „

Required the lat. and long. of the position where these tracks cross.

Ans. Lat. 0° 10′ N. Long. 2° 10′ W.

7. Construct a Mercator's chart, and, laying down the bearings and courses, find the latitude and longitude in from the following:

C. Spartivento (38° 52′ N., 8° 52′ E.) and C. Teulada (38° 52′ N., 8° 39′ E.) bore respectively N.E. by E. and N. by W.; afterwards sailed as under:

True Courses.	Distances.
S.W.¾W.	125 miles
E.S.E.	98 „
N.W. by N.	162 „
S. by E.	140 „
E.¼N.	118 „
N. by W.¼W.	74 „

The chart to be constructed on a scale of 1·4 inch to a degree of longitude, to extend from latitude 36° N. to 40° N., and from longitude 6° E. to 10° E.

Observations then placed the ship in latitude 37° 55′ N., longitude 9° 50′ E. Find, by projection, the set and drift of the current.

Find the compass course and distance from the observed position to St. Pancras, Cagliari (39° 13′ N., 9° 8′ E.), the variation being 14° W. and the deviation 4° 10′ E.

8. Construct a Mercator's chart on a scale of 5 inches to 1 degree of longitude, extending from 54° to 55° N., and from 4° to

6° W. At noon Laggon Head bore E.N.E. true, and Port Patrick Light S.E. by E.½E. true. Then sailed as follows:

True Courses.	Distances.
S. by E.	37 miles
N.E.¾E.	29 ,,
S. by W.½W.	34 ,,

The Calf of Man then bore W.S.W. true, and Peel Light N.W. true. Find, by protraction, the set and drift of the current, and the true courses and distances from the ship in her last position to Douglas, passing the Chicken and Langness on the port hand.

	Latitude.	Longitude.
Laggon Head	54° 58′ N.	5° 11′ W.
Port Patrick	54 50 N.	5 7 W.
Calf of Man	54 3 N.	4 50 W.
Peel Light	54 14 N.	4 42 W.
Chicken	54 2 N.	4 51 W.
Langness	54 3 N.	4 38 W.
Douglas	54 9 N.	4 28 W.

9. Construct a Mercator's chart on a scale of 3·8 inches = a degree of longitude, extending from lat. 53° to 54° N., and from long. 4° to 6° 30′ E.

A vessel sailed from lat. 53° 10′ N., long. 4° 8′ E., as follows:

Comp. Course.	Deviation.	Distance.	
N.E. by E.½E.	2° E.	33 miles	variation 16° W.
E. by N.¾N.	3 E.	42 ,,	
E.½S.	4 E.	14 ,,	

A light, in lat. 53° 50′ N., long. 6° 18′ E., was then seen bearing S.E. by S. (*mag.*), and after proceeding 7 miles farther on the same course (E.½S.) the light bore S. by W. (*mag.*). Required the position of the vessel and current experienced.

10. Draw a Mercator's chart extending from latitude 49° N. to latitude 50° 30′ N., and from longitude 4° W. to 7° W. The meridional diff. of latitude from 49° to 50° is to be represented by 5 inches. Lay down the following courses and find the latitude and longitude in.

The Eddystone bore S.S.W. and Rame Head N.W.; afterwards sailed as under:

True Courses.	Distances.
S. by W.	50 miles
W.S.W.	61 ,,
N.W.	38 ,,
N.E. by N.	27 ,,

St. Agnes light then bore West, and the Wolf N. 20° E. Find the set and drift of the current.

	Latitude.	Longitude.
Eddystone	50° 11′ N.	4° 16′ W.
Rame Head	50 19 N.	4 13 W.
S. Agnes	49 53 N.	6 20 W.
The Wolf	49 56 N.	5 49 W.

CHAPTER IV.

The Protractor—Station pointer—Cross bearings—Bearing and angle—Position determined by angles—Doubling the angle on the bow—Four-point bearings—Danger angle.

In laying down positions on charts, frequent applications are made of the Protractor and Station Pointer, the practical use of which should be thoroughly learned.

For purposes of navigation the protractor is usually made in one of the following forms :

Fig. 20.

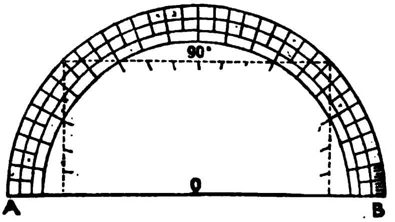
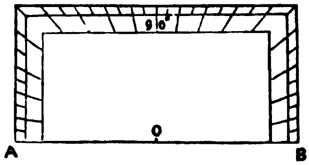


Fig. 21.



Semi-circular protractor

The semicircular protractor (fig. 20) is generally made of horn, so that the chart can be seen underneath it.

Rectangular protractor

The rectangular protractor (fig. 21) is simply a semicircular protractor with the edges made square, as illustrated, by removing all parts outside the dotted line on fig. 20.

In both forms the graduations on the edges represent angles measured from O A or O B with O as centre.

In using either form of the instrument the centre O should be placed on the *point* at which it is desired to lay off the given angle or course, and the edge A B on the *line* from which it is to be measured. Thus in laying off a magnetic course in degrees the centre O is placed over the centre of the compass on the chart, and the edge A B laid on the magnetic north and south line ; similarly a true bearing or course is laid off, with the edge A B placed on a meridian line.

Usually on rectangular protractors there are given lines of tangents, sines, secants, &c., by means of which trigonometrical problems may be resolved by *construction* without calculation ; this method of solution is, however, seldom practised in modern navigation, the results obtained being much less accurate.[1]

Scales of equal parts and diagonal scales for decimal division are also generally given.

Station pointer

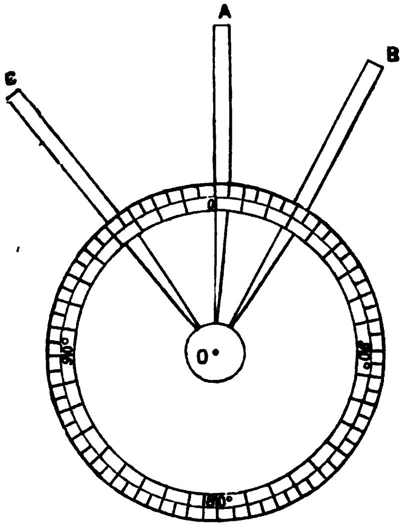
The station pointer is an instrument used to place on a chart a ship's position, determined by two *angles* observed at the position (*see* page 52). It may be compared to a combination of three flat rulers, two of which are movable about a common centre both right and left of the third, which is fixed, the angular distance through which they are moved being read off on an attached circular arc.

In fig. 22, A represents the fixed leg from which the movable legs B and C can be separated by angular distances equal to the observed angles taken at the ship with a sextant between three objects on shore.

The station pointer being then placed on the chart and moved until the edges of the legs O C, O A, O B pass through the positions of the objects observed, O will be the place of the ship at the time.

An excellent substitute for a station pointer[2] is afforded by a sheet of tracing-paper having a pencil line ruled upon it from any convenient point, from which point the observed angles are laid off with a protractor, one to the right and the other to the left, lines at these angles being ruled in pencil; the tracing-paper is then laid over the chart and moved until the three lines ruled upon it pass through the positions of the objects observed, when the original point from which the angles were laid off is the place of the ship at the time.

Fig. 22.



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[1] A description of the use of these lines will be found in Heather's *Treatise on Mathematical Instruments.* It would be an advantage if a diagonal scale of *inches* were more generally engraved on protractors.

[2] Station pointers are now supplied to every class of ships, and a supply of tracing paper is placed in every chart box, together with a pamphlet fully describing the use of the station pointer.

[3] If the right- and left-hand legs do not *both* shut up to zero, treat the *fixed* leg as either a right- or left-hand one according to circumstances.

In an extreme case prick through the positions on the chart and plot on the back.

Fixing Positions.

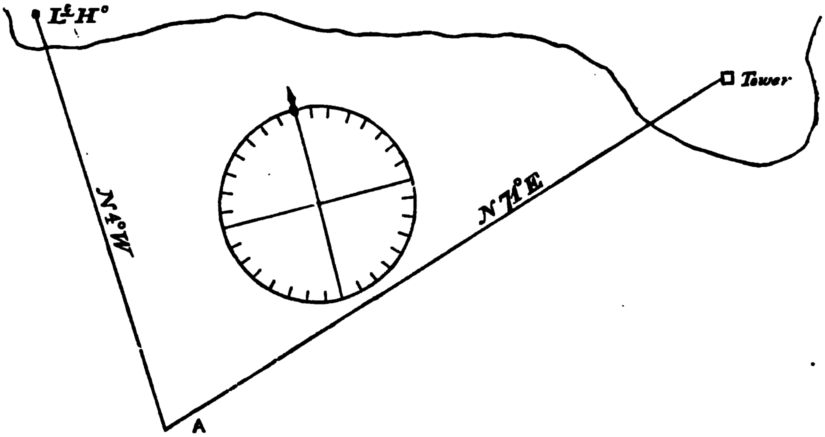
When in sight of land the position of a vessel is laid down on a chart by one or more of the following methods:

1. By bearings of two or more points on the land taken in succession as quickly as possible; this method is known as that by *cross bearings*.[1]

2. By the bearing of *one* point (or where circumstances admit of two points when in line) and the angle measured at the same time between it and another point; technically known as *bearing and angle*.

3. By measuring with a sextant two angles between three points on the shore; this method is quite independent of the use of the compass.

Fig. 23.



Cross bearings

In applying the first method the two bearings are observed at the standard compass and the direction of the ship's head noted. *Both* bearings are then corrected for the deviation due to that direction (applying the *same* correction to each), thus converting them into magnetic bearings. Parallel rulers are then laid over the nearest compass on the chart in the directions thus indicated, and moved until the edge lies in succession over each point observed; the intersection of the lines thus obtained gives the position of the ship at the time of observation.

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[1] The position determined from the bearings of only *two* objects should not be implicitly relied upon, since any error in the compass is not then apparent; the bearing of a third object should always be taken when possible. In torpedo boats, when bearings cannot conveniently be observed at the standard compass, they are taken with a *bearing plate*. From the distortion introduced into Mercator's charts by representing the meridians parallel to one another, it follows that an *observed* bearing (especially that of a distant object) will not, when laid down on such a chart, accurately pass through the position of the object. Practically, however, this correction (termed Convergency) need not, except in high latitudes, be applied to ordinary coasting charts, which are generally either of small area or on a small scale.

Example

1. Lighthouse bears N. 10° W. by compass and Tower N. 65° E. by compass (head S. by E.). The deviation for S. by E. being 6° E. (*see* Deviation Table, page 21), the above bearings become *magnetic* N. 4° W. and N. 71° E.; these are taken from the nearest compass on the chart and ruled through the Lighthouse and Tower (fig. 23), their intersection (A) being the position of the ship.[1]

In many ironclads the field of view from the standard compass is often much restricted by turrets, boats, &c.; in such a case, if wishing to fix the position by cross bearings of two points, only *one* of which is visible from the compass, the bearing of the other point is readily obtained by observing the bearing of *any* visible point, and applying to that the angle (measured by sextant) between it and the desired object.

Bearing and angle

In the second method of determining a position, viz. by bearing and angle, the bearing of *one* point (or of two when in line) is observed from the standard compass and at the same time the angle (of not less than 30°) measured between it and a second point. The bearing, corrected as before for the deviation due to the direction of the ship's head, is laid down on the chart as in the first method, and at *any* point on the line thus ruled the observed angle is laid off by protractor, and a line drawn in that direction. The parallel rulers are then moved parallel to this second line, until the edge lies over the point to which the angle was measured; the intersection of the line thus obtained with the line of bearing gives the position of the ship.

Example

2. Church bears N.W. by compass, and at the same time the angle at the ship measured between Church and Windmill is 63° (head S.S.W.). The compass bearing, corrected for the deviation due to S.S.W. (*see* page 21), becomes *magnetic* bearing N. 48° W.; this is laid off from the nearest compass as before; at *any* point O on this line (fig. 24) the angle 63° is laid off, and on moving the

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[1] In the case where one object (such as a light at night) is lost sight of at some time before another light is seen, if the course and distance during that interval are laid down from the supposed position on the first line of bearing, and a line is drawn parallel to this *first* bearing through the *supposed* position of the ship when the *second* bearing is taken, then the intersection of this parallel line with the second bearing gives the *true* position when that latter bearing was taken; this is precisely similar to the way in which the 'run' is dealt with in Sumner's method (*see* page 387).

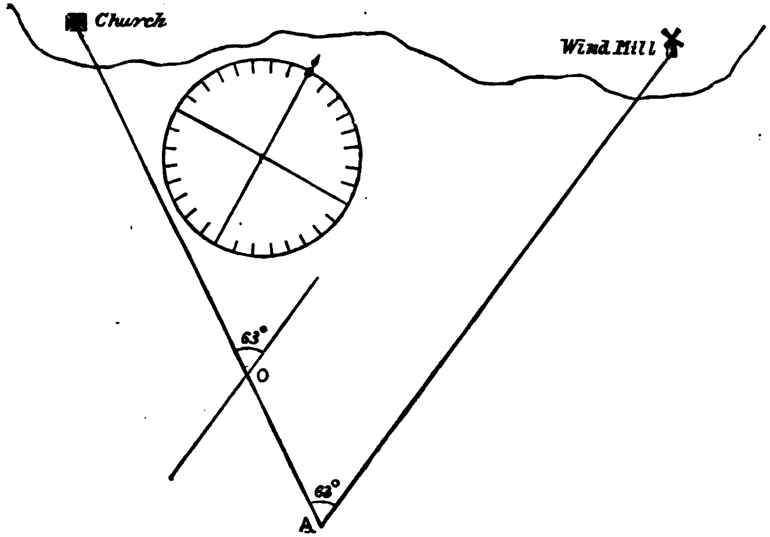
rulers parallel to the second line thus drawn, until their edge lies over Windmill, A will be position of the ship.

NOTE.—In fixing positions by either cross-bearings or bearing and angle, near objects should be observed in preference to those more distant, since small errors of observation or in laying down the lines on a chart misplace the position more with distant than with near objects.

Position by angles

The third method of determining a position, viz. that by *angles*, has the great advantage of being independent of compass bearings, for it may be taken for granted that an *angle* can always be more correctly measured by a sextant than a *bearing* can be observed, especially when a ship is rolling, and whereas compass bearings can only be taken at the standard compass, angles can be observed from any part of the ship, an obvious advantage when, as before remarked, in many cases in modern vessels of war there are objects which obstruct the field of view from the compass. As the latter consideration alone is rapidly leading to the more common use of this method we propose to illustrate it somewhat fully.

FIG. 24.



The information afforded by measuring *one* angle A O B between two fixed points A and B is that the observer is *somewhere* on the arc of a circle described through his position and those of the two points; if the observed angle is *less* than 90°, as in fig. 25, the segment of such a circle containing the observer's position must be *greater* than a semicircle, and if, as in fig. 26, the observed angle is *greater* than 90° such a segment will be *less* than a semicircle (Euc. III. 31). In both of the above cases for a certain given angle the observer's position *must* be on the arc A O B, but *may* be at *any* point on that arc, since the angles in the same segment of a circle are equal to one another (Euc. III. 21).

If, however, at the time of measuring the angle between A and B, another angle is measured between B and a third point C, and a similar circle described passing through B and C, containing the second observed angle, for similar reasons the observer's position must lie on the arc of *this* circle; hence the true position will be where the two circles cut one another.[1]

Three cases of this method of fixing positions may be distinguished—(1) that in which *both* observed angles are less than 90°, (2) where *both* are greater than 90°, and (3) where one is less and one greater than 90°.

CASE 1.—To describe these circles geometrically, suppose that an angle of 40° has been measured between A and B (fig. 27); then, since this is less than 90°, the centre of the required circle must lie between the observer and a line joining A B,[2] and if D is the centre, the angle A D B being the angle at the centre of a circle will be 80°, or double the angle at the circumference (Euc. III. 20). Hence each of the equal angles D A B, D B A will be 50°, or the complement of the observed angle.

FIG. 25.

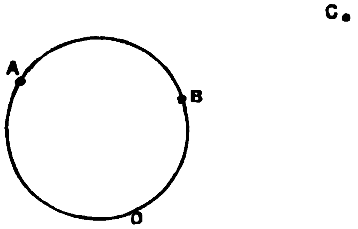
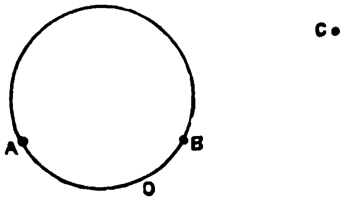


FIG. 26.



Therefore in this case if A and B be joined, and from each extremity of the line A B the complement of the observed angle be laid off *towards* the observer, the intersection of these lines will be a point D, from which as centre, with radius equal to D A or D B, a circle may be described the segment of which A O B will contain an angle of 40°.

In a precisely similar manner a circle may be drawn through B and C of which a segment B P C contains a second acute angle of 60°.

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[1] In practice a *third* angle is always observed as a check, or a bearing of one of the objects is taken.

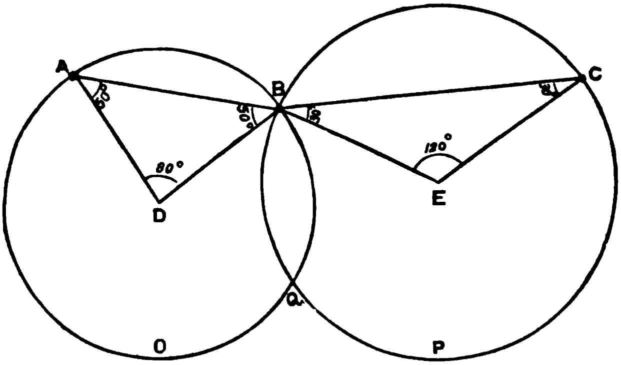
[2] The relative apparent positions of A and B with regard to right and left determine upon which *side* of A B the observer is situated.

If then the angles 40° and 60° be observed at the same instant between A and B and between B and C respectively, the intersection of these circles in Q must be the position of the observer.

CASE 2.—In the second case where both observed angles exceed 90° the centres of the circles will both lie on the sides of the lines joining the points remote from the observer, and the position required must be *within* the triangle formed by joining these points.

Suppose that an angle of 115° has been measured between A and B (fig. 28). The observer must be situated on the smaller segment of the circle passing through his position, A and B, and containing an angle of 115° (Euc. III. 31); the angle in the larger segment will therefore be 65°, or the supplement of 115° (Euc. III. 22). The angle at F, the centre, being double of this, or 130°, each of the equal angles F B A, F A B will be 25°, which is the excess of the observed angle 115° above 90°.

FIG. 27.



Hence in this case if A and B be joined, and from each extremity of the line A B the complement of the observed angle be laid off *from* the observer, the intersection of these lines will be a point F, from which as centre with radius equal to F B or F A a circle may be described the segment of which A Q B will contain an angle of 115°.

In a similar manner if at the time the angle between A and B was measured an angle of 120° was observed between B and C, a circle whose segment contained *that* angle may be described; the intersection of these circles in Q gives the position of the observer.

CASE 3.—In the third case, where one angle is less than 90° and the other greater, the first circle would be drawn as in Case 1 and the second as in Case 2.

A position can always be thus determined except in the case where the observer and the three objects lie near or on the arc of the same circle, when the two circles thus drawn will nearly or quite coincide.

The position of a ship may be always well determined under the following circumstances:

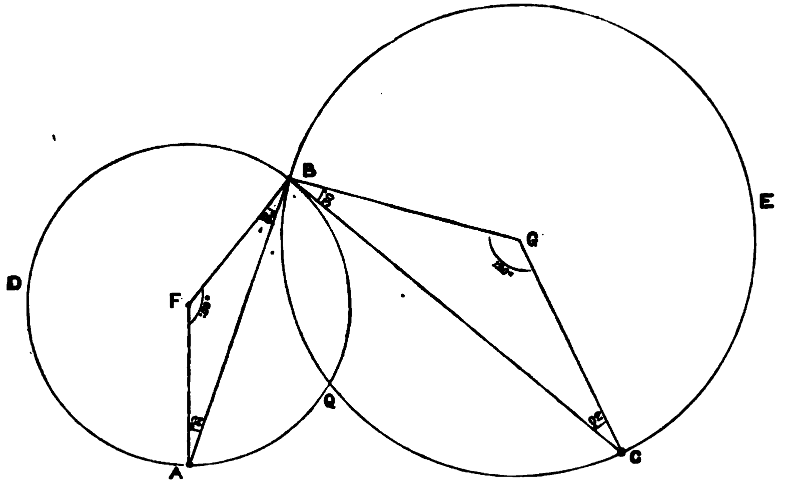
(1) When inside the triangle formed by joining the three points.

(2) When the centre object is nearer than either both or one of the others and each angle is not less than 40°.

(3) When the three objects are equidistant from the observer and neither angle is less than 70°.

(4) When one angle is large and the other small, provided always that in the small angle the outer object is much more distant than the centre one.

Fig. 28.



In the case of the middle object being *very much more distant* than the others, the 'fix,' as it is termed, may be good, but the points would require experience in selection.

If fixing the position with two objects in line and an angle to a third, the distance between the two objects should not be less than two-thirds of the distance of the nearer object from the observer.

A special case occurs when two of the objects appear in line (transit) when only *one* angle is required to fix the position; this angle, however, should not be less than 30°, and it is important to remark that an angle of 30° measured to an object at a certain distance will give as accurate a result as an angle of 90° measured to an object at twice that distance.

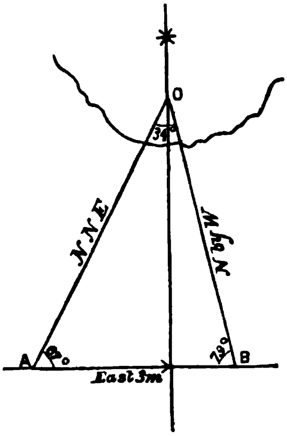
The foregoing may be regarded as the theory of this method of fixing a ship's position, the practical application being usually carried out by means of the station pointer or tracing-paper, as explained on page 51.[1]

The preceding methods of fixing the position of a ship necessarily presuppose that either *two* or *three* suitable objects are visible at the same time; a less exact but useful method, applicable when only *one* object can be seen, is founded on the following principle.

By the application of plane trigonometry, if the bearing of any object is observed at two different positions of a ship, and the course and distance between these positions known, the distances of the observed object from each position can readily be calculated.

Example

Fig. 29.



3. If an object O on shore be observed from a ship at A (fig. 29) to bear N.N.E. (true), and after steering East (true) 3 miles to B it bears N. by W. (true), the distance of O from A or B can be calculated, as in the triangle O A B one side A B is known and all three angles (the angle at O being the difference of the observed bearings).[2]

In practice this is done by ruling the two bearings on the chart and ascertaining by trial where the distance run on the known course will exactly fit between them.

Two important cases of this problem are worthy of note from their practical utility and general use.

1. The case when the angle between the second bearing and the ship's course is double the angle between the first bearing and course, known technically as *doubling the angle on the bow*.

2. The special case of the above when the first angle being 45°, or 4 points, the second angle is 90°, or 8 points—that is, the

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[1] The reader should further refer to the remarks by Admiral Wharton, R.N., F.R.S., the Hydrographer of the Admiralty, on this subject at page 26 of *Hydrographical Surveying*.

[2] Tables from which these distances can be taken by inspection were originally drawn up by Thos. Pyman, master mariner, 1802, and dedicated to Lord Nelson. They also form Table 15 of Inman's Tables.

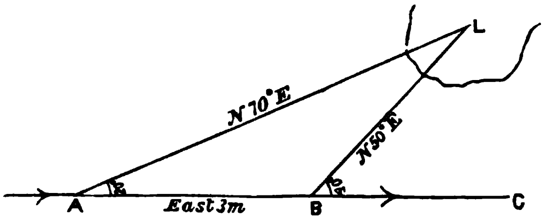
object is right abeam. This case is termed the *four-point bearing.*

Doubling angle on bow

To illustrate the first case, suppose a vessel steering East when a light L (fig. 30) is observed bearing N. 70° E.; the patent log is then read off, and again when the light bears N. 50° E.: the difference between these readings gives A B the distance run in the interval.

The angle L A B being 20° the light L is said to be 20° on the bow; in the second position B this angle being 40° (or double that at A), it is evident that since A L B = L B C − L A B = 20°, B L must equal A B, or the distance from the light at the second position equals the distance run.

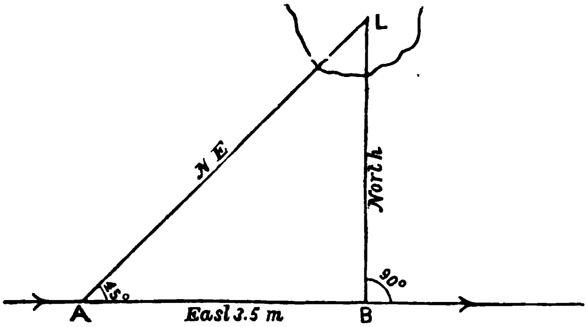
Fig. 30.



Four-point bearings

In the second case, where the angle between the bearing of the light and the ship's course is 45°, or 4 points (fig. 31), the distance from the light at B, the second position, will, as before, equal the distance run between the observations, but in this case B will be the point of nearest approach to L.

Fig. 31.



The above problem is perfectly general in its application, but the accuracy of result is vitiated by the action of any unknown current experienced between A and B; errors of observation affect the position of B least when the first angle between the direction of the light and the ship's course is 60°, in this case the two positions of the ship and that of the light form an equilateral triangle.[1]

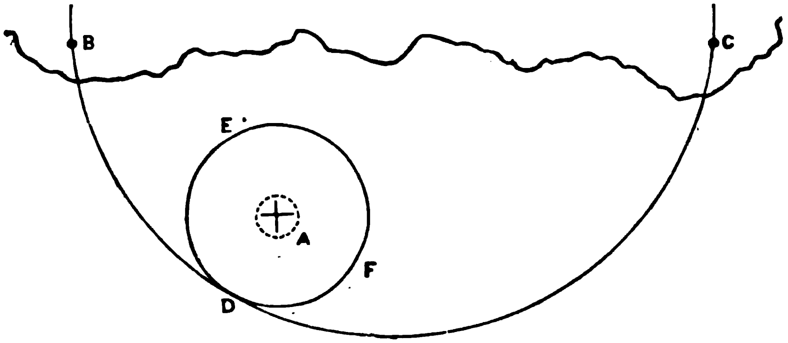
The position obtained by doubling the angle on the bow may be verified by repeating the observation one or more times, since the magnitude of the angles is immaterial provided that the second is double of the first. Similarly, the 'four point' bearing can be repeated by bringing the object four points on the quarter.

It is often of practical use when passing a point or light to know that the distance run between the times that it bears 26½° on the bow (2¼ pts.) and 45° on the bow (4 pts.) is equal to the distance which the vessel will be off when it is abeam, supposing that the same course is maintained.

Danger Angle

The danger angle is an application of the geometrical properties of the circle which enables a vessel to ensure passing a danger at any desired distance.

FIG. 32.



Let A in fig. 32 represent a danger which does not show above water, and which it is desirable to pass at a distance of half a mile outside, and let B and C represent two conspicuous objects on shore; if a circle D E F be described with centre A and radius half a mile, and another circle B C D be described passing through B and C and touching the circle D E F, then since at any point *within* the segment B D C the angle subtended by B and C will be *greater* and at any point *without* that segment *less* than the angle B D C (Euc. III. 21), a vessel in the vicinity of the danger can ensure passing it outside desired distance by steering so as to always keep the observed angular distance between B and C less than the angle B D C.

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[1] The bearings may be taken in points or degrees; with the vessel's head in the same direction throughout, since the deviation applicable to the course and *both* bearings is the same there is no need to correct *either*, since the object is that the *angle* in the second position should be double that in the first position.

To facilitate the practical application of this problem a dumb card or 'bearing plate' is employed.

The circles would be drawn on the chart before the vessel drew near, and the angle B D C, or danger angle, measured with a protractor.

Practically the circle B D C would be drawn by joining B C, bisecting it and erecting a perpendicular from the middle point, ascertaining by trial with compasses where on this perpendicular the centre of the required circle would lie (Euc. III. 1),[1] or else a point D would be marked on the chart at the required distance from A, and the angle B D C, measured with a protractor, considered to be the danger angle.

The danger angle, both horizontal and vertical, is often useful in passing *between* shoals, the vessel being steered so as to keep the angle on.

EXAMPLES.

Positions by Cross Bearings.

Upon chart No. 2615, plot the following positions :

1 Portland High Light N.N.W.½W. (magnetic)
Shambles light N.E. by E.½E. (magnetic)

2 Shambles light S. 30° W. (true)
Breakwater light N. 60° W. (true)

3 Portland High Light N.W. by N. (magnetic)
Shambles light N.E.½N. (true)

4 Shambles light S.S.W.¼W. (compass)
Breakwater light N.W. (compass)
Ship's head N.E.½E.

5 Bill of Portland W.S.W. (true)
Bindon Hill N. by E.¾E. (compass)
Ship's head S.S.W.

6 Shambles light W.N.W. (magnetic)
St. Alban's flagstaff N.N.E.¼E. (compass)
Ship's head N.W.¼W.

7 Portland High Light N. 57° W. (compass)
White Nore summit N. 20° E. (compass)
Ship's head W.S.W.

8 Portland High Light N. 32° W. (magnetic)
Shambles light N. 27° E. (magnetic)

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[1] The danger angle may also be obtained if an object of known height is visible by measurement of its angle of elevation. To facilitate the application of this *vertical* danger angle a valuable table has been calculated and published by Capt. Lecky, R.N.R., whose further illustrations of the application of the horizontal danger angle when entering a harbour (in *Wrinkles on Navigation*) should be referred to by the reader. Much discretion is, however, necessary in using a vertical danger angle, as the heights given on many charts are only approximate.

9 Bindon Hill N. (compass)
St. Alban's flagstaff E.½N. (compass)
Ship's head S.½W.

10 Bindon Hill N. (magnetic)
St. Alban's flagstaff E.½N. (magnetic)

11 Portland High Light W.¼S (compass)
Breakwater light N. by W. (compass)
Ship's head N.N.E.

12 Portland High Light S. 80° W. (compass)
Shambles light S. 21° W. (compass)
Ship's head N.N.W.

Note.—The Deviation Table on page 21 is to be used for all examples on this and following pages. The fixed part of Shambles light is the ball at masthead, and that of St. Alban's flagstaff is the lower part.

Positions by Bearing and Angle.[1]

Upon the plan of Valetta harbour, Malta (in Officers' Atlas), plot the following positions:

1. Ricasoli point S.S.W. (magnetic) 60° Dragut Point Light.
2. Ricasoli point 30° St. Elmo Light S. 65° W. (true).
3. Ricasoli point S.¼W. (compass) 32° Dragut point. Ship's head North.
4. Dragut point South (magnetic) 110° St. Julian's tower.
5. Dragut point 35° Sliema point W.¼S. (magnetic).

Upon chart No. 2615 plot the following positions :

6. Portland High Light N. 42° W. (true) 46° Shambles light.
7. Shambles light 80° Breakwater lighthouse N. 56° W. (compass). Ship's head N.N.E.
8. Bindon Hill 110° St. Alban's mast S. 66° E. (magnetic).
9. Bill of Portland S. 76° W. (compass) 60° Breakwater lighthouse. Ship's head N.N.W.
10. Portland High Light S. 76° W. (magnetic) 50° Breakwater lighthouse.

Position by Angles.

Upon chart No. 2615 plot the following positions, first by station pointer and secondly with tracing-paper:

1. Portland High Light 45° 30′, Shambles light 62° 40′ St. Alban's mast.

2. Portland High Light in line with Shambles light 98° 20′ Bindon Hill.

---

[1] The object to which the angle is measured is *written* right or left of the object whose bearing is observed according to its *actual* relative position as seen from the ship.

3. Shambles light 107° 40′, Portland High Light 110° 15′ Breakwater light.

4. Shambles light 110° 40′, Breakwater light 88° 0′ White Nore.

5. White Nore 54° 30′, Bindon Hill 77° 20′ St. Alban's flagstaff.

Note.—In using the station pointer place the middle leg on the middle object *at first*, and keep it over this until by suitably moving the instrument the three legs are over the three observed points.

In using the tracing-paper take a fresh point on the middle line each time, to avoid confusion of lines.

The places named are *written* in the same relative positions as they appeared from the ship.

Track of a Steam Vessel.

A vessel from an anchorage in Portland Harbour (with the Breakwater lighthouse bearing S.E½E. distant 7 cables, ship's head S.E.) steered E. 1·5 mile and then S.S.W. 3·4 miles, when Portland High Lighthouse bore W.½N. and the Breakwater lighthouse N.¾E. (head S.W. by S.).

Course was then shaped S.W by W.¾W., and when 3·8 miles had been run the position was fixed by cross bearings. Portland High Lighthouse N.¼E., Shambles light vessel E.½N. (head South).

The vessel then steered East, and on running 5·5 miles Shambles light vessel came in line with the Breakwater lighthouse and Portland Bill bore W. by N.½N. (head E.S.E.).

Course was now altered to E.N.E., and when 2 miles had been run the following bearings were observed:

Bindon Hill N.¾E. and Shambles light vessel W. by N.¾N. (head E.N.E.).

The vessel then steered N.¼W., bringing Shambles light vessel four points on the bow, and on running 2·2 miles on this course it came right abeam. Course was again altered to N. by W.¼W. for 4·6 miles, when the Breakwater lighthouse bore N.W. by W.½W. (head N. by W.), or four points on the bow; on continuing a further distance of 1·3 mile the Breakwater lighthouse came right abeam, after which the vessel proceeded to the anchorage.

Note.—The foregoing track and that following should be laid down on Admiralty chart No. 2615.

Track of a Sailing Vessel.

Leaving an anchorage in Portland Harbour with Breakwater lighthouse bearing E.N.E. and Castle (ruins) bearing N.W. by N.

(head North), a vessel under sail and proceeding to the eastward with an easterly wind makes the following compass courses (leeway ½ point on all courses) :

1. N. by E.½E.	2·1 miles
2. S. by E.½E.	6·0 „
3. N.N.E.½E.	3·9 „
4. S.S.E.¼E.	7·2 „
5. N.E.¾N.	5·2 miles
6. S.½E.	4·8 „
7. N.E.¾E.	2·5 „

At the end of each tack the positions were fixed by the following cross bearings :

1. Weymouth green lights in line and Breakwater lighthouse S.½E. (head N.N.E.).

2. Portland High Lighthouse W.½N. and Shambles light vessel S.E.¼E. (head S.S.E.).

3. Breakwater lighthouse W.¼S. and South Holworth N.¾W. (head N.E.).

4. Shambles light vessel N.W.¾W. and St. Alban's flagstaff N.E. by E.½E. (head S. by E.).

5. Bindon Hill (summit) N.N.W.¾W. and St. Alban's flagstaff E. by N. (head N.E.).

6. Shambles light vessel N.W. by W.½W and St. Alban's flagstaff N.E. (head S.).

7. Shambles light vessel W. and St. Alban's flagstaff N. by E.¾E. (head N.E.½E.).

NOTE.—After plotting each course the corresponding position by cross bearings is to be laid down and the next course laid off from *that* position.

CHAPTER V.

The sailings—Parallel sailing—Plane sailing—Middle latitude sailing—Departure—Mercator sailing—Traverse sailing—The day's work—Soundings.

The sailings

HAVING in the preceding chapters illustrated the methods by which, from the indications of the compass and log line, the true course and distance made good from day to day are obtained, and also how this course and distance is *graphically* represented on charts, we now proceed to the methods by which *mathematically* the daily position of a ship is *calculated* from the above data. These methods are known as the SAILINGS, and comprise Parallel sailing, Plane sailing, Middle latitude sailing, and Mercator sailing.

Of these, Middle latitude sailing is ordinarily employed for short distances, such as a day's run, and Mercator sailing for longer distances; Parallel sailing is rarely applicable, but the principles of both that and Plane sailing must necessarily be understood before explaining Middle latitude and Mercator sailings.

In practice the graphic method and that by calculation are *both* employed at the same time; on leaving port at the commencement of a voyage the position is fixed by one of the methods described in Chapter IV., and the courses and distances laid down on the chart each day, commencing from that position.

Departure course

The *calculated* positions also originate in the same way, but it is usual from the original fixed position at starting to take off the bearing and distance of some point of land whose latitude and longitude are known (technically termed *taking departure*), and consider the vessel as having sailed that distance from such point on an opposite course to the observed bearing.

Or more frequently the latitude and longitude of the position of the ship fixed outside the harbour are taken off the chart.

All problems on the sailings in practice resolve themselves into one or other of these two cases:

CASE 1.—Calculating the course and distance between two known positions.[1]

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[1] **In practice the course is only calculated to the nearest degree, and the distance to the nearest mile.**

CASE 2.—Finding the lat. and long. of a ship which has sailed on certain courses and distances *from* a known position.

These problems can be solved by plane trigonometry and also by inspection from the Traverse table given in all nautical tables, and of which a description will be found on p. 71; as this latter method is very short it is always advisable to use it as a check on the accuracy of calculation.

PARALLEL SAILING.

Parallel sailing[1] is the art of sailing on a parallel of latitude, or when the true course is either East or West, the vessel's position changing in longitude but not in latitude.

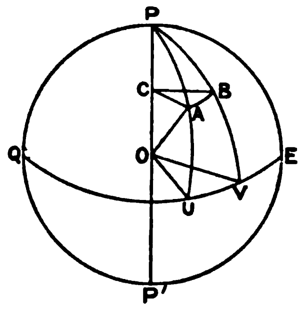
Before the invention of chronometers the *latitudes* of positions could be determined much more accurately than their *longitudes*, and it was the usual custom to approach a port by steering so as to get into the desired latitude, and then shape a true course East or West on a parallel of latitude, applying the principles of parallel sailing.

Although the precision with which geographical positions have now been determined renders this mode of procedure unnecessary, we retain parallel sailing as illustrating an important principle on which middle latitude sailing is founded.

Theory

When a ship sails on a parallel of latitude from a given meridian, *she changes her longitude only*; but *the distance between any two Meridians* varies according to the parallel sailed on.

FIG. 33.



Let P be one of the earth's poles (fig. 33), A and B two places on the same parallel of latitude.

Through A and B draw the meridians P A U and P B V, meeting the equator at U and V.

Then the *diff. long.* between A and B is the arc of the equator U V.

The arc A B is the *distance* between A and B.

The latitude of A or of B is represented by A U or B V.

Let O be the earth's centre and C that of the parallel A B; O U, O V, and O A, radii of the sphere; C A and C B, radii of the parallel of latitude.

---

[1] Described in Norwood's *Seaman's Practice*, 1637.

Then since by a property of geometry the circumferences of circles are to each other as their radii, and the same is true of equivalent arcs of those circles, it follows that—

$$\mathrm{UV} : \mathrm{AB} :: \mathrm{OU} : \mathrm{CA}$$
$$:: \mathrm{OA} : \mathrm{CA}$$

Therefore
$$\frac{\mathrm{AB}}{\mathrm{UV}} = \frac{\mathrm{CA}}{\mathrm{OA}}$$
$$= \sin \mathrm{COA} = \cos \mathrm{AOU}$$

But AOU, the angle measured by the arc AU, is the latitude of the point A,

Hence, therefore—

$$\frac{\text{Distance}}{\text{Diff. long.}} = \cos \text{lat.},$$

or

$$\text{Distance} = \text{diff. long.} \times \cos \text{lat.}$$

CASE 1.—1. Required the true course and distance from A to B. **Examples**

lat. A...80° N.	long. A...3° 50′ E.
„ B...80 N.	„ B...6 10 W.
long. from.........3° 50′ E.	dist. = diff. long. × cos lat.
„ in.............6 10 W.	L.[1] cos lat.........9·239670
10 0	log. diff. long......2·778151
60	„ dist.............2·017821
diff. long. = 600 W.	∴ dist. 104·2′.

The true course is evidently West.

Required the true course and distance from A to B in each of the following examples :

	Lat. A and B.	Long. A.	Long. B.	Answers. True Course & Dist.
2.	70° 10′ S.	15° 10′ E.	22° 15′ E.	East 144·2′
3.	50 48 N.	5 0 W.	5 0 E.	East 379·2
4.	50 10 N.	40 25 W.	50 10 W.	West 374·7
5.	48 10 N.	100 0 W.	110 0 W.	West 400·2
6.	75 13 N.	15 20 E.	0 0	West 234·7
7.	80 15 N.	179 0 W	176 0 E.[2]	West 50·8

CASE 2.—8. Sailed from A East (true) 1,000 miles; required **Examples**

the latitude and longitude in. Lat. A...32° 10′ S.; long. A...28° 42′ W.

---

[1] This is the abbreviation for *tabular* logarithm, which is the value of the true logarithm (l) increased by 10.

[2] As difficulty is sometimes experienced by the beginner in those cases where the longitude *from* and that *in* are of different names, we illustrate this example by the

diff. long. = dist. × sec lat.
lat. in = lat. from = 32° 10′ S.
L. sec lat.....................10·072372
log. dist............ ....3·000000
„ diff. long...............3·072372
∴ diff. long. = 1181′, or 19° 41′ E.
long. from.................28 42 W.
∴ long. in...... .... 9 1 W.

Required the latitude and longitude in in each of the following examples:

			Answers.	
True Course and Dist.	Lat. from	Long. from	Lat. in	Long. in
9. East 492·5′	52° 10′ N.	0° 29′ W.	52° 10′ N.	12° 54′ E.
10. East 1752	60 0 N.	5 10 W.	60 0 N.	53 14 E.
11. East 560	57 32 N.	13 5 W.	57 32 N.	4 18 E.
12. West 740	60 0 N.	50 0 W.	60 0 N.	74 40 W.

By traverse table

Since $\frac{\text{dist.}}{\text{diff. long.}}$ = cos lat. these elements may be considered as forming parts of a right-angled plane triangle, whence, as shown on page 71, if any two of them are given the third can be taken by inspection from the traverse table.

In example 1 with course 80° (equal to the lat.) and distance 150′ (equal to one-quarter of the diff. long., since the tables do not extend to 600′) the corresponding diff. lat. (equivalent to one-

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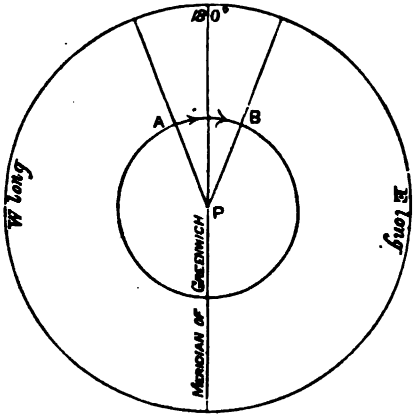
following figure, where an observer is supposed to be looking down on a globe from a point vertically over the North Pole.

Fig. 34.



quarter of the *distance*) is found to be 26′, which multiplied by 4 gives 104′, agreeing with the calculation on page 67 (fig. 35).

Similarly in example 8 with course 32° and diff. lat. 100′ (being one-tenth of the given distance) the corresponding distance is found to be 118′, which multiplied by 10 gives 1180′ as diff. long., agreeing with the results of calculation on page 68 (fig. 36).

Fig. 35.

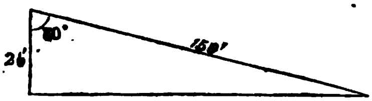
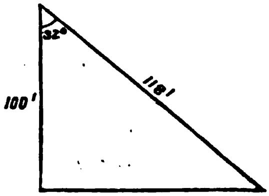


Fig. 36.



Plane Sailing.

Plane Sailing is the method of calculating a ship's place, and determining the course and distance traversed, by aid of the properties of a plane triangle. Plane sailing, however, does not assume the earth's surface to be a plane, nor does it involve any errors of importance even in great distances.

The relations, therefore, which subsist between the four quantities, course, distance, difference of latitude, and departure, can all be determined by the formulæ of plane trigonometry.

The *curved* lines on the sphere representing the diff. lat., departure (*see* page 70), and distance are considered to be *straight* lines of equal length to them, thus forming with the course a right-angled plane triangle.

Let P be one of the earth's poles (fig. 37); A and Z two places on the earth's surface; A Z, the rhumb line joining them.

Take A, B, C, D . . . W, X, Z, Z, a number of points on the rhumb line, and through these draw the meridians P E, P F, P G, . . . P L, P M, P N, P Q, and also B H, C I, D J . . . X R, Y S, Z T arcs of parallels of latitude intercepted by the adjacent meridians.

Departure

When the number of points, A, B, C, D . . . W, X, Y, Z is infinitely increased, the triangles A B H, B C I, &c., ultimately become equiangular right-angled plane triangles, since each contains the course, namely the angles H A B, I B C, J C D . . . T Y Z; also the sum of the arcs B H, C I, D J . . . X R, Y S, Z T becomes the departure.[1]

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[1] The reader should clearly recognise the difference between *departure* and *diff. long.*; departure is the sum of an infinite number of small arcs of parallels of latitude, and only becomes identical with diff. long. at the equator; it is also identical with the *distance* in parallel sailing.

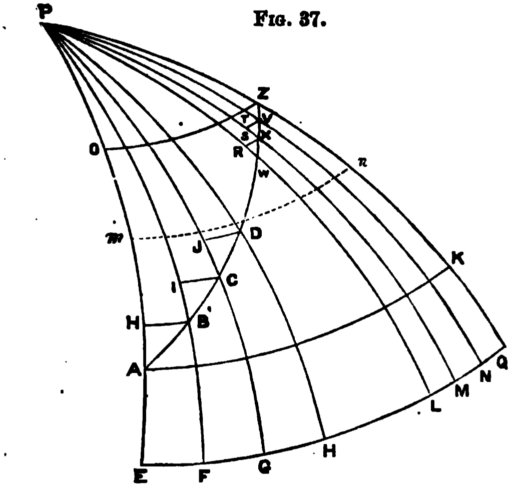
(1) *To prove that* DEPARTURE = DIST. × SIN COURSE.

In the triangle ABH, BH = AB . sin BAH = AB . sin course
„ BCI, CI = BC . sin CBI = BC . sin course
. . . .
„ XYS, YS = XY . sin YXS = XY . sin course
„ YZT, TZ = YZ . sin ZYT = YZ . sin course

By addition BH + CI + . . . + YS + ZT = (AB + BC + . . . + XY + YZ) sin course.

But BH + CI . . . YS + ZT = departure,
and AB + BC . . . XY + YZ = distance,
∴ departure = dist. × sin course.

Fig. 37.



(2) *To prove that* TRUE DIFF. LAT. = DIST. × COS COURSE.

The same construction being made,

In the triangle ABH, AH = AB . cos BAH = AB . cos course
„ BCI, BI = BC . cos CBI = BC . cos course
. . . .
„ XYS, XS = XY . cos YXS = XY . cos course
„ YZT, YT = YZ . cos TYZ = YZ . cos course

By addition AH + BI + . . . + XS + YT = (AB + BC + . . . + XY + YZ) cos course.

But AH + BI + . . . + XS + YT = true diff. lat.,
and AB + BC + . . . + XY + YZ = distance,
∴ true diff. lat. = dist. × cos course.

In all cases where the course is neither east nor west every successive mile of departure bears a relation to the diff. long., which varies as the latitude varies (*see* page 66, and Inman's Tables, Nos. 4 and 5).

The distance, diff. lat., and departure may evidently be represented by a right-angled plane triangle, one angle of which is equal to the course.

(3) *To prove that* tan course$=\dfrac{\text{departure}}{\text{true diff. lat.}}$

The same construction being made,

In triangle A B H, B H$=$A H . tan B A H$=$A H tan course

„ B C I, C I $=$B I . tan C B I$=$B I tan course

. . . .

„ X Y S, Y S$=$X S . tan Y X S$=$X S tan course

„ Y Z T, T Z$=$Y T . tan T Y Z$=$Y T tan course

By addition B H$+$C I$+$ . . . $+$Y S$+$T Z$=$(A H$=$B I$+$ . . . $+$X S $+$Y T) tan course

But B H$+$C I$+$ . . . $+$Y S$+$T Z$=$departure,

and A H$+$B I$+$ . . . $+$X S$+$Y T$=$true diff. lat.,

∴ departure$=$true diff. lat. × tan course,

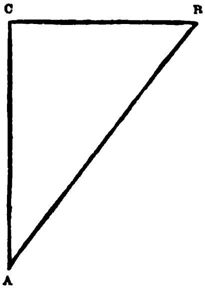
or

$$\text{Tan course} = \frac{\text{departure}}{\text{true diff. lat.}}$$

Traverse table

The traverse table contains the true difference of latitude and departure corresponding to every course from 0° to 90°; and for every distance from one nautical mile to 300.

FIG. 38.



It is constructed as follows. A course and distance being assumed, the true difference of latitude and departure may be computed for that course and distance: thus, in the triangle A B C, fig. 38, right-angled at C, if the angle C A B represents a given course, and A B a given distance, the side A C will be the true difference of latitude, and C B the departure corresponding to that course and distance.

Given course$=$25° and distance$=$26 miles; compute the corresponding true diff. lat. and departure.

In the triangle C A B, let A$=$25° and A B$=$26′,

then true diff. lat.$=$A C$=$A B . cos A$=$26′ . cos 25°$=$23·56

and dep.$=$B C$=$A B sin A$=$26′ sin 25°$=$10·99.′

When the difference of latitude and departure are computed in this manner from 0° to 45°, the diff. lat. and dep. for courses greater than 45° may be found by interchanging the titles to the columns. Thus, let it be required to find the diff. lat. and dep. for course 65°, and distance 26 miles:

diff. lat. for 65°$=$26′ cos 65°$=$26′ sin 25°$=$*dep.* for 25°

dep. for 65°$=$26′ sin 65°$=$26′ cos 25°$=$*diff. lat.* for 25°.

Thus it appears that diff. lat. and dep. for any course are the dep.

and diff. lat. respectively for the *complement* of that course; this will easily explain the reason why the quantities are tabulated in the following manner.

Form of Traverse Table.[1]

DISTANCE TWENTY-SIX MILES.			
Course	Diff. lat.	Dep.	Course
1°	26·0	0·5	89°
2	26·0	0·9	88
25	23·6	11·0	65
45	18·4	18·4	45
Course	Dep.	Diff. lat.	Course

The name Traverse table is given from its application to problems in Navigation, when the sides of the triangle considered are named as in the table, but it should be fully recognised that it is perfectly general in its application to the solution of *any* right-angled plane triangle.

For instance, suppose that the length of the hypothenuse is 200 feet and one of the acute angles is 40°; then on referring to the Traverse table (Inman's Tables, No. 3) and looking for the number 200 in the upper horizontal line, we find directly below it on the same line which has 40° on the left these numbers, 153·2 and 128·6, while 50° is on the right of the same line. This means that the other two sides of the triangle considered are 153·2 feet and 128·6 feet long (the longer side being of course opposite the larger angle), and the remaining angle is 50°.

Similarly if we know the two sides are 110·6 and 51·6 *miles* long, we look in the traverse table for the place where these numbers are side by side (Inman's Tables, No. 3), when the number on the upper line vertically above these numbers (122) gives the length of the hypothenuse in miles, while the angles 25° and 65° on the right and left are the values of the other angles of the triangle.

Note.—A convenient and practical use of the traverse table in setting courses may here be noticed.

Suppose a light visible 20 miles is just sighted on the horizon, and it is desired to pass five miles outside it. From the traverse table with 20 miles distance and five

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[1] A traverse table calculated for each of eight points is given in Eden's translation of Cortes's *Art of Navigation*, 1609.

miles departure the corresponding course is seen to be 15°, hence the required true course will be that number of degrees right or left of the true bearing of the light, as the case may be; an error of two miles more or less in the estimated distance will practically make but little difference.

Traverse sailing

The operation of reducing a number of different courses and distances by means of the Traverse table to the one course and distance they are all equivalent to is termed 'Traverse sailing.'

Examples on plane sailing

13. From a certain known position a vessel sails S. 42° W. true 56 miles. Required the diff. lat. and departure.

Ans. D. lat. 41·6′ S.; dep. 37·5′ W.

14. A vessel is 126 miles N. 81° E. true from a lighthouse. Required the diff. lat. and departure.

Ans. D. lat. 19·7′ N.; dep. 124·4′ E.

15. From a certain known position a vessel sails on the following true courses: N. 26° W. 41 miles, N. 82° W. 27 miles, and N. 4° W. 38 miles. Required the total diff. lat. and departure.

Ans. D. lat. 78·6′ N.; dep. 47·4′ W.

16. From a position in lat. 31° 20′ S., long. 4° 12′ E., a vessel sails on the following true courses: N.E. 81 miles, S.S.E. 27 miles, and W.S.W. 46 miles. Required the lat. arrived at and the departure.

Ans. Lat. 31° 5·2′ S.; dep. 25·1′ E.

MIDDLE LATITUDE SAILING.

On reference to fig. 37 it will be seen that the departure is evidently *intermediate* in value between the lengths of O Z and A K, and the assumption made in middle latitude sailing[1] is that it is *equal* to the length of *m n*, an arc of a parallel of latitude drawn midway between the latitude of A and that of Z, from which the name of Middle (or mean) latitude sailing is derived.[2]

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[1] Described in Norwood's *Seaman's Practice*, 1637, but first suggested by Gunter in 1628.

[2] The departure, being evidently *greater* than O Z and *less* than A K, must be *equal* to the arc of *some* parallel of latitude drawn between A and Z and intercepted between the meridians passing through these places; this latitude (either where the two places are on the same or opposite sides of the equator) may be found by equating the values of tan course as derived from Mid. lat. and Mercator sailing (*see* page 78). This gives

$$\cos \text{true mid. lat.} = \frac{\text{true diff. lat.}}{\text{mer. diff. lat.}}$$

The ordinary assumption that the mid. lat. is the arithmetical mean of the two lats. always gives diff. long. somewhat two small, but not sufficiently so to cause practical error in those cases to which it is recommended to restrict the employment of this sailing (*see* page 65).

Tables giving this correction to the mean of the two latitudes were published by Workman in 1805, and are also inserted in Inman's Tables (No. 6), but are not much

Upon this assumption it will be shown that, with a known course and distance between A and Z, the length of *m n* can easily be obtained, from which the length of E Q, the difference of longitude, is found, as in parallel sailing.

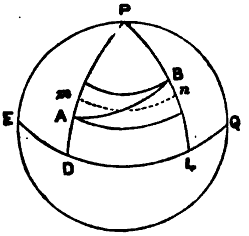
When the places considered are on the *same* side of the equator, the middle latitude employed is half the sum of their latitudes.

Where the places considered are on *opposite* sides of the equator, since the application of middle latitude sailing is generally limited to a day's run of from 300 to 400 miles, no sensible error is involved by considering the departure and diff. long. equal, but if longer distances are considered the departure must be found for the *true* mid. lat. obtained from the formula, cos true mid. lat. $=\dfrac{\text{true diff. lat.}}{\text{mer. diff. lat.}}$ (*see* note 2 on preceding page).

Theory

Let P be one of the earth's poles (fig. 39), A and B two places on the earth's surface. Through A and B draw the meridians P D and P L, meeting the equator at D and L. The diff. long. between A and B is the arc of the equator D L.

Fig. 39.



Let *m n* be the arc of the parallel of latitude which is equal to the departure between A and B.

Now, by the principles of spherical trigonometry (*see* page 66)

$$\frac{\text{the arc } m\ n \text{ of the parallel}}{\text{the arc D L of the equator}}=\cos \text{D } m$$

that is

$$\frac{\text{departure}}{\text{diff. long.}}=\cos \text{mid. lat.}$$

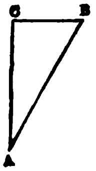
$$\therefore \text{dep.}=\text{diff. long.} \times \cos \text{mid. lat.}$$

Example

(17) Find the course and distance from A to B, having given

lat. A......14° 40′ N. long. A......56° 40′ E.
„ B......18 20 N. „ B......60 10 E.

Fig. 40.



In the right-angled *plane* triangle A B C (fig. 40), let A C represent the true diff. lat., and C B the departure (found from the above formula); then A B = distance, and angle A the course required.

In triangle C A B tan A $=\dfrac{\text{B C}}{\text{A C}}$,

used, since, except in very high latitudes, the error involved on the ordinary assumption is not great in those cases usually solved by Middle latitude sailing.

$$\text{or tan course} = \frac{\text{dep.}}{\text{true diff. lat.}} = \frac{\text{diff. long.} \times \cos \text{ mid. lat.}}{\text{true diff. lat.}};$$

diff. long. = 210′; mid. lat. = 16° 30′; true diff. lat. = 220′;

∴ course = N. 42° 28′ E.

In triangle CAB, $\frac{AB}{AC} = \sec A$; ∴ AB = AC . sec A,

or dist. = true diff. lat. × sec course; ∴ dist. = 298 miles.

From the same triangle CAB it is evident that the distance may be found from the formula dist. = dep. × cosec course. When the course is less than 45° the first formula should be used, and when the course is more than 45° the second formula, the reason being that when the course is very small or very large any errors which might exist in the departure or true diff. lat. would be multiplied by a large factor and become magnified in their effects on the corresponding distance.

Examples

CASE 1.—18. Required the true course and distance from A to B, by middle latitude method.

lat. A 50° 25′ N.   long. A 27° 15′ W.
„ B 47 12 N.   „ B 30 20 W.

$$(1)\ \text{tan course} = \frac{\text{diff. long.} \times \cos \text{ mid. lat.}}{\text{true diff. lat.}}$$

(2) dist. = true diff. lat. × sec course.

lat. A 50° 25′ N.	..........50° 25′ N.	long. A 27° 15′ W.	
„ B 47 12 N.	..........47 12 N.	„ B 30 20 W.	
3 13	2)97 37	3 5	
60	mid. lat. 48 48	60	
T. D. lat. 193 S.		diff. long. 185 W.	
L. cos mid. lat.	9·818681	L. sec course	10·072849
log. diff. long. ...	2·267172	log. T. D. lat...	2·285557
	12·085853	„ dist.........	2·358406
log. T. D. lat. ...	2·285557	∴ dist.	= 228·2′
L. tan course..	9·800296	∴ course	= S. 32° 16′ W.

Required the true course and distance from A to B in each of the following examples:

	Lat. from and Lat. in.	Long. from and Long. in.	Answers. Course and Dist.
19	lat. A 49° 52′ S.	long. A 17° 22′ W.	N. 26° 40′ E.
	„ B 42 13 S.	„ B 11 50 W.	513·6′
20	lat. A 21 15 S.	long. A 0 30 W.	S. 14° 37′ E.
	„ B 30 27 S.	„ B 2 10 E.	570·5′
21	lat. A 60 15 S.	long. A 14 55 E.	S. 32° 50′ E.
	„ B 65 36 S.	„ B 22 30 E.	382′

Examples

CASE 2.—22. Sailed S. 37° 10′ W. true from A, 472·6 miles. Required lat. in and long. in.

lat. A 27° 20′ S. long. A 25° 12′ W.

(1) diff. lat. = dist. × cos course.

(2) diff. long. = dist. × sec. mid. lat. × sin course.

L. cos course...9·901394	L. sin course...9·781134
log. dist. ....2·674494	log. dist. ....3·674494
„ T. D. lat. ...2·575888	L. sec mid. lat. 10·064531
∴ T. D. lat. 376·6′	log. diff. long. ...2·520159
or 6° 17′ S.	∴ diff. long. 331·3′
lat. from 27 20 S.	or 5° 31′ W.
„ in 33 37 S.	long. from 25 12 W.
2)60 57	„ in 30 43 W.
mid. lat. 30 28	

Required the latitude and longitude in each of the following examples having sailed from A as follows :

	True Course and Dist. from A.	Lat. A.	Long. A.	Answers. Lat. in	Answers. Long. in.
23.	N. 25° 42′ W. 427·3′	64° 10′ N.	40° 15′ W.	70° 35′ N.	48° 17′ W.
24.	S. 48 58 W. 175·2	47 10 N.	32 15 W.	45 15 N.	35 26 W.
25.	N. 34 48 W. 383·7	50 25 N.	3 40 E.	55 40 N.	2 24 W.

FIG. 41.

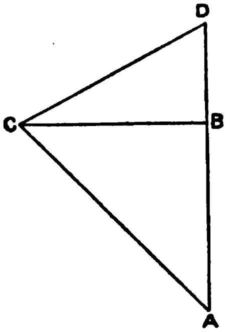
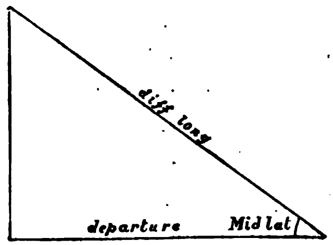


FIG. 42.



By traverse table

The formula of mid. lat. sailing can also be illustrated by the construction of two right-angled triangles in the following manner:

Let A B, B C, A C, and the angle at A, represent the diff., lat., dep., Distance, and Course, referring to two places A and C. From the point C in C B draw the line C D at an angle equal to the value of the mid. latitude between the two places A and C, and produce A B to meet it in D.

$$\text{Then} \quad \frac{\text{C B}}{\text{C D}} = \cos \text{B C D} = \cos \text{ mid. lat.}$$

$$\text{But} \quad \frac{\text{dep.}}{\text{D long.}} = \cos \text{ mid. lat.}$$

Hence, since C B, by construction, represents the departure, C D must also represent the D. long. corresponding to it.

In example 18, with course 49° (48° 48′) and distance 185′ (equal to the diff. long.), the corresponding diff. lat. is seen to be 121·4′, which is equal to the departure (fig. 42)

Then, on referring to the traverse table, with this departure and diff. lat. 193′, the corresponding course and distance is seen to be S. 32° W. 228 miles, agreeing with the results obtained by calculation.

Similarly, in example 22, the departure corresponding to a course of 30° and distance 118′ (or a quarter of the true distance) is looked out, and this departure turned into diff. long. as above.

MERCATOR SAILING.

Mercator sailing[1] is based on the same principles as those involved in the construction of Mercator's charts, and is a strictly accurate method of solving problems in navigation by the application of plane trigonometry.

Theory

In a Mercator's chart (figs. 43, 44) the meridians are drawn parallel to one another, and therefore the arcs B H, C I, D K, &c., the sum of which equals the departure, are increased and become

FIG. 43.

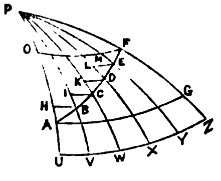
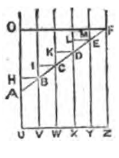


FIG. 44.



equal to the corresponding parts U V, V W, W X, &c., of the equator.

To preserve the similarity of the triangular spaces on the chart (fig. 44), corresponding to the triangles A B H, B C I, C D K, &c., on the globe (fig. 43), the sides A H, B I, &c., must be increased in the same proportion that B H, &c., have been increased (*see* page 31).

Hence when B H + C I + D K + &c., increases and becomes equal to the difference of longitude, the arcs A H + B I + C K + &c., will increase and become equal to what is called the *Meridional difference of latitude* (M. D. lat.), viz. A O.

---

[1] First described in Wright's *Errors in Navigation*, 1599.

Hence the mer. diff. lat., diff. long,, and the line into which the distance A F has been expanded, form a right-angled plane triangle in which

$$\frac{OF}{OA} = \tan OAF$$

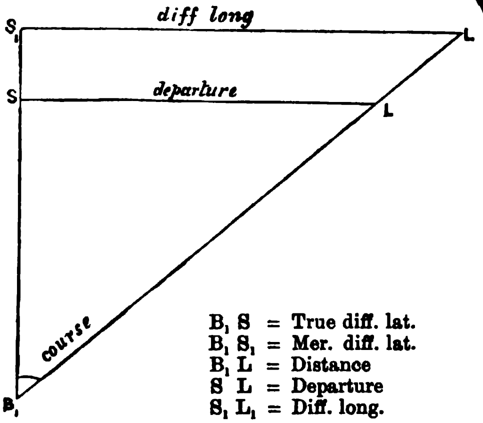
or

$$\frac{\text{diff. long.}}{\text{mer. diff. lat.}} = \tan \text{course}$$

$$\therefore \text{diff. long.} = \text{mer. diff. lat.} \times \tan \text{course}.$$

In the following figure let B_1 and L_1 represent the positions of two places as shown on a Mercator's chart, where the true distance between them (on the globe) has been increased on the chart in the same proportion as their true difference of latitude has been increased; if, while $B_1 S_1$ represents the meridional diff. lat. and $B_1 S$ the true diff. lat., a line is drawn through S parallel to $S_1 L_1$, then $B_1 L$ will be equal to the distance between the places (on the globe), the angle $S_1 B_1 L_1$ being, as before shown, the true course.

FIG. 45.



$B_1 S$ = True diff. lat.
$B_1 S_1$ = Mer. diff. lat.
$B_1 L$ = Distance
$S L$ = Departure
$S_1 L_1$ = Diff. long.

The triangles $L_1 S_1 B_1$ and $L S B_1$ being both right-angled plane triangles, the formulæ used in Mercator sailing are readily deduced.

Formulæ

$$\text{Case 1}\begin{cases}\text{Tan course} = \dfrac{\text{diff. long.}}{\text{mer. diff. lat.}}\\ \text{distance} = \text{true diff. lat.} \times \text{secant course}\end{cases}$$

$$\text{Case 2}\begin{cases}\text{true diff. lat.} = \text{distance} \times \text{cosine course}\\ \text{diff. long.} = \text{true diff. lat.} \times \text{tan course}\end{cases}$$

Examples

Case 1.—26. Required the true course and distance from A to B.

lat. A 45° 15′ N.	long. A 35° 26′ W.
„ B 47 10 N.	„ B 32 15 W.

(1) $\tan \text{course} = \dfrac{\text{diff. long.}}{\text{mer. diff. lat.}}$

(2) dist. = true diff. lat. × sec course.

	Mer. parts.	
lat. A...45° 15′ N.	3051·2 N.	long. A...35° 26′ W.
„ B...47 10 N.	3217·4 N.	„ B...32 15 W.
1 55	M. D. lat. 166·2 N.	3 11
60		60
T. D. lat. 115 N.		diff. long. 191 E.
log. diff. long. + 10..12·281033		L. sec course 10·182767
„ M. D. lat.........2·220631		log. T. D. lat....2·060698
L. tan course......10·060402		„ dist..........2·243465
∴ course = N. 51° 58′ E.		∴ distance = 175′.

27. Required the true course and distance from A to B.

lat. A...41° 20′ S.	long. A...12° 10′ W.
„ B...49 50 S.	„ B... 1 5 E.

	Mer. Parts.	
lat. A...41° 20′ S.	2728	long. A...12° 10′ W.
„ B...49 50 S.	3459	„ B... 1 5 E.
8 30	M. D. lat. 731	13 15
60		60
T. D. lat. 510		diff. long. 795 E.
log. diff. long. + 10..12·900367		L. sec course 10·169491
„ M. D. lat......... 2·863917		log. T. D. lat...2·707570
L. tan course......10·036450		„ dist............2·877061
∴ course = S. 47° 24′ E.		∴ distance = 753′.

Required the true course and distance from A to B in each of the following examples, by Mercator's method:

	Lat. from and Lat. in.	Long. from and Long. in.	Answers. Course and Distance.
28	lat. A 49° 52′ S.	long. A 17° 22′ W.	course N. 26° 36′ E.
	„ B 42 13 S.	„ B 11 50 W.	dist. 513·3 miles.
29	lat. A 49 10 N.	long. A 29 17 W.	course N. 37° 48′ W.
	„ B 56 45 N.	„ B 39 5 W.	dist. 576 miles.
30	lat. A 50 48 N.	long. A 1 10 E.	course N. 41° 55′ W.
	„ B 52 35 N.	„ B 1 25 W.	dist. 144 miles.
31	lat. A 58 24 N.	long. A 4 12 W.	course N. 32° 34′ E.
	„ B 63 17 N.	„ B 2 13 E.	dist. 347·6 miles.

	Lat. from and Lat. in.	Long. from and Long. in.	Answers. Course and Distance.
32	lat. A 2° 37′ N.	long. A 100° 42′ W.	course S. 75° 12′ W.
	„ B 0 0	„ B 120 36 W.	dist. 614·4 miles.
33	lat. A 3 30 N.	long. A 33 40 E.	course S. 42° 32′ E.
	„ B 4 10 S.	„ B 40 42 E.	dist. 624 miles.

Examples

Case 2.—34. Sailed from A, N. 37° 10′ E. true, 472·6 miles. Required the latitude and longitude in.

lat. A 27° 20′ N. long. A 25° 12′ E.

(1) true diff. lat. = dist. × cos course.

(2) diff. long. = mer. diff. lat. × tan course

L. cos course	9·901394		L. tan course	9·879740
log. dist..........	2·674494		log. M. D. lat.	2·641474
„ T. D. lat.	2·575888		„ diff. long.	2·521214
∴ T. D. lat. = 376·6′			diff. long. = 332·1′	
or 6° 17′ N.		Mer. Parts.	or 5° 32′ E.	
lat. from	27 20 N.......	1706 N.	long. from	25 12 E.
„ in...	33 37 N.......	2144 N.	„ in...	30 44 E.
M. D. lat.		438		

35. A ship in latitude 27° 0′ S. and longitude 123° W. sailed S.S.E.½E. (or S. 28° 7′ 30″ E.) true 150 miles. Required the latitude and longitude in.

L. cos course	9·945430		L. tan course ...	9·727957
log. dist.	2·176091		log. M. D. lat....	2·176091
„ T. D. lat.	2·121521		„ diff. long. ...	1·904048
T. D. lat.	= 132·3′		∴ diff. long. ...	= 80·1′ or
or	2° 12′ S.	Mer. Parts.		1° 20′ E.
lat. from	27 0 S.	1683 S.	long. from	123 0 W.
„ in	29 12 S.	1833 S.	long. in......	121 40 W.
M.D. lat.		150		

Required the latitude and longitude in each of the following examples, having sailed from A as follows:

	True Course and Dist. from A.	Lat. A.	Long. A.	Answers.	
				Lat. in.	Long. in.
36.	N. 26° 36′ E. 513·5′	49° 52′ S.	17° 22′ W.	42° 13′ S.	11° 50′ W.
37.	S. 48 58 W. 175·2	47 10 N.	32 15 W.	45 15 N.	35 26 W.
38.	N. 29 10 E. 373·4	52 10 N.	17 32 W.	57 36 N.	12 15 W.
39.	S. 37 7 E. 370·0	70 14 S.	25 30 E.	75 9 S.	38 5 E.
40.	N. 47 47 E. 272·4	50 15 S.	15 10 E.	47 12 S.	20 16 E.

By traverse table

Problems in Mercator sailing can be solved by the traverse table, but, as this sailing is generally employed when calculating courses and distances between places far apart, the restricted limits of that table would involve some error in its application.

Comparison of sailings

In drawing a comparison between the results obtained by solving problems in navigation by Middle latitude and Mercator sailings it will be observed that, if the application of the former method (which is only approximately accurate) is restricted to an ordinary day's run of less than 350 miles, no appreciable error is involved when the latitude is less than 50° (north or south). This may be illustrated by solving the following two problems in both ways and comparing the results:[1]

41. A vessel sails from lat. 44° 00′ N., long. 30° 00′ W. on a true course N. 10° E. 350 miles. Required the lat. and long. in.

42. Find the true course and distance between the following positions: lat. 43° 30′ N., long 30° 00′ W., and lat. 55° 10′ N., long. 31° 30′ W.

In higher latitudes Mercator sailing, which is a rigidly accurate method, should be used in computing the ship's position, except when the course is large;[2] in all cases when the latitude exceeds 50° the diff. long. should be found for each course *separately*.

In working the course and distance from a ship's position to that of a port of destination, or any similar question where a considerable distance is involved, Mercator sailing should be employed, except when the places are nearly in the same latitude, when mid. lat. sailing is more accurate, for the reason given in the footnote.

NOTE.—Mercator sailing to be *rigidly* accurate should take into consideration the fact of the real form of the earth being that of an oblate spheroid and not that of a sphere; this is allowed for in the accurate construction of *charts*, but that such correction is unnecessary in *navigation* is seen by a comparison between the course and distance between the Lizard and Barbados, computed on both assumptions:[3]

On the sphere	S. 50° 41′ 00″ W.	3,499 miles
On the spheriod	S. 50 51 50 W.	3,502 „

Current sailing

When navigating in those localities where the direction and rate of the tidal streams are well known, the course is shaped so as to make allowance for the probable distance through which the vessel will be set by these streams.

---

[1] These examples are selected as comprising the limits of ordinary navigation of men-of-war; the errors of mid. lat. sailing in lower latitudes and with larger courses will of course be less. The results will *exactly* accord if the *corrected* mid. lat. be employed (*see* footnote, page 73).

[2] This was first pointed out by Workman in 1805, and arises from the fact that when the course is *large* any error in the mer. diff. lat. is greatly augmented when it is turned into diff. long., being multiplied by the tangent of an angle near 90°.

In mid. latitude sailing it will have been remarked that it is the *departure* which is converted into diff. long., and in Mercator sailing the mer. diff. lat.

[3] From *Robertson's Navigation*, ed. 1796.

Thus in fig. 46 if a vessel is steaming at the rate of 10 knots in a locality where the tide is probably running 3 knots East (true), it is evident that in order to steer North (true) the course from A must be shaped so much to the *westward* of North that if A C be taken as 10 knots B C will equal 3 knots.

In this simple case the desired course may be taken by inspection from the traverse table.

As another illustration, if the true course across a channel is W.S.W. and the distance 70 miles, a vessel steaming 10 knots will probably experience during the passage *both* tidal streams, and if we suppose that during 6 hours the stream sets North with an average velocity of 4 knots and then South at 2 knots, the whole drift of the vessel will be about 22 miles to the northward.

FIG. 46.

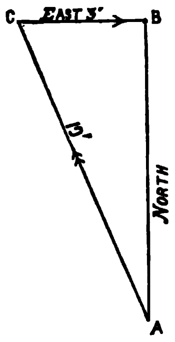
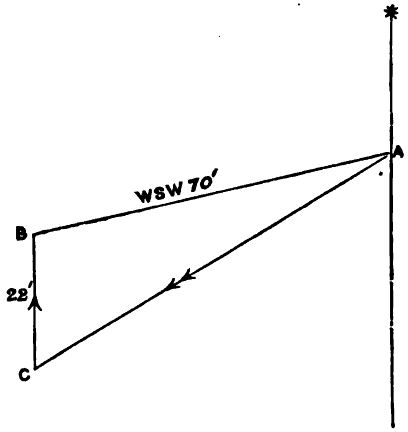


FIG. 47.



It is evident, then, that if A B (fig. 47) represents the true course W.S.W. and distance 70 miles, a vessel starting from A must steer southward of W.S.W., so that if B C be drawn equal to 22 miles and the course shaped for C, she will really arrive at B, though having been apparently directed towards C.

At noon each day from the results of observation and calculation a form is filled up similar to that below.

Course made good	Distance		Latitude	Longitude	Variation allowed	True Bearing and Distance
	Made good	Through Water				
Current			Obs. D. R.	Chr. D. R.		

Ship's Log Book

The data from which these particulars are obtained is kept as a permanent record on board every ship in the 'Ship's Log Book,' together with all other details affecting her navigation, such as the deviation, leeway, wind and weather, bearings and angles observed, and the results of astronomical calculations hereafter described.

Day's work

The necessary computations for doing this are collectively known as the DAY'S WORK. The course and distance made good are calculated (usually by Middle latitude method) between the astronomical positions at noon of the day and that of the preceding day (latitude obs. long. chr.).

The position by dead reckoning (latitude D. R., longitude D. R.) is ascertained by applying to the astronomical position of the previous noon the differences of latitude and longitude caused by sailing on each course.

Current

If a vessel could be accurately steered and the distance run exactly determined, whatever difference existed between the position by dead reckoning and that by astronomical observations would be solely attributable to the action of a current; it is the custom to work or take from the traverse table the course and distance between these two positions, and consider this as the effect of current during the preceding day, but it should be remembered that this result is the united effect of the *true* current and errors incidental to imperfect navigation; it is more correct in practice to ascertain by calculation the effect of the current between each time of obtaining astronomically the ship's position, such a precaution being especially necessary when in localities where the currents are known to be strong and variable.

The terms *set* and *drift* are applied to currents in the same sense as *course* and *distance*; an easterly current meaning one which would set a vessel *to* the eastward.[1]

The column headed *True Bearing and Distance* is filled in by calculating (usually by Mercator sailing) the course and distance between the astronomical position at noon and that of either the place of destination or of some headland, lightship or danger lying near the future track of the ship.

The distance through the water is either the sum of the hourly distances run shown in the log, or the total distance shown by patent log.

---

[1] Our present knowledge of the prevailing currents on the surface of the ocean is embodied in the 'Wind and Current Charts,' published by the Admiralty.

Most useful supplements to these are the more detailed charts of particular localities published at various times by the Meteorological office.

In ships steaming at high speed on courses causing a great alteration of longitude allowance for the change of time must be made in the hourly estimate of speed, but not, of course, in the indication of the patent log.

The variation allowed is that taken from the Variation chart.

Practical rule

The method of working a day's work is illustrated by the following rules and example: this latter is selected as an exercise in applying *every* correction to the courses, which in the present case should be estimated in *points*, the large amount of leeway (known only by *estimation*) rendering working to *degrees* superfluous; under circumstances where there is *no* leeway the latter method should always be adopted.

Rule (1) Correct each course for variation, deviation, and leeway; thus find the true courses, and arrange the same in a tabular form, as in the example, page 86. Add together the hourly distances sailed on each course, and insert the same in this table opposite the true course.

(2) Take out of the traverse table the true difference latitude and departure for each course and distance, putting them down in the columns headed with the same letters as in the course. Previously to opening the traverse table, fill up the columns of true difference latitude and departure not wanted by drawing horizontal lines; this will frequently prevent mistakes.

(3) If the ship does not sail from a place whose latitude and longitude are known, her bearing by compass and estimated distance from some near object, as a church spire, &c., must be ascertained, and its latitude and longitude taken from the chart. Then the ship is supposed to sail from this known object to her point of departure, her course being the opposite to the bearing of the object from the ship. This course (departure course) must be corrected like the rest for variation and deviation (due to the direction of the ship's head when the bearing was taken), and inserted in the table as an actual course, with the distance of the object at a distance (*see* also page 86).

(4) *To find the latitude in.* The quantities in the four columns of true difference latitude and departure being added up separately, the difference between the north difference of latitude and south difference of latitude, with the name of the greater, will give the *whole* true difference of latitude. The departure is found in a similar manner. Apply true difference latitude to latitude from, so as to obtain the latitude in.

(5) *To find the longitude in.* From the formula diff. long. =

departure × sec mid. lat. (*see* page 74) the diff. long. is found, and from this the longitude in.[1]

The following example, worked out in detail, will be sufficient to explain the operations directed in the following rules:

Example

43. On April 27, at noon, a point of land in 36° 30′ S. and longitude 120° 20′ W. bore by compass E. by N.½N. (ship's head being S. by W. by compass), distant 14 miles; afterwards sailed as by the following log account. Required the latitude and longitude in, on April 28, at noon.

Hours	Knots	Courses	Winds	Lee-way	Deviation	April 27. Remarks
1	2·5	S.W. by W.¼W.	S. by E.	2¼		P.M.
2	3·4					
3	2·3					
4	3·2	W.½S.	S. by W.	2½		
5	4·4					Variation of
6	2·3					compass 20°
7	2·3	N.W. by W.	S.W.	2		E.
8	3·3					
9	4·0					
10	5·4					
11	4·2					
12	4·4					Midnight
1	3·3	N.W.¾N.	W.S.W.	2½		A.M.
2	3·3					
3	3·5					
4	4·2					
5	6·3	W.¼N.	S. by W.	1½		
6	3·7					
7	2·5					
8	5·0					
9	10·0	W.¼S.	S. by E.	0		
10	10·3					
11	8·0	E. by N.½N.	„	0		
12	8·0					Noon

(1) The column in the above table headed 'Deviation' should be formed from the table of deviations (page 21) previously to correcting the courses.

---

[1] In high latitudes (greater than 60°) the diff. long. on *each* course should be found *separately*.

It may in any case be taken by inspection from Table 4, Inman's Tables.

(2) From a table such as below, by writing down the headings, points, courses, &c., over the seven columns which are to be filled in with the corrected courses, &c.

Points	Courses	Distance	Diff. Lat.		Departure	
			N.	S.	E.	W.

(3) *To correct the departure course* which is W. by S.½S. (the opposite bearing to E. by N.½N.).

pts. qrs.
Compass course............6 2 r.S.
variation............1 3 r.
ship's head S. by W. ∴ dev. 0 0 (*See* table, page 21.)
—— 1 3 r.
true course............8 1 r.S.
or 7 pts. 3 qrs. left of N., or W.¼N. dist. 14′.

Insert this course and distance in table below.

Points	Courses	Dist.	Diff. Lat.		Departure	
			N.	S.	E.	W.
7¾	W.¼N.	14·0	0·7	...	...	14·0
8	W.	8·2	...	...	...	8·2
6	W.N.W.	9·9	3·8	...	...	9·2
3¼	N.W.¾N.	23·6	19·0	...	...	14·1
¾	N.¾W.	14·3	14·1	...	...	2·1
6½	W. by N.½N.	17·5	5·1	...	...	16·7
7¾	W.¼S.	20·3	...	1·0	...	20·3
5¾	S.E. by E.¾ E.	16·0	...	6·8	14·5	...
			42·7	7·8	14·5	84·6
			7·8			14·5
		T.D. lat. 34·9 N.			Dep. 70·1 W.	

First Course.—S.W. by W.¼W.

pts. qrs.
Compass course........ 5 1 r. S.
variation..........1 3 r.
deviation.........1 1 l.
—— 0 2
5 3 r. S.
leeway (wind S. by E.)...2 1 r.
true course..............8 0 r. S. or West 8·2′.

The distance 8·2′ is found by adding up the hourly distances until the course is altered, at 4 o'clock. Insert this course and distance in the table.

In a similar way each of the remaining courses is corrected for variation, deviation, and leeway.

(4) *To find difference latitude and departure* for each course and distance, by traverse table.

Enter the traverse table, and take out difference latitude and departure corresponding to 7¾ points, and distance 14 (look out rather 7¾ points and 140 distance, the diff. lat. and dep. for which are 6·9 and 139·8; move the decimal points one place to the left), and put down the results to the nearest tenth, which are 0·7 and 14·0.[1] Insert them in the spaces left unmarked under N. and W.

The second course being W. true 8·2′, the departure will be 8·2 (the same as the distance).

With the third course 6 points and distance 9·9 (looking for 99, and making the proper change in decimal points) the diff. lat. is 3·8′ and dep. 9·2′.

In a similar manner find difference latitude and departure for the other courses.

When the four columns are added up, it appears that the ship has sailed N. 42·7′ and S. 7·8′; therefore upon the whole the true difference latitude is 34·9′ N.; and her departure has been 14·5′ E. and 84·6′ W.; hence the departure made good in the 24 hours is 70·1 W.

Latitude in

(5) *To find the latitude in.* Apply the true difference latitude to the latitude from, in the usual manner, to obtain the latitude in.

Longitude in

(6) *To find the longitude in.* With the latitude from and

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[1] In practice, with such a large amount of leeway, working to decimals would of course be superfluous.

latitude in find middle latitude; then from the formula diff. long. = departure × sec mid. lat. the diff. long. is found, and thence the long. in. Thus:

To find latitude in.	To find longitude in.
T.D. lat.... 0° 35′ N.	L. sec mid. lat...10·093148
lat. from...36 30 S.	log. departure......1·845718
„ in.......35 55 S.	„ diff. long......1·938866
2)72 25	∴ diff. long. = 87′
mid. lat....36 12	or 1° 27′ W.
	long. from . 110 20 W.
	„ in.......111 47 W.

Soundings In this chapter we have considered various methods of finding a ship's position, all of which depend entirely on the accuracy with which the course and distance are estimated, and from which the deduced position of a vessel at any time may be much in error.

Apart from astronomical observations the only means available for judging of the accuracy of the estimated position is (when in soundings and out of sight of land) by comparing the depth of the water with that shown on the chart.

Although a *single* sounding does not usually afford sufficient information to check the assumed position of the ship, the comparison of a continuous *line* of soundings with the depths indicated on the chart will generally at once indicate any considerable error in the supposed position of the vessel. Such a line of soundings can be obtained without stopping the ship by an apparatus patented by Sir W. Thomson in 1874.

Such a line of soundings is most conveniently employed in practice by plotting them on tracing-paper, which may be moved over the chart parallel to the course until a position is found where the depths *obtained* correspond with those given on the chart.

Formulæ used in the sailings.[1]

PLANE SAILING
Departure = distance × sin course.
True diff. lat. = distance × cos course.

PARALLEL SAILING
Distance = diff. long. × cos latitude.
Diff. long. = distance × sec latitude.

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[1] **The student might of course derive these various formulæ himself from the *fundamental* formulæ in each of the sailings, but they are given in their present form for convenient reference.**

Formulæ used in the sailings.—cont.

MID. LAT. SAILING
$$\left\{\begin{array}{l} \text{Tan course} = \dfrac{\text{diff. long.} \times \cos \text{mid. lat.}}{\text{diff. lat.}} \\ \text{Distance} = \text{true diff. lat.} \times \sec \text{course.} \\ \text{Diff. lat.} = \text{distance} \times \cos \text{course.} \\ \text{Diff. long.} = \text{dist.} \times \sin \text{course} \times \sec \text{mid. lat.} \end{array}\right.$$

MERCATOR SAILING
$$\left\{\begin{array}{l} \text{Tan course} = \dfrac{\text{diff. long.}}{\text{mer. diff. lat.}} \\ \text{Distance} = \text{true diff. lat.} \times \sec \text{course.} \\ \text{Diff. lat.} = \text{distance} \times \cos \text{course.} \\ \text{Diff. long.} = \text{mer. diff. lat.} \times \tan \text{course.} \end{array}\right.$$

PRACTICAL EXAMPLES.

In the following series of examples on the sailings the first half of each series relate to positions in the North Atlantic Ocean, and the second half to those in the South Atlantic Ocean. If these charts are accessible, the positions, courses, &c., should be laid down on them and the results compared with those obtained by calculation and by traverse table.

The deviation is to be taken from the table on page 21.

Parallel Sailing.

44. Find the true, magnetic, and compass courses[1] and also the distances between the first and second of each of the following positions:

	Lat.	Long.		Lat.	Long.	Variation.
1.	51° 10′ N.	15° 5′ W.	and	51° 10′ N.	25° 10′ W.	32° W.
Ans.	True co. West; mag. N. 58° W.; comp. N. 38° W.; dist. 379′.					
2.	40 50 N.	20 10 W.	„	40 50 N.	39 50 W.	27 W.
Ans.	True co. West; mag. N. 63° W.; comp. N. 42° W.; dist. 893′.					
3.	30 20 N.	60 14 W.	„	30 20 N.	41 20 W.	10 W.
Ans.	True co. East; mag. S. 80° E.; comp. N. 80° E.; dist. 979′.					
4.	36 19 S.	20 2 W.	„	36 19 S.	1 2 E.	20 W.
Ans.	True co. East; mag. S. 70° E.; comp. East; dist. 1018′.					
5.	45 2 S.	60 1 W.	„	45 2 S.	41 41 W.	10 E.
Ans.	True co. East; mag. N. 80° E.; comp. N. 60° E.; dist. 777′.					
6.	53 4 S.	45 12 W.	„	53 4 S.	60 28 W.	14 E.
Ans.	True co. West; mag. S. 76° W.; comp. N. 82° W.; dist. 550′.					

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[1] If laid down on the chart the *first* magnetic and compass course is the required, the variation for the position sailed from being taken from the Variation chart.

The courses are required to the nearest degree and the distances to the nearest mile.

Find the lat. and long. of position arrived at, having sailed as follows from the following positions:

	Lat.	Long.	True Course.	Distance.
7.	50° 50′ N.	18° 41′ W.	West	311 miles
Ans.	Lat. 50° 50′ N.; long. 26° 53′ W.			
8.	47 22 N.	21 4 W.	West	517 „
Ans.	Lat. 47° 22′ N.; long. 33° 47′ W.			
9.	20 20 N.	31 5 W.	West	1,066 „
Ans.	Lat. 20° 20′ N.; long. 50° 2′ W.			
10.	35 4 N.	60 12 W.	East	1,216 „
Ans.	Lat. 35° 4′ N.; long. 35° 27′ W.			
11.	30 12 N.	59 50 W.	East	497 „
Ans.	Lat. 30° 12′ N.; long. 50° 15′ W.			
12.	25 17 N.	70 2 W.	East	2,744 „
Ans.	Lat. 25° 17′ N.; long. 19° 27′ W.			

Middle Latitude Sailing.

45. Find the true, magnetic, and compass courses and also the distances between the first and second of each of the following positions:

	Lat.	Long.		Lat.	Long.	Variation.
1.	49° 20′ N.	49° 8′ W.	and	50° 30′ N.	43° 30′ W.	34° W.
Ans.	True co. N. 72° E.; mag. S. 74° E.; comp. N. 86° E.; dist. 229′.					
2.	45 10 N.	20 15 W.	„	41 12 N.	26 30 W.	26 W.
Ans.	True co. S. 49° W.; mag. S. 75° W.; comp. N. 83° W.; dist. 362′.					
3.	31 44 N.	50 12 W.	„	34 7 N.	44 20 W.	18 W.
Ans.	True co. N. 64° E.; mag. N. 82° E.; comp. N. 62° E.; dist. 328′.					
4.	36 18 S.	47 45 W.	„	40 50 S.	49 58 W.	3 E.
Ans.	True co. S. 21° W.; mag. S. 24° W.; comp. S. 29° W.; dist. 291′.					
5.	42 10 S.	54 7 W.	„	48 37 S.	51 3 W.	9 E.
Ans.	True co. S. 18° E.; mag. S. 27° E.; comp. S. 40° E.; dist. 408′.					
6.	49 2 S.	55 49 W.	„	48 19 S.	49 2 W.	10 E.
Ans.	True co. N. 81° E.; mag. N. 71° E.; comp. N. 53° E.; dist. 272′.					

Find the lat. and long. of position arrived at, having sailed as follows from the following positions:

	Lat.	Long.	Course.		Miles.	Variation.
7.	48° 50′ N.	41° 10′ W.	true	N. 47° E.	315	
Ans.	Lat. 52° 25′ N.; long. 35° 7′ W.					
8.	45 40 N.	39 50 W.	compass	S. 54 E.	410	30° W.
Ans.	Lat. 43° 6′ N.; long. 29° 58′ W.					
9.	39 20 N.	20 15 W.	true	S. 63° W.	380	
Ans.	Lat. 36° 27′ N.; long. 27° 24′ W.					
10.	40 10 S.	2 16 W.	compass	N.W. by W.	406	26 W.
Ans.	Lat. 42° 2′ S.; long. 10° 54′ W.					

	Lat.	Long.	Course.		Miles.	Variation.
11.	45° 20′ S.	1° 58′ W.	true	N. 59° E.	320	
	Ans. Lat. 42° 35′ S.; long. 4° 23′ E.					
12.	50 6 S.	21 12 W.	compass	N.W. by N.	394	11 W.
	Ans. Lat. 47° 13′ S.; long. 30° 8′ W.					

Mercator Sailing.

46. Find the true, magnetic, and compass courses and also the distances between the first and second of each of the following positions:

	Lat.	Long.		Lat.	Long.	Variation.
1.	49° 58′ N.	5° 12′ W.	and	32° 15′ N.	64° 52′ W.	21° W.
	Ans. True co. S. 68° W.; mag. S. 89° W.; comp. N. 68° W.; dist. 2874′.					
2.	46 40 N.	53 7 W.	„	32 43 N.	16 40 W.	29 W.
	Ans. True co. S. 63° E.; mag. S. 34° E.; comp. S. 49° E.; dist. 1871′.					
3.	23 9 N.	82 22 W.	„	49 58 N.	5 12 W.	4 E.
	Ans. True co. N. 66° E.; mag. N. 62° E.; comp. N. 47° E.; dist. 3986′.					
4.	22 55 S.	43 6 W.	„	33 56 S.	18 29 E.	20 W.
	Ans. True co. S. 78° E.; mag. S. 58° E.; comp. S. 76° E.; dist. 3306′.					
5.	55 59 S.	67 14 W.	„	33 56 S.	18 29 E.	20 E.
	Ans. True co. N. 70° E.; mag. N. 50° E.; comp. N. 37° E.; dist. 3809′.					
6.	55 59 S.	67 14 W.	„	37 2 S.	12 17 W.	21 E.
	Ans. True co. N. 63° E.; mag. N. 42° E.; comp. N. 30° E.; dist. 2508′.					

Find the lat. and long. of position arrived at, having sailed as follows from the following positions:

	Lat.	Long.	Course.		Miles.	Variation.
7.	50° 20′ N.	40° 29′ W.	true	N. 27° E.	316	
	Ans. Lat. 55° 2′ N.; long. 36° 33′ W.					
8.	53 15 N.	30 14 W.	compass	N.N.W.½W.	280	35° W.
	Ans. Lat. 54° 4′ N.; long. 37° 59′ W.					
9.	55 50 N.	12 40 W.	true	N. 54° W.	341	
	Ans. Lat. 59° 10′ N.; long. 21° 13′ W.					
10.	50 9 S.	40 12 W.	true	S. 34° W.	322	
	Ans. Lat. 54° 36′ S.; long. 45° 7′ W.					
11.	55 37 S.	62 27 W.	compass	N.E. by E.½E.	411	17 E.
	Ans. Lat. 54° 47′ S.; long. 50° 30′ W.					
12.	56 14 S.	50 8 W.	true	S. 49° W.	220	
	Ans. Lat. 53° 50′ S.; long. 54° 57′ W.					

Various Examples on the Sailings.

The following examples should be worked both by Middle latitude and Mercator sailings, and the results compared.

47. Required the true course and distance between the following positions:

	Lat.	Long.		Lat.	Long.
1.	45° 2′ N.	20° 19′ W.	and	61° 30′ N.	10° 12′ W.
2.	50 8 N.	49 12 W.	„	30 2 N.	30 12 W.

	Lat.	Long.		Lat.	Long.
3.	20° 22′ N.	45° 24′ W.	and	40° 30′ N.	20° 10′ W.
4.	10 10 S.	30 7 W.	„	30 15 S.	5 22 E.
5.	31 17 S.	20 18 W.	„	51 6 S.	10 40 W.
6.	48 16 S.	10 8 E.	„	56 12 S.	31 4 W.
7.	4 20 N.[1]	31 2 W.	„	4 20 S.	24 46 W.
8.	5 17 N.	15 23 W.	„	5 17 S.	21 2 W.
9.	31 5 N.	30 12 W.	„	2 10 S.	23 8 W.
10.	27 40 S.	5 12 E.	„	1 58 N.	24 16 W.

EXAMPLES ON DAY'S WORKS.[2]

48. On October 23, at noon, the eastern lighthouse on Ushant, in latitude 48° 28′ N., and longitude 5° 4′ W., bore by compass S.E. (ship's head North), distant 15 miles (variation 22° W.); afterwards sailed as by the following log account. Required the latitude and longitude in on October 24, at noon.

Ans. Lat. 48° 25′ N.; long. 6° 43′ W.

Hours	Knots	$\frac{1}{10}$ths	Course	Wind	Leeway pts.	Oct. 23
1	5	4	N. by E.½E.	N.W.	0	Noon
2	5	2				
3	5	8				
4	6	1				
5	6	5	S.S.W.	W.N.W.	0	
6	7	3				Variation 22° W.
7	7	0				
8	7	2				
9	6	8	N.W. by W.	W.S.W.	0	
10	6	5				
11	6	1				
12	5	8	S. by W.¾W.	West	0	Midnight
						Oct. 24
1	6	0	S. by W.¾W.	West	0	A.M.
2	6	5				
3	6	8				
4	6	4	N.N.E.	N.W.	0	
5	6	0				

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[1] The middle latitude in examples 7 and 8 is taken practically as half of *either*, and in examples 9 and 10 as half the *greater* latitude.

[2] In those days' works where there is no leeway the courses should be corrected in *degrees*; the first three examples relate to the North Atlantic, and the last three to the South Atlantic Ocean. If possible the tracks should be *plotted* on those charts.

Hours	Knots	$\frac{1}{10}$ths	Course	Wind	Leeway pts.	Oct. 23
6	6	5				
7	6	8				
8	7	2	N.W.	W.S.W.	0	
9	7	6				
10	7	9				
11	8	1				
12	8	5				Noon

49. On May 1, at noon, Cape Ortegal, in latitude 43° 45′ N., and longitude 7° 56′ W., bore by compass S.S.W.½W. (ship's head N.½W.), distant 25 miles, variation 20° W.; afterwards sailed as by the following log account. Required the latitude and longitude on May 2 at noon.

Ans. Lat 43° 17′ N.; long. 7° 3′ W.

Hours	Knots	$\frac{1}{10}$ths	Course	Wind	Leeway pts.	May 1
1	3	2	S.S.E.½E.	S.W.	2¼	Noon
2	3	4				
3	3	5				
4	4	0				
5	4	2				
6	4	3	W.N.W.	Do.	2½	Variation 20° W.
7	4	4				
8	4	5				
9	5	2				
10	5	6				
11	5	7				
12	6	1				Midnight
						May 2
1	7	2	South	West	¼	A.M.
2	7	1				
3	7	3				
4	8	3				
5	8	4				
6	7	5	S.E.¾E.	S.S.W.	1½	
7	7	2				
8	7	1				
9	6	7				
10	6	5				
11	6	3				
12	6	0				Noon

50. On May 8, at noon, a vessel by astronomical observations was found to be in lat. 48° 10′ N., and long. 10° 10′ W.; afterwards she sailed as by the following log account. Required the dead reckoning latitude and longitude at noon on May 9.

Ans. Lat. 48° 2′ N.; long. 10° 7′ W.

Hours	Knots	$\frac{1}{10}$ths	Course	Wind	Leeway pts.	May 8
1	3	0	N.N.W.½W.	N.E.	0	Noon
2	3	2				
3	3	4				
4	2	7				
5	2	4	E. by S.¾S.	Do.	0	
6	2	4				Variation 23° W.
7	3	6				
8	3	2				
9	3	2				
10	4	0	E.S.E.	South	0	
11	4	2				
12	4	5				Midnight
						May 9
1	5	2	N.N.E.¼E.	N.W.	0	A.M.
2	6	2				
3	7	2				
4	7	5				
5	8	0				
6	5	2	W.S.W.	Do.	0	
7	6	2				
8	6	4				
9	5	4				
10	6	2				
11	6	3				
12	7	0				Noon

51. On March 4, at noon, a vessel was in the following position; lat. obs. 29° 40′ S., and long. chr. 20° 10′ W.; afterwards she sailed as by the following log account. Required the latitude and longitude in, on March 5 at noon.

Ans. Lat. 29° 30′ S.; long. 20° 32′ W.

Hours	Knots	$\frac{1}{10}$ths	Course	Wind	Leeway pts.	March 4
1	3	5	N.N.W.½W.	N.E.	1¾	Noon
2	4	1				
3	4	3				
4	2	7	E.S.E.	Do.	2	
5	3	0				
6	3	2				Variation 20° W.
7	4	0				
8	5	6	S.¾W.	E.S.E.	2¼	
9	5	2				
10	5	5				
11	4	5				
12	4	6				Midnight
						March 5
1	4	7	N.E.¼N.	Do.	1½	A.M.
2	4	2				
3	4	4				
4	3	7				
5	3	2				
6	3	5	W.½N.	S.S.W.	1¼	
7	4	2				
8	3	6				
9	3	4				
10	9	5	N. by E.	South	0	
11	10	2				
12	10	3				Noon

52. On June 4, at noon, a vessel by astronomical observations was in lat. 52° 10′ S., long. 50° 50′ W., and afterwards sailed as follows. Required the dead reckoning position at noon, June 5.

Ans. Lat. 54° 1′ S.; long. 52° 18′ W.

Hours	Knots	$\frac{1}{10}$ths	Course	Wind	Leeway pts.	June 4. Remarks
1	8	2	W.½N.	S.S.W.	0	P.M.
2	8	0				
3	8	0				
4	7	8				
5	7	0				
6	7	2	W. by S.	South	0	
7	7	5				Variation 11° E.
8	7	5				
9	6	8				
10	7	0				

Hours	Knots	$\frac{1}{10}$ths	Course	Wind	Leeway pts.	June 4. Remarks
11	7	0				
12	7	2	S. by E.½E.	E. by S.	0	Midnight
						June 5
1	7	5	S. by E.½E.	E. by S.	0	A.M.
2	7	5				
3	7	8				
4	8	0				
5	8	0	S.E.	E.N.E.	0	
6	8	2				
7	8	2				
8	7	8				
9	7	8				
10	7	5	E. by S.½S.	South	0	
11	7	5				
12	7	0				Noon.

53. On May 10, at noon, a vessel by astronomical observations was in lat. 40° 50′ S., long. 50° 25′ W., and at noon on May 11 by similar observations she was found to be in lat. 42° 19′ S., long. 49° 40′ W. Between these periods she sailed as by following log account. Required the lat. and long. by D.R. at noon of May 11, the course and distance made good, current experienced, and the course and distance to the Eddystone, Falkland Islands, in lat. 51° 11′ S., long. 59° 1′ W.

Ans.
Lat. D.R. 42° 4′ S.; long. D.R. 49° 56′ W.
Co. and dist. made good S. 21° E. 95′.
Current S. 38° E. 19′.
Eddystone bears S. 36° W. 656′.

Hours	Knots	$\frac{1}{10}$ths	Course	Wind	Leeway pts.	May 10. Remarks
1	7	5	E.½S.	S. by E.	¼	P.M.
2	7	5				
3	7	0				
4	5	0	S.W.¼S.	S.S.E.	1¾	
5	4	2				
6	3	8				
7	4	0				Variation 6° E.
8	4	0				
9	5	0	S.S.W.¼W.	S.E.	1	
10	6	0				

Hours	Knots	$\frac{1}{10}$ths	Course	Wind	Leeway pts.	May 10. Remarks
11	6	5				
12	6	5				Midnight
						May 11
1	6	0	S.S.W.¼W.	S.E.	1	A.M.
2	6	0				
3	5	8				
4	5	2				
5	7	5	E.¼N.	South	0	
6	8	5				
7	9	0				
8	9	0				
9	6	0	E. by N.¾N.	S.E.	1¾	
10	4	2				
11	3	8				
12	4	0				Noon

APPENDIX.

SWINGING SHIP FOR DEVIATIONS OF THE COMPASS.[1]

Swinging ship

THE term swinging ship is one now generally accepted amongst seamen to denote the process by which a ship is turned in azimuth, and steadied with her head sufficiently long on each successive point of the standard compass to take the compass bearing of an object, the magnetic bearing of which is known or can be ascertained.

Swinging ship for deviations of the compass was initiated by Captain Matthew Flinders, R.N., in a report made by that officer to the Lords Commissioners of the Admiralty in 1811, as a result of his observations when surveying the coasts of Australia in H.M.S. 'Investigator.' In consequence of this report, certain of H.M. ships were swung at Sheerness, Portsmouth, and Plymouth, Flinders superintending the observations at the two first-named ports.

Every ship or vessel, without distinction of size or material of construction, requires a table of deviations of the standard compass, showing the amount for every direction of the ship's head before going to sea, and it is expedient that the interval between leaving a dock or any other position in which the ship has been kept in a fixed direction and the swinging for deviation should be as long as convenient.

Standard compass

The term standard does not belong exclusively to any particular *form* of compass, but refers to that compass which is mounted in the best position in the ship with regard to surrounding iron, and that most suitable for taking bearings; consequently it is that by which the ship's course is invariably directed.

It is generally found convenient, and indeed necessary in thick weather, that ships should be swung in harbour or at anchor. There is, however, much to be said in favour of swinging ships under way, when the sea is moderate, the vessel's steam power making the observer independent of possible strong tides and the assistance of tugs or hawsers. Both methods will now be described.

Swinging ship in harbour

The observer should first examine the caps and pivots of the standard and steering compasses, to see that they are free from dirt or injury, and the cards working freely.[2]

Then, either with the aid of a tug or hawsers, the ship should be slowly turned and steadied on each point or alternate point of the compass, the observer noting the compass bearing of either a distant terrestial object, the sun, or an auxiliary compass on shore. The differences between the compass bearings and magnetic bearing are the deviations, termed easterly when the north point of the compass is drawn to the right of the magnetic north, westerly when drawn to the left.

By distant object

If a terrestrial object be observed, it should not be nearer than four miles, so that its magnetic bearing may remain practically the same from all points of the circle described by the ship in swinging round, unless a correction for parallax be applied to the mean magnetic bearing of the object for each direction of the ship's head.

---

[1] By Captain Creak, R.N., F.R.S., Superintendent of Compasses, Admiralty.

[2] After slightly disturbing the card (1° or 2°) it should return to rest on exactly the same point as before disturbance.

By the sun

When the sun is observed, the apparent time must be noted when each bearing is taken; the sun should be as low as possible in altitude, and the compass bowl kept horizontal.

By reciprocal bearings

If an auxiliary compass on shore is necessary from the absence of a suitable distant object, it should be set up in a place free from local attraction; signals should be arranged for the simultaneous observation of the bearings from the standard compass to the shore compass, and *vice versâ*; the shore bearings should immediately be signalled to the ship as taken.

The accuracy or otherwise of the observed deviations depends very much on the rate of swinging the ship. If the ship's head is *steadied* on every point of the compass for about half a minute, very good results may be obtained; but if, on the contrary, the ship is kept constantly though slowly turning, it is better to swing her completely round, first to starboard, then to port, and use the mean of the results as the correct deviation. These remarks are specially applicable to heavily armoured vessels with thick steel decks.

Swinging under way
By reciprocal bearings

In places where there is sufficient room to turn a ship under steam, and easy communication can be maintained with the compass on shore, swinging under way will be found very convenient, as all oppositions of tide and wind are obviated. Spithead is a familiar example of this.

By distant object

When the ship can be turned in a small circle, a terrestial object may be used, but not at a less distance than ten miles.[1]

By the sun

At sea in moderate weather, with the sun at as low an altitude as possible, a ship under steam may be swung with great facility, and under the actual conditions in which she is proceeding on her passage from port to port.

Graphic construction of table of deviation

By means of a simple graphic method a smaller number of observations than 32 made whilst a ship is swinging by the tide at her anchors, or under way at sea, may be utilised in making a complete table of deviations.

Napier's diagram

It is not necessary that the observations should be made for any special direction of the ship's head, as long as they are fairly distributed round the compass. For this purpose Napier's diagram will be found very useful when the deviations are large and changing rapidly in amount from point to point, for converting compass courses into magnetic courses and *vice versâ*. Full descriptions of the construction and use of this diagram are given in the Admiralty Manual for Deviations of the Compass,[2] and on page 24 of this book.

Ruled foolscap

For the smaller deviations usually found in a corrected compass an efficient diagram may be drawn on a sheet of foolscap ruled with thirty-three equidistant lines, each of which may be taken to represent a point of the compass. A vertical line representing magnetic north and south should be drawn down the centre and the degrees of deviation on any convenient scale measured from it along or parallel to the horizontal lines, according to the direction of the ship's head at the time of observation. Easterly deviations should be plotted on the right of the vertical line, westerly to the left. A flowing curve being drawn through each point plotted on the paper, the deviations for every direction of the ship's head may be determined by measurement from the vertical line.

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[1] If only a *very* distant object is available, a correction should be applied to its magnetic bearing if taken from the chart (*see* footnote on page 52).

[2] The deviation table used throughout this work is graphically represented on Plates 4 and 5, both on Napier's diagram and ruled foolscap.

Straight-line method

A complete deviation table having been formed by one of the methods as already explained, a graphic representation from which the compass course to be steered may be rapidly deduced for each given magnetic course will be found a great assistance to the officer of the watch when, as in the case of the tactical manœuvres of a squadron, the course is frequently changed. For this purpose Archibald Smith's straight-line course table is clear and simple in construction.

The diagram consists of three vertical columns. The first contains the standard compass courses, the third the magnetic courses, both of which are given in points, quarter-points, and degrees. The central column is for drawing straight lines from the compass courses to the corresponding magnetic courses.

In practice it is only required to look in one column for the required magnetic course, and follow the straight line to the compass course to be steered.[1]

General remarks

The deviations of the compass in iron or composite vessels are liable to change from various causes, such as lapse of time, change of geographical position, running on one course for several days together, firing heavy guns, or subjection of the vessel to violent strains or concussion.

The deviation table therefore obtained at the port of departure should not only be supplemented by daily observations on the course the ship may be steered, but upon any considerable change of latitude, or when approaching shores where, from climatic reasons, few opportunities of observation are likely to occur, a new table should be formed.

It will be found a useful practice to occasionally swing the ship at sea for every point of the quadrant of the compass in which the ship is steering. This will give timely warning of any changes taking place in the deviations generally, and confidence, when it is required, to make a large alteration of course after steering in one direction for several days.

The route from England to Singapore *viâ* the Suez Canal serves as an example illustrative of these latter remarks, there being two changes from courses near the south to those near the east, and a final return to southerly courses.

Compass journal

As a means of reference in cloudy weather and for obtaining a full knowledge of the changes of deviation to which a vessel is liable, it is important that a compass journal should be kept. In this journal every observation for deviation should be entered and each table of deviations on a suitable form pasted in a place provided.

A compass journal in an established form is kept in the Royal Navy, and journals of a similar description are kept in a large number of vessels in the mercantile navy.

NOTE.—The accuracy with which the observations for deviation have been made is tested by the *regularity* of the resulting curve on either Napier's diagram or ruled foolscap; the advantage of the former lies in the facility with which, when the deviation is large, a given magnetic course can be converted into a compass course.

Another test for accuracy of observation is supplied by the fact that under ordinary circumstances, if the maximum deviation does not exceed 20°, that at North (by compass) should be nearly equal and opposite in sign to that at South (easterly deviation being considered + and westerly −); similarly that at East should be nearly equal and opposite in sign to that at West, and the mean of deviations at N.E. and S.W. should also equal in amount the mean of those at N.W. and S.E.

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[1] The deviation table used throughout this book is thus illustrated by the straight-line method on Plate 6.

GREAT CIRCLE SAILING.

Great circle and rhumb line

The shortest distance between two places on a globe is evidently found by stretching a thread from one to the other on its surface; such an arc extended in both directions would be a great circle, and great circle sailing is the term applied to the art of sailing on such an arc.

To clearly understand the properties of great circle sailing the reader should, both on a globe and chart of the Atlantic Ocean, draw the great circle and rhumb line between two well-known places, such as Cape Horn and the Cape of Good Hope, transferring from one to the other the latitudes and longitudes of different points on each track; this is illustrated by the following figures, where H and C denote the positions of Cape Horn and the Cape of Good Hope, represented on fig. 48 on the globe, and on fig. 49 on the corresponding Mercator's chart.

The following observations (applicable to all great circle tracks in either north or south latitudes) should then be made:

1. The rhumb track (shown as a broken line) is always nearer the equator than the corresponding great circle.

2. The great circle between two places in *different* hemispheres and not on the same meridian will be represented on a Mercator chart by a curve of double flexure intersecting the equator on the same point as the rhumb track between them.

3. The great circle track intersects every meridian at a different angle, and on the chart forms a curve concave to the equator, so that while the course from H to C is on the rhumb line exactly opposite to that from C to H, on the great circle they are generally not so.

4. On the equator and on any meridian the great circle and the rhumb line coincide.

5. A curve of equal length to the rhumb line (shown as a dotted line) can be drawn on the polar side of the great circle; this practically can only *approximately* be drawn by taking a point on the chart on the polar side of the great circle where that is most widely separated from the rhumb line at that distance of separation from it, and drawing by hand a curve passing through it and the two given places.

6. If making a voyage on a great circle the port of destination is *always* right ahead, but if on a rhumb line it only comes right ahead immediately before arriving there.

These facts were known at a very early period when, owing to the imperfection of charts, a vessel's track was frequently drawn on a globe, but the difficulty in most parts of the world of preserving a desired course in a sailing vessel for any length of time prevented the general application of the principles until the introduction of steamers.[1]

Calculation of great circle

In fig. 48 the three arcs S H, S C, C H (hard line), being portions of great circles, form a spherical triangle in which S H, S C are the complements of the latitudes of Cape Horn and the Cape of Good Hope, and the included angle is their difference of longitude; these all being known, the other elements of the triangle H S C are easily calculated by spherical trigonometry.

---

[1] Sebastian Cabot is said to have projected a voyage across the North Atlantic Ocean on the arc of a great circle in 1495; in ancient works on Navigation it was termed 'globular' sailing.

Of these elements H C is the distance, S C H is the first true course (if proceeding from the Cape of Good Hope), and S H C the supplement of the final true course.

Since practically it is impossible for a vessel to continually change course, it was formerly customary to calculate the latitudes and longitudes of points on this great circle track separated by convenient intervals and steer for such points by Mercator sailing.

This calculation may be made as follows:

Assuming a point A distant 500 miles from the Cape of Good Hope, and joining S A by an arc of a great circle (a meridian); in the triangle S A C are known S C, C A, and the angle S A C, to find S A (the complement of the latitude of A) and the angle C S A (the diff. long. between the Cape of Good Hope and A).

Since it may be shown that the point on the equator where a great circle intersects it is the pole of the meridian which passes through its vertex, the course on which a vessel steering on a great circle crosses the equator is equal to the colatitude of the vertex.

In this way the positions of various points on the great circle can be calculated and then transferred to a Mercator's chart, the course being altered on arriving at each of these points.

Fig. 48.

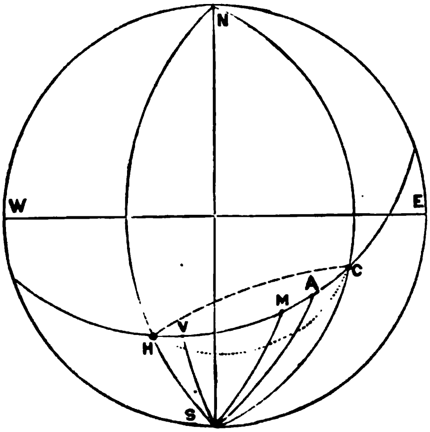
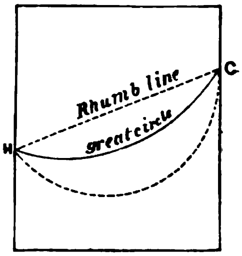


Fig. 49.



The preceding calculations are all that are essential in *computing* a great circle track, but there are two points connected with it deserving of notice.

On referring to fig. 48 it is evident that on every great circle there must be two opposite points of maximum separation from the equator (termed vertices), and that these points must be midway between the points of intersection of the great circle and the equator. By spherical trigonometry it therefore follows that if a meridian be drawn through the vertices it will cut the great circle at right angles.

Vertex

It is usual to refer to that nearest the place of greatest latitude as *the* vertex.

The vertex may or may not be situated *between* the two places, but if they are in the same latitude it lies *midway* between them. The latitude and longitude of the vertex are determined by drawing S V perpendicular on H C, when in the right-angled triangle S V H, S H, and the angle S H V being known, S V or the colatitude of the vertex is readily found, and also the angle H S V, which is the difference of longitude between Cape Horn and the vertex.

Point of maximum separation

If two places are on the same side of the equator, the ship on the great circle track between them is always in a *higher latitude* than if following the rhumb course; hence, since both tracks coincide at their extremities, there must be some point on the great circle at which its distance from the rhumb line, measured on a meridian, is greater than at any other point. This point is at the maximum separation in latitude.

When the two places are on opposite sides of the equator there are two points of maximum separation in latitude, one to the northward of the rhumb line in north latitude, the other to the southward of the rhumb line in south latitude.

The point of maximum separation necessarily falls between the places. When the vertex also falls between the places the point of maximum separation falls between the vertex and the place most distant from this in longitude. If the two places are in equal latitudes the vertex and point of maximum separation in latitude coincide.

The point of maximum separation may be also defined as the point on a great circle where two vessels leaving a port and pursuing one the great circle and the other the rhumb line, but changing their longitude at equal rates, are most widely separated in latitude.

The latitude and longitude of the point of maximum separation can be calculated by remembering that at this point the great circle course and the rhumb course must be the same. If, then, M in fig. 48 be this point, we have in the right-angled triangle S V M, S V, and the angle S M V (equal to the rhumb course) known, from which S M, the colatitude of M and V S M, or the difference of longitude between M and V (the vertex) can readily be computed.[1]

To obviate the necessity for laborious calculations several methods are practised by which the great circle track can without any calculation be drawn on a Mercator's chart; of these methods three will be described.

Towson's method

Towson's linear index and accompanying tables, published in 1847 by the Admiralty, enable any great circle to be *accurately* drawn on a chart.

The linear index gives by inspection the latitude and longitude of the vertex, and the tables give the true courses at every degree of longitude from the vertex.

Godfray's method

Godfray's great circle chart (Plate 8) is a chart of those southern latitudes where great circle sailing is most advantageously adopted, drawn on the gnomonic projection[2] (*see* page 89). From the principle of construction of such charts *all* great circles are represented by straight lines, and hence the track between two places may be laid down on a Mercator's chart by joining them on the gnomonic chart and transferring to the former the latitudes where this line cuts the successive meridians; this chart is of course available for drawing great circle tracks in similar *northern* latitudes.

If, however, great circle charts were desired which would include the equatorial regions, Godfray points out that we should merely have to take for our centre some point in the equator. In such a chart the meridians would be straight lines, and the parallels of latitude would be represented by hyperbolas; also, if a point not in the equator were taken for the centre, the meridians would be converging straight lines, and the parallels would be hyperbolas and ellipses.

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[1] The solutions of problems in great circle sailing by calculation afford valuable exercises in spherical trigonometry, but are of little practical utility; the student is referred for such examples to Goodwin's *Problems in Navigation and Nautical Astronomy*. *See* also Lecky's A, B, C and D tables.

[2] Originally published in *Cambridge Philosophical Transactions* for 1858. In the French Navy, charts on the same projection are used for this purpose, but the tangent plane being taken as touching the *equator*, the meridians are represented by parallel straight lines, and the parallels as hyperbolas.

Airy's method

Airy's method, though not so accurate as the preceding, is sufficiently so for practical purposes, and is probably the most simple yet devised; it was originally communicated to the Royal Astronomical Society by the late Astronomer Royal in 1858,[1] and afterwards published by the Board of Trade.

The rules given are here reproduced.

1. Join the two places on a Mercator's chart by a straight line. Find its middle point. Draw thence a perpendicular to that line on the side next the equator, and, if necessary, continue it beyond the equator.

2. With the middle latitude (between the two places) enter the following table, and take out the corresponding parallel.

3. The centre of the required arc (described either with beam compasses or a pencil attached to a thread) will be the intersection of this parallel with the perpendicular, the radius being the distance from the centre to either place.

Middle Latitude.		Corresponding Parallel.	Middle Latitude.		Corresponding Parallel.
20°		81° 13′	52°		11° 33′
22		78 16	54		6 24
24		74 59	56		1 13
26		71 26			———
28		67 38	58		4 0
30		63 37	60		9 15
32	Opposite Name.	59 25	62		14 32
34		55 5	64	Same Name.	19 50
36		50 36	66		25 9
38		46 0	68		30 30
40		41 18	70		35 52
42		36 31	72		41 14
44		31 38	74		46 37
46		26 42	76		52 1
48		21 42	78		57 25
50		16 39	80		62 51

Economy in distance and time

The economy in *distance* in Great circle over Mercator sailing is greatest in high latitudes between places not differing much in latitude, and is least between such as are situated nearly on the same meridian, one in north latitude and the other in south.

The economy in *time* can only be estimated by observing whether the great circle track passes through regions more favourably situated with regard to winds and currents than that traversed by the Mercator track, and it is advisable in all cases, when making a voyage, to draw all three tracks (*see* page 102) on the Wind and Current chart of the month in question.

The contrast between great circle and rhumb courses would be most noticeable if it were possible to navigate the Arctic ocean between two places in the same north latitude but differing 180° in longitude; by great circle the course would be first north and then south, while by the rhumb line the course would be east or west.

Since navigation cannot safely be performed in very high latitudes, a modified form of great circle sailing was proposed by Mr. Towson in 1858, termed *composite sailing*, in which a vessel sails on an arc which *touches* the parallel of highest desired latitude; this may be drawn by the linear index and tables, or laid off on Godfray's chart by drawing tangents to the parallel of maximum latitude (as shown on Plate 8), and transferring this track to the Mercator's chart.

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[1] The mathematical theory will be found in the *Monthly Notices* for that year.

Composite sailing

In illustration of the foregoing it will be instructive to determine by calculation, as well as by the more practical methods described, the elements of a great circle route from the Cape of Good Hope to Hobart (A to B fig. 50).

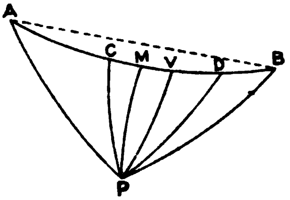
Lat. Cape of Good Hope 34° 22′ S. $\therefore$ λ or colat $=$ 55° 38′ Long. 18° 29′ E.
Lat. Hobart 42 54 S. $\therefore$ λ' $=$ 47 6 „ 147 22
$\lambda - \lambda' =$ 8 32 D $=$ 128 53

1. To find the distance A B—

$$\text{vers AB} = \text{vers}(\lambda - \lambda') + \text{vers}\,\theta,$$

where $\text{hav}\,\theta = \sin\lambda \sin\lambda' \,\text{hav APB}$

Fig. 50.



9·9167
9·8648
9·9106
$\text{hav}\,\theta =$ 9·6921

vers θ 0984300
vers $(\lambda - \lambda')$ 0011070

$\theta =$ 89° 6′ vers A B 0995370 $\therefore$ A B $=$ 89° 44′
or 5384′.

2. For the position of the vertex V (l being its latitude and A P V its diff. long. from A)—

$$\cos l = \frac{\sin\lambda \sin\lambda' \sin \text{APB}}{\sin \text{AB}}$$

9·9167
9·8648
9·8913
0·0000
9·6728

Lat. of vertex l, 61° 55′ S.

$\cos \text{APV} = \cot l \cot \lambda$

9·7272
9·8350
9·5622

A P V $=$ 68° 36′
Long. A 18 29 E.
Long. V 87 5 E.

$\cos \text{AV} = \dfrac{\cos\lambda}{\sin l}$

9·7517
9·9456
9·8061

A V $=$ 50° 13′
or 3013′

3. To find the initial and terminal courses—

$\sin \text{A} = \dfrac{\cos l}{\sin \lambda}$

9·6728
9·9167
9·7561

A $=$ 34° 46′

$\sin \text{B} = \dfrac{\cos l}{\sin \lambda'}$,

9·6728
9·8648
9·8080

B $=$ 40° 0′

Hence the initial course is S. 34°46′ E.
and the terminal course S. 140 0 E.
or N. 40 0 E.

4. To find the position of the point of maximum separation, M (referred to the vertex). Note.—Rhumb-course, R = 85° 9′.

(1)	(2)	(3)
$\cos \text{lat.} = \frac{\cos l}{\sin R}$;	$\sin VPM = \frac{\cos R}{\sin l}$;	$\sin MV = \frac{\cot l}{\tan R}$
9·6728	8·9271	9·7272
9·9984	9·9456	11·0713
9·6744	8·9815	8·6559
lat. M 61°48′S	VPM = 5° 30′	MV = 2° 36′
	Long. V 87 5 E.	or 156′
	Long. M 81 35 E.	

5. To find the position of any point, C or D, referred to the vertex. Let VC, and also VD = 20°.

(1)	(2)	(3)
$\sin \text{lat.} = \cos VC \sin l$;	$\tan VPC = \frac{\tan VC}{\cos l}$;	$\tan PCV = \frac{\cot l}{\sin VC}$
9·9730	9·5611	9·7272
9·9456	9·6728	9·5341
9·9186	9·8883	10·1931
lat. C or D 56° 0′ S.	VPC = 37° 43′	PCV or PDV = 57° 20′
	Long. V 87 5 E.	
	Long. C 49 22 E.	
	Long. D 124 48 E.	

Also G. C. course at C is S. 57° 20′ E., and at D, S. 122° 40′ E.

The course and distance on the rhumb-line track, by Mercator's sailing, between the Cape and Hobarton, is S. 85° 9′ E., 6056 miles.

Hence, in this instance, the distance on the great circle track being only 5384 miles, a saving of distance would be effected of 672 miles; but, on the other hand, in order to attain this, the ship would have to go as far south as 61° 55′ South latitude, the practicability of which would depend on the season of the year and other considerations.

CALCULATION OF MERIDIONAL PARTS.

The discovery of the correct principle on which charts adapted to purposes of navigation could be drawn has been justly considered as only second in importance to that of the mariner's compass.

Plane chart

The errors of the plane chart (on which the meridians were in all latitudes drawn at the same distance apart as they are at the equator, and the degrees of latitude everywhere made equal to one another) were well known at an early period of the art of navigation, but, as no means could be devised of correcting these errors, it was customary (especially in the higher latitudes) to place the ship's position on large globes, and actually *measure* the various courses and distances required.

Mercator's chart

To remedy this great practical inconvenience, a chart was published in 1556 by Gerard Mercator, having the meridians represented by parallel lines, and the degrees of latitude lengthened from the equator towards the poles, but not increasing in their true proportion.

It is supposed that he was not aware of the mathematical principles on which the lengths of these degrees should be increased, and took them by actual measurement from a globe.[1]

In 1599 Mr. Edward Wright, in a work entitled 'The Correction of Certain Errors in Navigation,' clearly explained these principles, and gave accurate rules for the construction of charts.

Before giving Wright's formulæ, the excellent popular description he gave in that work of the nature of a Mercator's chart will be repeated: 'Suppose a spherical globe representing the world inscribed in a concave cylinder to swell like a bladder equally in every part (that is, as much in longitude as in latitude) until it joins itself to the concave surface of the cylinder, each parallel increasing successively from the equator towards either pole, until it is of equal diameter to the cylinder, and consequently the meridians widening themselves until they are everywhere as distant from each other as they are at the equator. Such a spherical surface is thus by extension made cylindrical, and consequently a plane parallelogram surface, since the surface of a cylinder is nothing else but a plane parallelogram surface wound round it. Such a cylinder, on being opened into a flat surface, will have upon it a representation of a Mercator's chart of the world.'

FIG. 51.

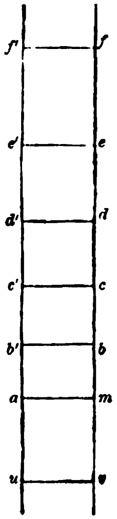
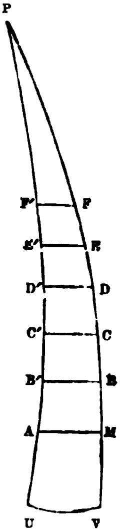


FIG. 52.



An Approximate Method of Calculating the Meridional parts for any Latitude.

Calculation of meridional parts

In order to construct a Mercator's chart we must know the lengths of the lines corresponding to the latitude of every point on the globe, at small distances from each other. These lengths, computed for every minute of latitude from 0° to 90°, form the table of meridional parts. It may be approximately obtained in the following manner:

Suppose the meridians P V, P U, and parallels A M, B′B, &c. (fig. 52), are drawn sufficiently near to each other that the quadrilateral surfaces of the earth, A M B B′ B′ B C C, &c., thus formed, may be considered without any practical error to be *plane surfaces*; this may be done if the arcs M B, B C, &c., be not taken greater than one minute: then these quadrilateral surfaces being expanded into $a\,m\,b\,b'$, $b'\,b\,c\,c'$ (fig. 51), similar to them on the chart, we can prove that

$$\begin{aligned} m\,b &= \text{MB} \;.\; \text{sec lat. M}, \\ b\,c &= \text{BC} \;.\; \text{sec lat. B}, \\ c\,d &= \text{CD} \;.\; \text{sec lat. C}, \\ d\,e &= \text{DE} \;.\; \text{sec lat. D}, \\ \&\text{c.} &= \&\text{c.} \end{aligned}$$

For by trigonometry, $\frac{\text{AM}}{\text{UV}} = \cos \text{MV} = \cos \text{lat. M}$,

$$\therefore \text{ sec lat. M} = \frac{\text{UV}}{\text{AM}} = \frac{u\,v}{\text{AM}} = \frac{a\,m}{\text{AM}}.$$

---

[1] On this chart the first meridian (from which the longitudes were reckoned) was that passing through Corvo Island, one of the Azores, where at this time there was no variation.

But since the quadrilateral surfaces on the globe and chart are similar,

$$\therefore \frac{bm}{\text{BM}} = \frac{am}{\text{AM}} = \text{sec lat. M},$$

$$\therefore bm = \text{BM} \ . \ \text{sec lat. M},$$

$$\begin{aligned} \text{Similarly } bc &= \text{BC} \ . \ \text{sec lat. B}, \\ cd &= \text{CD} \ . \ \text{sec lat. C}, \\ de &= \text{DE} \ . \ \text{sec lat. D}, \\ \text{\&c.} &= \text{\&c.} \end{aligned}$$

Let lat. of $\text{M} = l$, and $\text{BM} = \text{BC} = \text{CD} = \text{DE} = 1$ minute; then lat. $\text{B} = l + 1'$, lat. $\text{C} = l + 2'$, lat. $\text{D} = l + 3'$; $\therefore$ adding, we have

$\sec l + \sec (l+1') + \sec (l+2') + \sec (l+3') = bm \times bc + cd + de = me =$ meridional difference of lat. between M and E.

If the point M be on the equator, then $l = 0$, and the above expression gives the value of the meridional parts for 4 minutes: thus

$$bm + bc + cd + de—\sec 0' + \sec 1' + \sec 2' + \sec 3',$$

or the merid. parts for $4' = \sec 0' + \sec 1' + \sec 2' + \sec 3'$.

Hence, generally, we have for any lat. l

$$\text{Mer. parts for } l^{\circ} = \sec 0' + \sec 1' + \sec 2' + \ . \ . \ . \ + \sec (l^{\circ} - 1').$$

A nearer approximation to the value of the meridional parts for 4 minutes would of course be obtained by taking the parts MB, BC, CD, &c., still smaller, as 1 second; for then the meridional parts for 4 minutes would be found from the expression,

$$\text{MP for } 4' = \sec 0'' + \sec 1'' + \sec 2'' + \sec 3'' + \ . \ . \ . \ + \sec 3'\,59''.$$

From similar expressions to these was the first table of meridional parts computed.

If we suppose the meridional parts for 70° to have been found to be = 5965·92 (by the above method), let it be required to calculate the meridional parts for 70° 10′.

MP for 70° 10′ = sec 0′ + sec 1′ + sec 2′ + sec 3′ + . . . + sec 69° 59′ + sec 70° + sec 70° 1′ + . . . + sec 70° 9′

= 5965·92 + sec 70° + sec 70° 1′ + . . . + sec 70° 9′.

Log. sec.	70°	0′	......0·465948......	∴ nat. sec.	=	2·9238
„	70	1	......0·466296......	„	=	2·9262
„	70	2	......0·466643......	„	=	2·9285
„	70	3	......0·466991......	„	=	2·9308
„	70	4	......0·467339......	„	=	2·9332
„	70	5	......0·467688......	„	=	2·9355
„	70	6	......0·468037......	„	=	2·9379
„	70	7	......0·468386......	„	=	2·9403
„	70	8	......0·468735......	„	=	2·9426
„	70	9	......0·469085......	„	=	2·9450

29·3438

Mer. parts for 70° = 5965·92

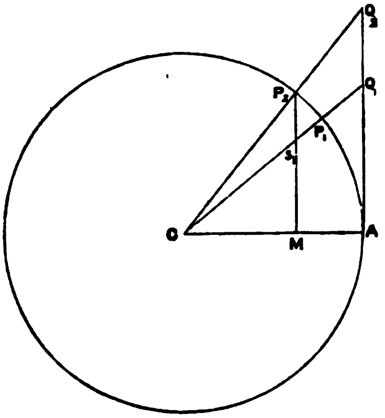
∴ mer. parts for 70° 10′ = 5995·2638

Given the meridional parts for 70° 10′ = 5995·2638, calculate the meridional parts for 70° 15′. *Ans.* Mer. parts = 6010·03.

Graphic representation of meridional parts

The relation of the true and meridional differences of latitude between two places near each other may be *graphically* shown, as follows:

FIG. 53.



Let C (fig. 58) represent the centre of the globe, $P_1\,P_2$ two points near one another on the same meridian, $A\,Q_1\,Q_2$ a tangent to this meridian at the equator; then $P_1\,C\,A = l =$ lat. of P_1, and $P_2\,C\,P_1 = d\,l =$ a small increment of lat. From P_2 draw $P_2\,S_1\,M$ perpendicular to $C\,A$; then $P_2\,S_1$ will represent the *meridional* diff. lat. corresponding to the *true* diff. lat. $P_2\,P_1$

$$Q_2\,Q_1 = C\,A\,\{\tan(l + dl) - \tan l\} = C\,A\left\{\frac{\tan l + \tan dl}{1 - \tan l\,.\,\tan dl} - \tan l\right\}$$

$= C\,A\,\dfrac{\sec^2 l\,.\,\tan dl}{1 - \tan l \tan dl} = C\,A\,.\,\sec^2 l\,.\,dl$; for since dl is small, $\tan l\,.\,\tan dl$ may be neglected in comparison with unity, and for $\tan dl$ we may write dl;

$$\therefore\ Q_2\,Q_1 = P_2\,P_1\ .\ \sec^2 l \text{ since } P_2\,P_1 = C\,P_2 \text{ (or } C\,A)\ .\ dl.$$

But in Mercator's projection the corresponding projection to $P_2\,P_1$ is $P_2\,P_1 \sec l$. Hence $Q_2\ Q_1$ is greater than the arc of Mercator's projection in the ratio of $\sec l$ to 1.

By similar triangles $P_2\,S_1 : Q_2\,Q_1 :: C\,S_1 : C\,Q_1$.

$$\therefore\ P_2\,S_1 = Q_2\,Q_1\,\frac{C\,S_1}{C\,Q_1} = Q_2\,Q_1\,\frac{C\,M}{C\,A}$$

$$= Q_2\,Q_1\,\frac{C\,M}{C\,P_2} = Q_2\,Q_1 \cos l \text{ (approximately).}$$

$$\text{But } Q_2\,Q_1 = P_2\,P_1 \sec^2 l;$$

$$\therefore\ P_2\,S_1 = P_2\,P_1 \sec l,$$

and hence $P_2\,S_1$ represents the arc of Mercator's projection corresponding to the arc of the meridian $P_2\,P_1$. (From *Naval Science*, 1872.)

DEFINITIONS IN NAVIGATION.

Sphere **Centre** A SPHERE is a solid, bounded by a surface, every point of which is equally distant from a fixed point called the *centre*.

Radius A radius of a sphere is a straight line joining the centre to any point in the surface.

Diameter A diameter of a sphere is a straight line passing through the centre and terminated both ways by the surface.

Section A section of the surface of a sphere made by a plane is a circle.

Great circle A great circle is the section of the surface of a sphere made by a plane passing through the centre.

Small circle A small circle is the section of the surface of a sphere made by a plane not passing through the centre.

Axis The axis of a circle of a sphere is that diameter of the sphere which is perpendicular to the plane of the circle.

Poles The poles of a circle are the extremities of its axis.

Secondary A secondary to a circle is a great circle passing through its poles.

Spherical angle A spherical angle is the angle between two great circles.

A spherical angle is *measured*—

Measure of a spherical angle 1. By the angle between the tangents to the circles at the point of their intersection.

2. By the angle between the planes of the circles.

3. By the arc which the circles intercept on the great circle to which they are secondaries.

Parallel A parallel to a great circle is a small circle whose plane is parallel to the plane of the great circle.

Earth's axis The axis of the earth is that diameter about which it revolves with a uniform motion from west to east.

Earth's poles The poles of the earth are the extremities of its axis.

The equator The equator is the great circle whose axis and poles are the axis and poles of the earth.

Meridians Meridians are great circles which pass through the poles of the earth.

Meridian of a place The meridian of a place on the earth is that meridian which passes through the place.

The first or prime meridian The first or prime meridian is that fixed meridian by reference to which the longitude of places on the earth is measured.

By general consent the meridian of Greenwich is now taken as the prime meridian.

Parallels of latitude Parallels of latitude are small circles whose planes are parallel to the plane of the equator.

Longitude of a place The longitude of a place is the arc of the equator intercepted between the first meridian and the meridian of the place.

Longitude is reckoned east or west from the first meridian, and may be of any magnitude from 0° *to* 180° *E. or W.*

Latitude of a place The latitude of a place is the arc of a meridian intercepted between the equator and the place.

Latitude is reckoned north or south from the equator, and may be of any magnitude from 0° *to* 90° *N. or S.*

Difference of latitude The difference of latitude between two places is the arc of a meridian intercepted between their parallels.

This is called T.D. latitude, in opposition to Mer. D. lat.

Difference of longitude The difference of longitude between two places is the smaller arc of the equator intercepted between their meridians.

True latitude The true latitude of a place is the angle between a perpendicular to the earth's surface at that place and the plane of the equator.

Reduced or central latitude The reduced or central latitude of a place is the angle between the earth's radius at that place and the plane of the equator.

The difference between the true and reduced latitudes is always small, being due to the fact that the earth is not quite, though very nearly, a sphere. If the earth be regarded as a sphere, the definition of true and reduced latitude will agree with the definition of latitude given above.

Nautical mile A nautical mile is equal to the mean length of a minute of latitude, and is reckoned at 6080 feet.

Rhumb line A rhumb line is a curve which cuts all meridians at the same angle.

Course The course of a ship on a rhumb line is the angle which the rhumb line makes with any meridian.

Distance The distance between two places is the length of the rhumb line joining them expressed in nautical miles.

Departure The departure between two places is the amount of easting or westing made in sailing between them on a rhumb line.

If an infinite number of points be taken on the rhumb line, the meridian through each of these points cuts off an arc of the parallel of latitude through the next point. The sum of all these arcs is called the *departure*.

Middle latitude The middle latitude of two places on the same side of the equator is half the sum of their latitudes.

The term is not strictly applicable to places on opposite sides of the equator.

Traverse When a ship sails on two or more courses she is said to describe *a traverse*.

Mercator's chart A Mercator's chart is a chart constructed on the following principles:

1. Meridians are represented by parallel straight lines. Hence (*a*) a rhumb line, since it cuts all meridians at the same angle, is represented by a straight line; (*b*) each parallel of latitude is expanded to the same length as the equator.

2. At any place the small arc of latitude is increased in the same proportion as the small arc of the parallel. Hence the scale of the chart at any place is equal to the scale of the chart at the equator × sec. lat.

Meridional parts The meridional parts of a certain latitude give the length expressed in nautical miles of the line on the Mercator's chart which represents the latitude.

Mer. diff. lat. The meridional difference of latitude between two places is the length expressed in nautical miles of the line on the Mercator's chart which represents the difference of latitude.

Plane sailing Plane sailing is the method of solving problems in navigation in the case of a ship sailing on a rhumb line by the use of the plane triangle which represents the relations between the course, distance, departure, and T.D. latitude.

Traverse sailing Traverse sailing is the method of solving problems in navigation in the case of a ship which is sailing on several courses.

Mercator's sailing Mercator's sailing is the method of solving problems in navigation, in the case of a ship sailing on a rhumb line, by the aid of Mercator's chart and the table of meridional parts constructed for it.

Parallel sailing Parallel sailing is the method of solving problems in navigation when a ship is sailing due east or west—that is, on a parallel of latitude.

Mid. lat. sailing Middle latitude sailing is the method of solving problems in navigation, in the case of a ship sailing on a rhumb line, on the assumption that she would make the same departure by sailing on a parallel of middle latitude.

The latitude in which the meridian distance is exactly equal to the departure is sometimes called true mid. lat.

Great circle sailing Great circle sailing is the method of solving problems in navigation in the case of a ship sailing on a great circle.

The distance on the great circle is the shortest distance between two places.

Vertex of a great circle The vertex of a great circle is the point in it which is nearest to the pole.

At the vertex the meridian and the great circle intersect at right angles. It must be noted that a great circle between two places has two vertices. A convention is made that the vertex is used which is on the same side of the equator as the place of higher latitude.

Leeway Leeway is the angle between a ship's fore and aft line and her wake.

Set and drift of a current The set of a current is its direction, and is named after the point to which it is running.

The drift of a current is the distance through which it flows in a given time.

Magnetic meridian The magnetic meridian is the great circle in which the compass needle lies when unaffected by local attraction.

Variation The variation is the angle between the magnetic meridian and the meridian of the place.

Deviation The deviation is the angle between the compass needle and the magnetic meridian.

Deviation is the effect of local disturbing forces.

Compass error The compass error is the angle between the compass needle and the meridian of the place.

The compass error is the algebraic sum of variation and deviation.

Dip of the needle

The dip of the needle is the angle between the horizontal plane and a magnetic needle freely suspended at its centre of gravity.

True course steered

The true course steered is the angle between the ship's fore and aft line and the meridian of the place.

True course made good

The true course made good is the angle between the rhumb line sailed and the meridian of the place.

True bearing

The true bearing of an object is the angle between the meridian and the great circle which passes through the object and the observer.

NOTE.—*In the expression* 'true bearing and distance,' *which is used in the logs of H.M. ships,* 'true bearing' *signifies* true course *to the place indicated.*

The courses steered or made good and the bearings of objects are termed true, magnetic, or compass, according as they are referred to the meridian of the place, the magnetic meridian, or the direction of the compass needle.

Cross bearings

Cross bearings are the bearings of two objects taken from the same place.

The intersection of the lines of bearing fixes the point of observation on a chart.

PART II.—NAUTICAL ASTRONOMY.

CHAPTER VI.

Elementary astronomy—Solar system—Fixed stars—Finding the stars—Ecliptic—Equinoctial—Declination—Right ascension—Horizon—Altitudes—Azimuth—The sextant—Correction of altitudes—Dip—Refraction—Semi-diameter—Parallax.

Definition

THE arts of determining the latitude and longitude of a ship's position, the error and rate of a chronometer, and the error of the compass by observations of the various heavenly bodies, are collectively known as Nautical Astronomy.

This branch of the art of Navigation is (as the name implies) merely a special application of Practical Astronomy, and the problems involved are those practised at astronomical observatories, adapted to the instruments and circumstances peculiar to navigation.

The solution of these problems is in all cases obtained by an application of the principles of Spherical Trigonometry, and it will be assumed that the reader is conversant with the solution of spherical triangles.

A knowledge of the positions and movements of the principal heavenly bodies is a necessary preliminary to the study of nautical astronomy.

The earth is one among several large and a vast number of small bodies, mostly of nearly spherical forms, whose movements being controlled by and directed round the sun, occasion their being, collectively with that luminary, known as the Solar System.

Solar system

These bodies comprise seven more or less resembling the earth in size and shape, some 350 very small similar bodies known as asteroids or planetoids, thirty-six large and many small comets, and a vast number of extremely small bodies known as meteorites, which occasionally become visible as shooting stars.

To confine our attention at first to the planets, which are the only members of the solar system (besides the sun and moon) sufficiently conspicuous to be useful for the general purposes of navigation, the following characteristics are common to all:

Revolution 1. They all move round the sun in the same direction—viz. opposite to that in which the hands of a watch move, and the oval curves they describe (known as their orbits) are *nearly* circles, having the sun in one focus.[1]

The planets nearer the sun move round it more rapidly than those more remote,[2] and every planet moves most quickly over that part of its orbit which is nearest the sun. Many of the planets are attended by moons similar to our own, which generally move round them in the same direction that the planets themselves move round the sun.

The exceptions to this rule are the satellites of Uranus and Neptune, which revolve in the opposite direction.

This form of movement is termed *revolution.*

Rotation 2. Observation shows that, while revolving round the sun, each of the planets turns round its axis, similarly to the earth, those most distant from the sun turning more rapidly and those nearer at about the same rate as our globe. This form of movement is termed *rotation*, and is also performed by the sun itself.

The period of a complete revolution forms the *year*, and that of a complete rotation the *day* of each planet.

The above movements, which would appear simple if seen from the sun, are complicated, as seen from the earth, by the fact of our globe being itself in motion. Every observed movement is, however, readily accounted for by the accepted principles of astronomy (first clearly explained by Kepler in 1594), which also enable accurate *predictions* of the places of the heavenly bodies to be made many years in advance.

Elements The table on page 117 exhibits the chief elements of the sun and principal planets.

The following illustration of the relative sizes and distances of the members of the solar system is given by Herschel in his 'Outlines of Astronomy.'

The following laws governing the revolutions of the planets round the sun were first discovered by Kepler in 1594–1619, and are known as Kepler's Laws: (1) They revolve in ellipses (generally closely resembling circles) having the sun in one focus. (2) If a line be supposed always to connect the centre of the sun with that of a planet such a line will, by the movement of the planet, pass over equal *areas* in equal times. (3) The squares of the times of revolution of each of the planets are in the same proportion to one another as the cubes of their mean distances from the sun.

[2] Thus, the mean velocity of Mercury in its path round the sun is twenty-nine miles per second, that of the earth eighteen miles, and that of Neptune, the outermost known planet, three miles per second.

Name	Mean Distance from Sun	Period of Revolution round Sun	Diameter	Apparent Diameter
	Miles	Days	Miles	
Sun . .	—	—	852,580	31′ 32″ to 32′ 36″
Mercury .	35,393,000	88	2,960	5″ „ 12″
Venus .	66,131,000	225	7,510	10″ „ 67″
Earth .	92,560,000	365¼	7,918	—
Mars . .	139,312,000	687	4,920	4″ to 30″
Jupiter .	475,693,000	4,332	85,390	30″ „ 50″
Saturn .	872,135,000	10,759	71,900	15″ „ 21″
Uranus .	1,753,851,000	30,686	33,000	4″
Neptune .	2,746,271,000	60,127	36,600	2″

'Choose any well-levelled field or bowling-green. On it place a globe two feet in diameter, this will represent the Sun; Mercury will be represented by a grain of mustard-seed, on the circumference of a circle 164 feet in diameter for its orbit; Venus, a pea on a circle 284 feet in diameter; the Earth, also a pea, on a circle of 430 feet; Mars, a rather large pin's head, on a circle of 654 feet; the asteroids, grains of sand in orbits of from 1,000 to 1,200 feet; Jupiter, a moderate-sized orange in a circle nearly half a mile across; Saturn, a small orange on a circle of four-fifths of a mile; Uranus, a full-sized cherry, or small plum, upon the circumference of a circle of more than a mile and a half; and Neptune, a good-sized plum on a circle about two miles and a half in diameter.'

Fixed stars

The fixed stars, numbering many millions, are all situated at immense distances beyond the limits of the solar system, the nearest (*a Centauri*) being so remote that, even with very delicate astronomical instruments, scarcely any change in its place can be detected as seen from positions at opposite points of the earth's orbit, though separated by a distance of 185 millions of miles.[1]

Constellations

From the earliest times the fixed stars have been arranged in fanciful groups of figures of men, women, and animals, termed Constellations, of which 67 are recognised as affording a convenient mode of distinguishing one group from another by name; the great mass of the stars, however, is congregated in the Milky Way, estimated to contain at least eighteen millions.[2]

The fixed stars are further classified according to their brilliancy or magnitude, of which classes the first six are visible under favourable circumstances to the naked eye, the other magnitudes are invisible except through telescopes.

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[1] The distance of this star is considered to be not less than 210,000 times the distance of the sun from the earth. A ray of light from such a star would take more than three years to reach the earth.

[2] Exclusive of the Milky Way, there are about 3,500 stars visible in England to the naked eye, 15 of which are of the first magnitude, about 50 of the second, and 120 of the third, but of these only some 2,000 are visible at any *one* time. The total number of stars in the *whole* heavens visible to the naked eye is about 7,000.

Star charts

A list of the positions of 349 of the stars most useful for purposes of navigation,[1] and visible to the naked eye, is published in the 'Nautical Almanac,' and the reader should make himself familiar with the situation of those of the first magnitude (twenty in number), from which, on reference to a star chart, the others can easily be recognised.[2]

Finding the stars

The positions of the most important stars are readily obtained by referring them to lines of approximate direction in the heavens, pointed out by the principal stars in such well-known constellations as the Great Bear (Ursa Major) and Orion, the names and appearances of which should be first studied.[3]

The following general description will then enable the reader to find the principal stars visible in the latitude of London.

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[1] For the purposes of practical astronomy, much more extensive catalogues of stars have been prepared. The earliest catalogue of stars was that by Hipparchus in B.C. 125, which gave the positions of 1,080 stars; one of the most extensive was that of Lalande, in 1800, giving the places of 47,390 stars.

[2] Jeans' *Handbook for the Stars* is generally used in the Royal Navy for this purpose, and, by its general and particular star charts, affords every necessary information. The positions of the planets are best learned by occasionally marking their places on such charts. The only four of practical use in navigation are Venus, Mars, Jupiter, and Saturn.

[3] These constellations are selected for this purpose because in England the Great Bear never sets, and Orion is remarkable to the southward during the winter evenings. *Proper* names have been given to the most conspicuous of the stars; besides which, each star in a constellation is distinguished by a Greek or Roman letter, and sometimes, when the number is great, by a figure or number, and also by a character such as α^2, combining both a letter and figure. The first letter of the Greek alphabet (α) is usually given to the brightest in the constellation, the second letter (β) to the next brightest, and so on to ω, the last letter. The Greek letters being exhausted, the *Roman* letters are then used in the same order. These letters were formerly sufficient to distinguish all the stars in each constellation, but afterwards, as the power of the telescope increased, the number of stars in each group was found far to exceed that of the letters of these alphabets: they were then distinguished by means of numerals. It is easy now to understand what is meant by α of the Little Bear, Aldebaran, or α Tauri, 61 Cygni, α^2 Capricorni, &c.

The small letters of the Greek alphabet are principally used for distinguishing the brightest of the fixed stars. They are as follows:

α	alpha	ι	iota	ρ	rho
β	beta	κ	kappa	σ	sigma
γ	gamma	λ	lambda	τ	tau
δ	delta	μ	mu	υ	upsilon
ε	epsilon	ν	nu	φ	phi
ζ	zeta	ξ	ksi	χ	chi
η	eta	ο	omicron	ψ	psi
θ	theta	π	pi	ω	omega

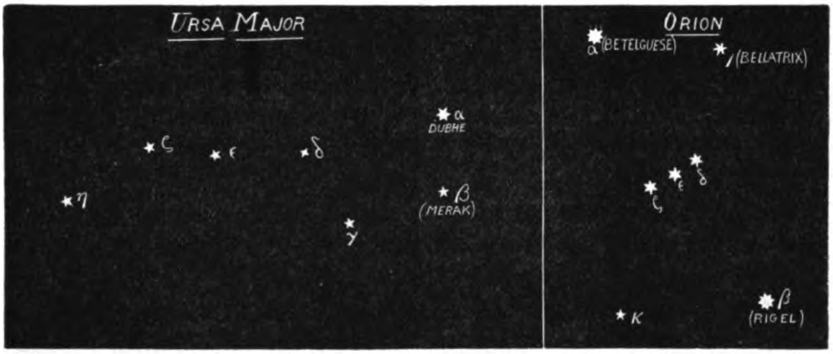
A line from β Ursæ Majoris (Merak) through α Ursæ Majoris (Dubhe) points to the Pole-star (Polaris), which is at the same distance from Dubhe as the last star (η) in the tail of the Great Bear.

α and β Ursæ Majoris are known as the Pointers.

The Pole-star is not *at* but *near* the Pole, the latter point is on the line joining Polaris and the middle star in the tail of the Great Bear, so that when the last-mentioned star is vertically above or below Polaris both are on the meridian of the observer.

A line from the first in the tail of the Great Bear (ε) through the Pole-star points to Cassiopea, a constellation shaped like an irregular W, and about as far distant from the Pole as the square of the Great Bear.

FIG. 54.



Arcturus (α Bootis) is found by a line through Polaris and the last star in the tail of the Great Bear drawn away from the pole.

If the curve formed by the three stars in the tail of the Great Bear be continued, it passes first through Arcturus and then through Spica (α Virginis).

The line joining Polaris and ζ Ursæ Majoris also points to Spica.

Arcturus, Spica, and β Leonis form an equilateral triangle.

The twins (Castor and Pollux, or α^2 and β Geminorum) are two conspicuous stars midway between the Great Bear and Orion; a line joining ε Orionis and β Orionis points towards them.

α Cygni is found by drawing a line from the Twins through the Pole-star, and extending it for an equal distance on the other side of the Pole.

Capella (α Aurigæ) lies on a line drawn from the Pole-star at right angles to the Pointers (away from the Great Bear) at a distance of about 45°.

α Coronæ is pointed out by a line joining δ of the Great Bear and the last star in the tail of the Great Bear; it is the most conspicuous of several which form a semicircle.

Vega (α Lyræ) is found by extending the line from Capella through the Pole-star to about an equal distance on the opposite side of the Pole.

Thus, while β Ursæ Majoris and Dubhe point to the Pole-star, Vega and Capella are at right angles to this line and about equidistant from the Pole-star.

Vega, Arcturus, and the Pole-star form a right-angled triangle having the right angle at Vega.

Altair (α Aquilæ) is the central and brightest of three stars in a line, found by drawing a line from the Pole-star midway between Vega and α Cygni, and extending it for an equal distance beyond them.

Altair, α Lyræ, and α Cygni form a right-angled triangle having the right angle at α Lyræ.

The line of Orion's belt extended in a S.E. direction points to Sirius (α Canis Majoris), and extended in a N.W. direction it passes south of Aldebaran (α Tauri) and nearly through a conspicuous group known as the Pleiades.

Sirius, Orion's belt, Aldebaran, and the Pleiades are nearly equidistant from one another.

Procyon (α Canis Minoris) is recognised by its forming with Sirius and the north-eastern star in Orion (α) an equilateral triangle.

The square of Pegasus is a conspicuous irregular square pointed to either by a line joining the south-west and north-east stars in Orion (α and β), or by a line from the Pole-star through the western part of Cassiopea.

This square together with β and γ Andromedæ and α Persei forms a figure much resembling the Great Bear on a larger scale.

The position of the first point of Aries (*see* page 123) is nearly on a line passing through the two eastern stars in the square of Pegasus, and extended to the southward for a distance about equal to that between those stars.

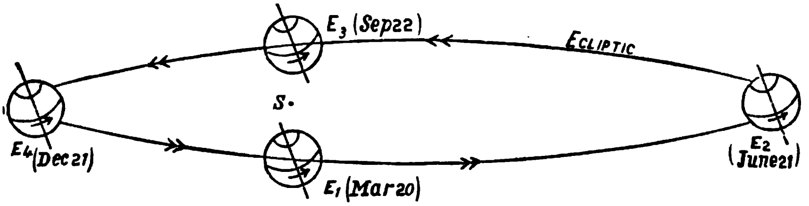
Regulus (α Leonis) lies near a line drawn through δ Orionis and Procyon, extended about the same distance that these stars are apart.

Revolution of the earth

Before giving the definitions of terms used in nautical astronomy, we must consider more closely the apparent effects to an observer on the earth of its movement of revolution round the sun.

In fig. 55, S represents the sun, and E_1 E_2 E_3 E_4 the positions of the earth at the commencement of the astronomical seasons of spring, summer, autumn, and winter respectively, the observer being supposed to look down from the north side of the plane of the orbit;[1] the axis of the earth is not at right angles to that plane, but inclined to it at an angle of about 66° 33′, and hence the plane of the *equator* and that of the orbit are inclined to one another at an angle of about 23° 27′ (90° − 66° 33′).

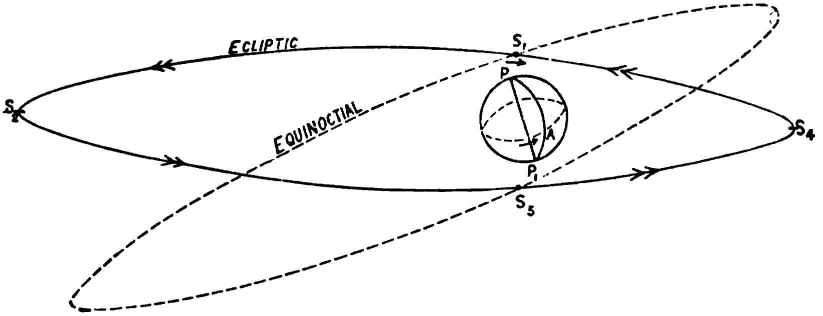
Fig. 55.



Celestial concave

The observer, being unable to judge of the relative distances of the heavenly bodies, sees them apparently on the interior surface of a very large sphere, of which his eye is the centre; this is termed the Celestial Concave.[2]

Fig. 56.



Poles of the heavens

The Poles of the heavens are the points of the celestial concave towards which the earth's axis is directed. That pole which is above the horizon is termed the Elevated pole.

From the point of view of the *earth* being stationary and the sun revolving round it once a year, it will be convenient to compare fig. 56, drawn on this supposition, with fig. 55, which illustrates the true case.

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[1] The earth at E_4 is about 3,000,000 miles nearer to the sun than when it is at E_2

[2] The *derivation* of the terms used in nautical astronomy will be found in Harbord's *Glossary of Navigation*. For convenience of reference the definitions are brought together and formally stated in the terms sanctioned for use in H.M.S. *Britannia* on pages 421–424.

S_1 S_2 S_3 S_4 are the apparent places of the sun as *really* seen from E_1 E_2 E_3 E_4.

A good general conception of the earth's motion round the sun, and of its effects in producing the alternations of the seasons, may be obtained by running a long pin through the centre of a spherical ball, so as to represent the earth and its axis, and carrying the ball by hand in a circular or slightly elliptical orbit round a lamp or candle as a centre, meant to represent the sun, the horizontal plane representing the ecliptic. If the axis of the ball be inclined to it at an angle of about 67° (the complement of the obliquity 23°), the upper end representing the north pole and the lower the south pole, it will be observed that as the ball is moved horizontally round the lamp, always keeping the axis parallel to itself, the upper and lower sides alternately receive a greater or less portion of light and heat: the one may be taken to represent the conditions of summer, the other those of winter. There will also be two opposite positions of the ball, neither extremity of the axis being turned towards the light, in which the two halves or hemispheres of the ball would be equally illumined. These positions would correspond to that of the earth at the commencement of spring and autumn, the sun being in the equinox.

If, in addition to its motion in its orbit round the lamp, the ball were at the same time made to rotate round its own axis, the explanation of the phenomena of day and night would be obtained. It would be observed that each half of the ball would be successively illumined, also that when the ball was in such a position as to turn one extremity of its axis towards the light, its upper half would be wholly illumined, while the lower half would be in a corresponding condition of darkness. When the ball arrived at the opposite point of its circuit these conditions would be reversed; the part formerly illumined would now be deprived of light, and the part formerly in darkness would receive the light. Thus would be illustrated the conditions of the long polar day prevailing in the summer time and of the corresponding long night experienced in the winter season in the polar regions.

The phenomena of alternate seasons, summer and winter, spring and autumn, and the changes of day and night, produced by the combined motions of revolution round the sun and rotation round their own axes, are actually experienced by all the planetary bodies of the solar system.

Ecliptic

Owing to the *real* movement of the earth in the direction of the double arrows (fig. 55), the centre of the sun will appear, in the course of a complete revolution or year, to describe a circle on the celestial concave in an *opposite* direction (fig. 56); this circle is termed the Ecliptic. The centre of the sun is of course always *on* the ecliptic; and since the planes of the orbits of the moon and principal planets are but little inclined to that of the ecliptic, they at all times are *near* it.[1]

Equinoctial or Celestial equator

The plane of the earth's equator, being supposed to extend to the celestial concave, would form upon it another great circle which would cut the ecliptic in two opposite points.

Obliquity of ecliptic

The great circle thus traced on the heavens is termed the equinoctial,[2] and is evidently inclined to the ecliptic at the same angle that the plane of the earth's equator is inclined to that of its orbit; this angle (23° 27′) is termed the obliquity of the ecliptic.[3]

Equinoctial points

The axis of the earth remaining for all practical purposes parallel to itself during each and every revolution, the two opposite points of intersection of the ecliptic and equinoctial are practically *fixed* points in the celestial concave; they are known as the first points of Aries and Libra, since in very early ages they occupied positions in those constellations. On March 20, when the earth is at E_1, the centre of the sun occupies on the celestial concave the same apparent position as the first point of Aries, which is therefore termed the vernal equinoctial point; and for a similar reason, when the earth on September 22 is at E_3, the sun apparently occupying the first point of Libra, that is termed the autumnal equinoctial point.

Thus the first point of Aries is indicated by the position of the sun where it moves from south to north of the equinoctial; it is usually denoted by the symbol ♈.

Similarly, the first point of Libra, denoted by the symbol ♎, is at the position occupied by the sun when it moves from north to south.

---

[1] The principal planets, except Mercury, are at all times situated within 4° of either side of the ecliptic. Mercury at times is 7° north or south of it, and the Moon 5°.

An imaginary belt of the heavens extending 8° on both sides of the ecliptic is termed the Zodiac (*see* Inman's Tables, No. 19).

[2] This great circle is sometimes termed the Equinoctial and sometimes the Celestial Equator; the former term will be retained in this book, to avoid confusion with the *terrestrial* equator. The obliquity is more accurately 23° 27′ 9″ (1898), and is subjec to small changes extending over very long periods.

[3] A difficulty may occur to the reader that the plane of the equator, when the earth is at E_2 (fig. 55), is not *identical* with that when the earth is at E_4, and that therefore the intersection of this with the *fixed* plane of the ecliptic will not be the *same* in the two cases. This is strictly true, and would be appreciable at a *finite* distance, but is not so at the *infinite* distance of the celestial concave.

Solstitial points

The great circles of the ecliptic and equinoctial *intersecting* each other at the equinoctial points must be most widely separated at positions midway between them; these are known as the solstitial points, and the sun appears to occupy those positions on the celestial concave when the earth is at E_2 (June 21) and E_4 (December 21).

Co-ordinates of position

The equinoctial being a fixed circle on the celestial concave, and the first point of Aries a fixed point on that circle,[1] the positions of heavenly bodies may be expressed in precisely the same way as those of places on the surface of the earth are expressed by reference to the equator and meridian of Greenwich.

The terms Declination and Right Ascension are therefore applied to *heavenly* bodies in the same sense that latitude and longitude are applied to positions on the earth.

Circles of declination

Circles of Declination are great circles which pass through the poles of the heavens and correspond to terrestrial meridians; they are sometimes termed Hour Circles.

The circle of declination passing through the first points of Aries and Libra is known in practical astronomy as the equinoctial colure, and that at right angles to this passing through the Poles of the equinoctial and ecliptic as the solstitial colure.

Six o'clock hour circle

The circle of declination passing through the east and west points is termed the six o'clock hour circle.

Parallels of declination

Parallels of Declination are small circles whose planes are parallel to that of the equinoctial.

Declination

The Declination of a heavenly body is the arc of a circle of declination intercepted between the equinoctial and the place of the body; it is thus similar to *latitude* on the globe. Declinations are measured both north and south of the equinoctial, increasing from 0° at the equinoctial to 90° at each Pole.

Polar distance

The Polar Distance of a heavenly body is the arc of a circle of declination intercepted between the elevated Pole and the place of the body, and is therefore the complement of the declination.

Right ascension

The Right Ascension of a heavenly body is the arc of the equinoctial intercepted between the first point of Aries and the circle of declination which passes through the place of the body; it is measured eastward from the first point of Aries, or against the movement of the hands of a watch, increasing to 360° or twenty-four hours (*see* page 180). Thus, the right ascension of the sun on March 20 is 0°, or 0 hrs.; on June 21, 90°, or six hours ; on September 22, 180°, or twelve hours ; and on December 21 it is 270°, or eighteen hours.

---

[1] The first point of Aries is not really a fixed point, but moves yearly with a motion (approximately) of 50″ to the westward, or from S_1 towards S_4 (fig. 56).

The term *Right* ascension was originally used as a distinction from *Oblique* ascension, which in old works on navigation was defined as 'an arc of the equinoctial intercepted between ♈ and the east point of the horizon where the heavenly body is rising.' Ascensional difference was defined as the 'difference between right and oblique ascension,' while oblique descension was 'the arc of the equinoctial between ♈ and the west point of the horizon where the body is setting.'

The fixed stars are so distant that their apparent positions are unaffected by the annual change of position of the earth, and their declinations and right ascensions practically only change by a small yearly amount, dependent principally on the annual movement of the first point of Aries.

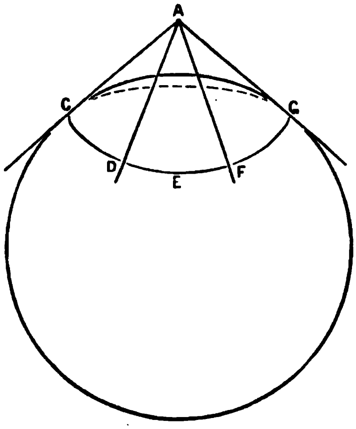
Figures explaining the foregoing definitions will be given after some further illustrations of the terms Horizon, Altitude, and Azimuth.

The positions of heavenly bodies with regard to the first point of Aries and the equinoctial are obtained by the fixed instruments of astronomical observatories; but observations at sea cannot at first be made to depend upon the ecliptic and equinoctial, and must be referred to some *visible* line or circle.

Horizons and zenith

This visible object is the Horizon, which at sea appears as an unbroken circle, at the centre of which is the observer. The following definitions apply to this part of the subject, and are illustrated by figs. 57 and 58.

FIG. 57.

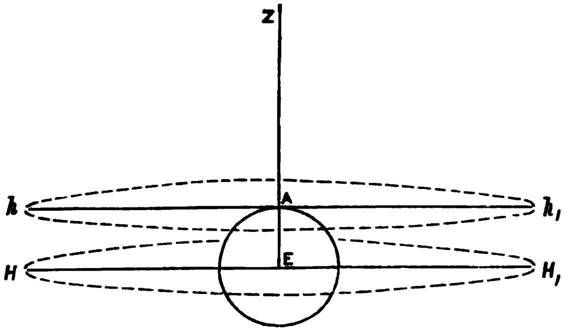


The *apparent* or *visible* horizon (C D E F G, fig. 57) is that actually seen by an observer (A) elevated above the surface of the earth ; and its distance from the observer will evidently vary with his elevation.

The *sensible* horizon is that circle on the celestial concave the plane of which touches the earth at a point vertically beneath where the spectator stands, as h A h_1 (fig. 58) ; and a circle parallel to this and passing through the centre of the earth is termed the *rational* horizon: thus H E H_1 is the rational horizon. These two circles, however, practically form one and the same great circle on the celestial concave. This may be readily conceived when we consider that the distance of any two points on the surface of the earth will make no sensible angle at the *infinite* distance of the celestial concave; therefore either of these two circles is to be understood by the word 'horizon' when considering the various heavenly bodies.

The sensible horizon and a plane parallel to it passing through the eye of the observer may be considered the same for any elevations above the earth's surface with which we have to deal.

FIG. 58.



The Zenith of an observer is that point in the celestial concave which is the extremity of the line drawn perpendicular to the rational horizon through the place of the spectator, as Z (fig. 58).

The Nadir (a term not often used) is the point in the celestial concave opposite to the zenith.

Rising and setting

The Horizon and Zenith are of course different for different positions on the earth, and also are both carried round by the daily rotation of the earth on its axis; the observer, being unconscious of this motion, perceives the heavenly bodies apparently moving in the opposite direction.

Thus, in fig. 59, if A be an observer on the surface of the earth and Z his zenith, the dotted circle, supposed to be the edge of a plane tangent to A and extending to the celestial concave, will represent the sensible horizon.

The direction of rotation of the earth being that shown by the arrows, a star (B) will appear to be rising above, and a star (C) to be setting below the horizon, while D will be at some elevation or altitude above it.

Vertical circles

Great circles passing through the zenith are called *circles of altitude* or *vertical* circles, and the *altitude* of a heavenly body may be defined as its angular distance from the horizon measured on a vertical circle.

Altitude

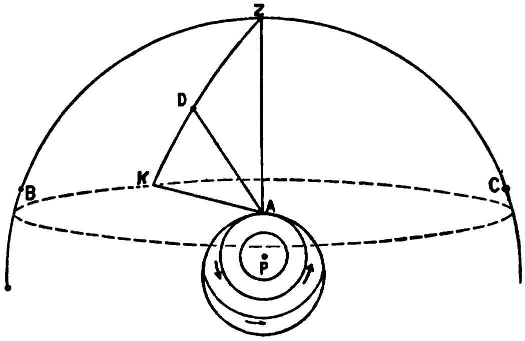
The complement of the altitude is termed the *zenith distance.*

Celestial meridian

That circle of altitude which passes through the Poles of the heavens is the *celestial meridian*, being in fact the plane of the terrestrial meridian extended to the celestial concave. The points of the horizon through which the celestial meridian passes are called the *north* and *south* points. A circle of altitude at right angles to the meridian is termed the *prime vertical.* This last circle intersects the horizon in two points, called the *east* and *west* points. The east and west points are the Poles of the celestial meridian, and the north and south points the Poles of the prime vertical.

Prime vertical

Fig. 59.



Azimuth

The *azimuth*, or bearing of a heavenly body, is an angle subtended by the arc of the horizon intercepted between the north or south points[1] and the circle of altitude passing through the place of the body; or it is the corresponding angle at the zenith between the celestial meridian and the circle of altitude passing through the body.

In fig. 59, Z K represents part of a circle of altitude, and the altitude of a star (D) will be the angle D A K, or the corresponding arc D K of a vertical circle; the zenith distance being the angle D A Z, or the corresponding arc Z D.

Astronomical diagrams

The heavenly bodies are seen on the celestial concave by an observer on the earth *within* it. Since, however, no *one* figure could show the whole interior of a spherical surface, astronomical diagrams are usually drawn on the supposition that the observer is *outside* the celestial concave.

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[1] In Practical Astronomy azimuths are reckoned only from the south point, and increase to 360° through west, north, and east.

1. If the point of view is supposed to be immediately over the zenith of the observer, the representation obtained is said to be on the *plane of the horizon.*

2. If the point of view is above the pole of the heavens, the representation is said to be on the *plane of the equinoctial.*

3. If the point of view is in the plane of the equinoctial, and at either the east or west point, the representation is described as being on the *plane of the celestial meridian.*

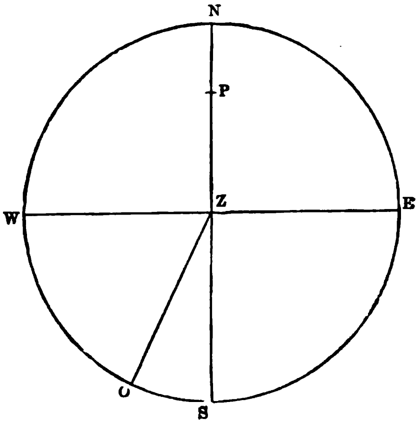
Of these three *graphical* modes of representing the positions of heavenly bodies, that on the plane of the horizon is most frequently used, but some problems will be found more clearly illustrated by one or other of the projections on the plane of the equinoctial or meridian.[1]

Graphical illustrations

The definitions of Right Ascension, Declination, &c., will now be illustrated by figures drawn on the *plane of the horizon.*

In fig. 60, let N W S E represent the horizon of the spectator; then the point Z, considered as the pole of the horizon, will be the zenith, and portions of great circles W E, N S, Z O, will be parts of *circles of altitude* or vertical circles, usually represented (though not quite accurately) as straight lines. Let the circle of altitude Z N pass through the pole of the heavens P; then N Z S is the *celestial meridian* of the spectator, whose zenith is Z; and the circle of altitude W E, drawn at right angles to the celestial meridian, is the *prime vertical.* If P be the north pole, then the point of the horizon (N) intersected by the celestial meridian is the *north* point, and the opposite point of the horizon (S) is the *south* point; the points of the horizon W, E, intersected by the prime vertical, are the *west* and *east* points.

FIG. 60.



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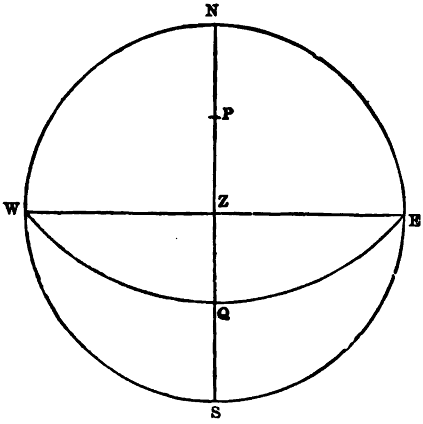
[1] These methods of drawing astronomical diagrams are all upon the *stereographic* projection of the celestial sphere.

To represent on the plane of the horizon, the pole of the heavens and the equinoctial for a given latitude.

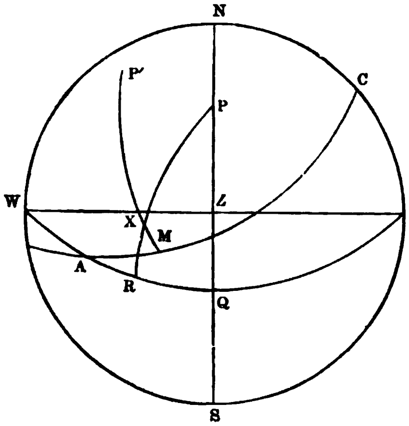
Fig. 61.



In fig. 61, let N W S E represent the horizon, Z the zenith, N S the celestial meridian, and W E the prime vertical; take Z Q = the given latitude, and through the points W, Q, E draw a great circle W Q E; this will represent the equinoctial. For since the horizon and equinoctial are both perpendicular to the celestial meridian, the points where these intersect each other must be 90° distant from every part of the meridian; that is, the equinoctial must cut the horizon in the east and west points. From Q measure Q P = 90°; then P will represent the pole of the heavens, since the pole of the equinoctial is the pole of the heavens.

To represent the right ascension, declination, and polar distance of a heavenly body.

Fig. 62.

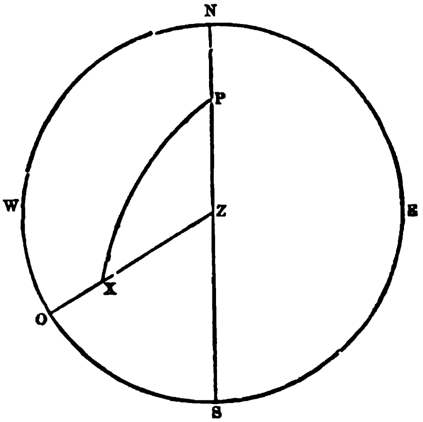


Let N W S E represent the horizon (fig. 62), P the pole, Z the zenith, and X the place of a heavenly body. Draw the equinoctial W Q E and let A be the first point of Aries; through A draw A C to represent the ecliptic inclined at an angle of 23° 27′ to W Q E. Through X draw a circle of declination, P R. Then A R is the *right ascension*, X R is the *declination*, and P X is the *polar distance* of X.

To represent the altitude, zenith distance, and azimuth or true bearing of a heavenly body.

Let N W S E represent the horizon (fig. 63), N S the celestial meridian, P the pole, and Z the zenith. Let X be the place of a heavenly body; through X draw the circle of altitude, Z O, and circle of declination, P X. Then X O is the *altitude* of the heavenly body, and Z X its *zenith distance*. The angle P Z X or S Z X, or the arc of the horizon N O, or S O, is its *azimuth* or true bearing.

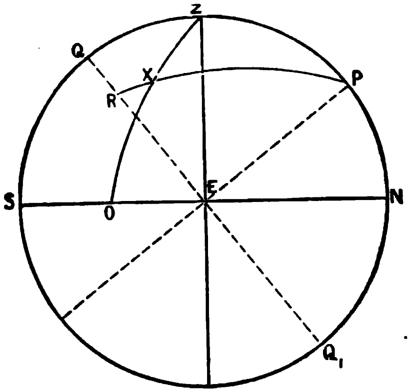
FIG. 63.



The following figure illustrates some of the preceding definitions by a diagram drawn on the plane of the meridian (fig. 64), the point of sight being the east point, and all great circles passing through that point being correctly represented by straight lines.

E represents the east point; N P Z S the meridian, N and S being the north and south points; P is the pole and Q Q_1 the equinoctial; Z is the zenith and S N the horizon; P X is the polar distance of a heavenly body X, and X R its declination; Z X is the zenith distance of X, and X O its altitude; while the angle S Z X or P Z X is the azimuth of X.

FIG. 64.



In a similar way the student should when possible represent the examples for exercise throughout this work by figures drawn on each of the three planes; that of the Equinoctial is illustrated on page 172.

Systems of co-ordinates From the preceding definitions it will be observed that the position of a heavenly body at any instant of time is defined either by its right ascension and declination, or by its altitude and azimuth; the former elements belonging to what is sometimes termed the Equinoctial system, and the latter to the Horizon system.

Our knowledge of the position of a ship at sea depends upon comparisons made between *observed* positions of heavenly bodies with regard to the horizon or to one another, and those they are known to occupy at the same time, as seen from the centre of the earth. The *observed* positions must be referred to the horizon system, but the *known* positions must equally depend on the equinoctial system, since they are obtained from *predictions* which could not possibly be made for the horizon of *every* place, and can only be expressed in terms of elements which are not affected by the position of an observer.

These predicted positions are given in the *Nautical Almanac*.

Celestial latitude and longitude

In practical astronomy the positions of heavenly bodies with regard to the *ecliptic* and the first point of Aries are often required to be known; these are expressed as their geocentric latitudes and longitudes, but for nautical purposes these elements (termed the Ecliptic system) are not employed. If in fig. 62 P′ represents the pole of the ecliptic, and a great circle P′ M be drawn passing through P′ and a heavenly body X, then X M is the *latitude* and A M the *longitude* of X.

The latitudes of heavenly bodies are reckoned in precisely the same way as terrestrial latitudes, but celestial longitude is reckoned *eastward* from the first point of Aries continuously through 360° along the ecliptic.

The position of a *planetary* body as seen from a spectator's place on the earth's surface, after the application of the necessary corrections, gives its *geocentric* place, or its position as viewed from the earth's centre. From this by calculation can be obtained its *heliocentric* place, or its position as it would appear if seen from the sun's centre. The *stars* are at such a vast distance that their geocentric and heliocentric places are the same.

CONVERSION OF RIGHT ASCENSION AND DECLINATION INTO LONGITUDE AND LATITUDE, AND THE CONVERSE.

In figure 65 let A be the first point of Aries; A E the equinoctial, and P its pole; A L the ecliptic, and P′ its pole.

Let S be a star, P S D the circle of declination, and P′ S I the circle of latitude, passing through it. Join A and S by the arc of a great circle.

Let A D, the star's right ascension $= a$
S D, its declination $= \delta$
A I, its longitude $= L$
S I, its latitude $= \lambda$

Also, let L A E, the obliquity of the ecliptic $= \omega$; S A D $= \phi$; and A S $= x$.

Then in the right-angled triangle S A D—

$$\tan \phi = \frac{\tan \delta}{\sin a} \qquad (a)$$

$$\sin \phi = \frac{\sin \delta}{\sin x} \text{ or } \sin x = \frac{\sin \delta}{\sin \phi} \qquad (b)$$

And

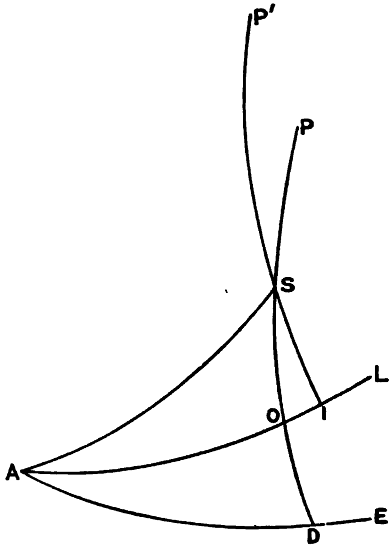
$$\cos \phi = \frac{\tan a}{\tan x} \text{ or } \tan x = \frac{\tan a}{\cos \phi} \qquad (c)$$

Similarly in the right-angled triangle S A I—

$$\sin (\phi - \omega) = \frac{\sin \lambda}{\sin x} \text{ or } \sin x = \frac{\sin \lambda}{\sin (\phi - \omega)} \qquad (d)$$

$$\cos (\phi - \omega) = \frac{\tan L}{\tan x} \text{ or } \tan x = \frac{\tan L}{\cos (\phi - \omega)} \qquad (e)$$

FIG. 65.



Then from (a) ϕ becomes known, and equating the values of $\tan x$ from (c) and (e)—

$$\frac{\tan L}{\cos (\phi - \omega)} = \frac{\tan (a)}{\cos (\phi)}$$

$$\therefore \quad \tan L = \frac{\tan a \cos (\phi - \omega}{\cos \phi}$$

Again, equating the values of $\sin x$ from (b) and (d) we have—

$$\sin \lambda = \frac{\sin \delta \sin (\phi - \omega)}{\sin \phi}$$

From which formulæ we can obtain the geocentric longitude and latitude of a heavenly body, its right ascension and declination being known.

The Nonagesimal, called also the 'mid-heaven,' is the highest point, or 90th degree of the ecliptic reckoned from its intersection with the horizon at any time, and its altitude is equal to the angle that the ecliptic makes with the horizon at their intersection, or equal to the distance of the zenith from the pole of the ecliptic. This point is frequently mentioned in discussions, in which the places of the heavenly bodies being referred to the ecliptic, their latitudes and longitudes are elements of the calculation.

We now proceed to a detailed description of the principal instrument employed in nautical astronomy for determining the positions of heavenly bodies at any instant of time.

THE SEXTANT.

Sextant

The sextant[1] is an instrument designed for the measurement of angles, and especially adapted for use at sea, where the constant motion of a ship renders *fixed* instruments for that purpose of no avail; the chief use of the sextant for nautical purposes is to measure altitudes of the heavenly bodies.

Laws of reflection

The construction of this instrument is based on the principles of the laws of reflection of light from plane mirrors, which are—

(1) The angle of reflection is equal to the angle of incidence.

(2) The incident and reflected ray are both in the same plane, which is perpendicular to the reflecting surface.

The following are the names of the principal parts of a sextant:

Names of parts of sextant

In fig. 66, *m m′*, the index glass, or movable reflector, is a plane mirror attached to M C, the radius bar, and movable round the centre M. *f f′* is the horizon glass or fixed reflector, half of which is a plane mirror and half unsilvered glass. O is the telescope,[2] screwing into the collar H K, which is supported on what is termed the up and down piece, and can by means of a screw at the back be drawn in or out from the body of the instrument. V is the vernier plate, immediately above or below A B, the graduated arc or limb. N (not shown) is the clamp screw behind C, and T the tangent screw; S being a magnifying glass for reading the graduated arc, and W and X coloured shades.

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[1] A manuscript description of the principles of construction of the sextant was left by Sir Isaac Newton; the first instrument actually made was by Hadley in 1731. Originally the arc only subtended an angle of 45°, and the instrument was therefore termed a quadrant, being only available for measuring angles of 90° and less.

[2] Sextants are usually fitted with two telescopes, one of which inverts objects while the other does not. The object of an inverting telescope is to obtain considerable magnifying power without much loss of brightness, the loss of light in an erecting telescope being occasioned by the necessary introduction of additional lenses, each of which absorbs a certain amount of the light passing through it emanating from the object looked at.

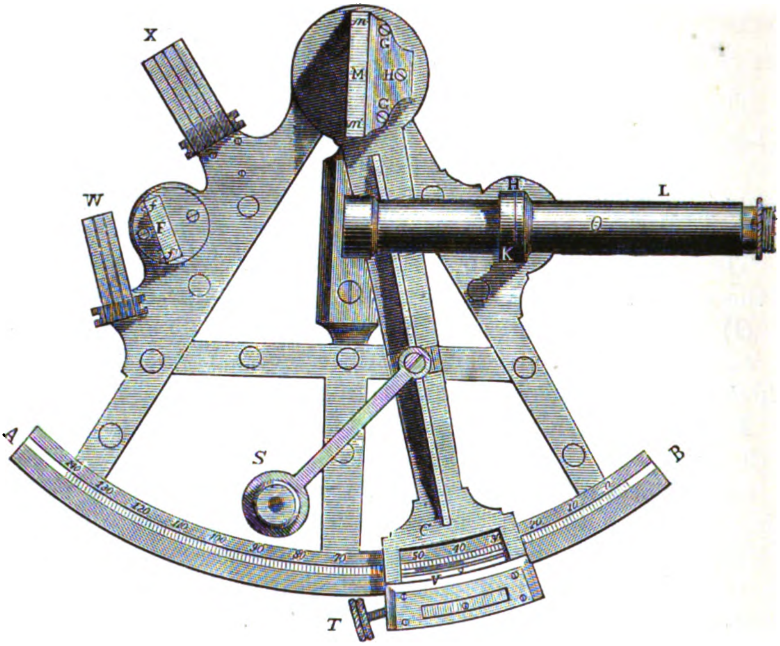
A telescope having a large field glass for observations of stars is now generally supplied also.

A greatly improved form of sextant, admitting of measuring angles up to 180°, was formerly constructed by Pistor & Martins, Berlin. In this instrument a *prism* was substituted for the mirror of the horizon glass.

It is, however, extremely difficult to cut a prism sufficiently accurately.

The theory of the construction of the sextant may be explained by means of the following figure (fig. 67).

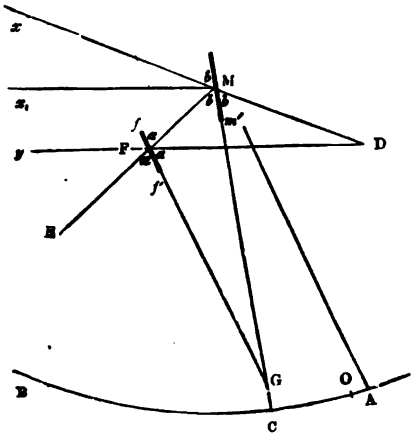
FIG. 66.



Theory of construction

Suppose a ray of light proceeding from the object x (fig. 67) in the direction x D to impinge on the surface of the movable reflector M at the angle x M m; then the ray will be reflected back in the direction M F, making an angle F M m' with the movable reflector equal to the angle x M m. Again, at the fixed reflector F, the ray M F will suffer another reflection in the direction F D, making, with the reflecting surface ff', the angle D F f' = angle M F f. If we suppose an observer's eye to be placed at D, and another ray of light to proceed from the object y along the same line y F D, the rays from the two objects x and y will thus appear to come to the spectator from the same point y; the image of the object x having been transmitted to him from the quicksilvered part of F, and the direct image of y through the part of F which is left transparent for that purpose. The angular distance between x and y, which is required to be found, is the angle x M x_1 = D,

FIG. 67.



and this angle D will be proved (*see* page 136) to be double of the angle A M C, measured by the arc A C; A M being supposed to be drawn parallel to the surface $f f'$ of the fixed reflector F. Hence, if the arc A B, supposed to be the sixth part of a circle,[1] or to subtend 60° at M, be so graduated that it shall contain twice that number, or 120°, then the *reading* on the arc A C will be the value of the angle at D; and this is the method adopted in dividing the arc of the sextant.

To observe, therefore, the angle between any two objects, x and y, the observer[2] at D looks directly at the left-hand or lower object y (according as the angle to be measured is horizontal or vertical) through the fixed reflector F; he then moves the radius bar M C, attached to the movable reflector M, in the plane passing through D and the two objects, until he sees the ray proceeding from x in the same direction as the object y. Then the reading on the arc A C measures the angle at D, the angular distance between the two objects x and y.

NOTE.—x and y are supposed to be sufficiently distant for two rays from either of them to each of the mirrors M and F to be sensibly parallel.

If measuring an angle between two objects differing in either distinctness or brightness, the least distinct or bright should be looked at *directly*, since by reflecting an object it necessarily becomes fainter; if this direct object is on the right of the other, the sextant should be held in an inverted position with the face downwards.[3]

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[1] Hence the original name of *sextant*, though such instruments now generally measure more than 120°.

[2] The observer's eye is seldom exactly at the point D, but at some other point in the line D F; this, however, will make no appreciable difference when the objects x and y are at a considerable distance from the spectator, as the sun or moon; in all cases the angle *really* measured is that subtended at M between the two observed celestial objects.

[3] In sextants used for surveying operations two index glasses are sometimes fitted to the same instrument, thus admitting of two angles being measured at the same instant, or of a left-hand object being brought to the right without inverting the

Proof that the arc A C *or inclination of the mirrors to one another measures the angle at* D *between the two objects* x *and* y.

Produce M F (fig. 67) to E and ff' to cut the line M C in G; then the angles x M m and D M G are equal, being vertical angles; also the angle x M m is equal to the reflected angle F M G; let therefore these three equal angles at M be represented by the same letter b; in the same manner the three angles, denoted by a, formed at F by the reflected ray, may be shown to be equal.

Now, in the triangle M D F, the exterior angle $\text{EFD} = \text{FMD} + \text{MDF}$, or $2a = 2b + \text{D}$.

$$\therefore\ a = b + \tfrac{1}{2}\text{D};$$

also in the triangle F G M, the exterior angle $a = b + \text{FGM}$.

$$\therefore\ b + \tfrac{1}{2}\text{D} = b + \text{FGM}, \text{ or } \tfrac{1}{2}\text{D} = \text{FGM} = \text{GMA},$$

since F G is parallel to M A.

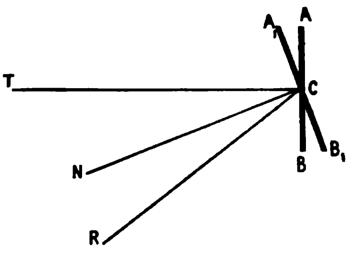
But the arc A C, which measures the angle G M A, is divided into double the number of degrees due to its length, the divisions being supposed to commence at the point A; therefore the angle *read* on the arc A C measures the angle D, the angular distance between the two objects.

The fact that the direction of a ray of light reflected from the surface of a plane mirror is changed by double the angle through which the mirror is turned, is of such importance that another illustration is now given.

Let A B (fig. 68) represent a plane mirror perpendicular to the plane of the paper and capable of rotation round an axis C, also perpendicular to the paper.

Let T C represent an incident ray which, when the mirror is in the position A B perpendicular to T C, is reflected back in the same line.

Fig. 68.



On turning the mirror through an angle A C A_1 the reflected ray takes the direction C R, making an angle N C R with N C the perpendicular on the mirror equal to the angle of incidence T C N. Hence the deviation T C R of the reflected ray is double the angle T C N or A C A_1 through which the mirror has been turned; and on

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sextant. This form of instrument (termed a double sextant) might be also very useful in navigation.

rotating the mirror through a further angle θ, the reflected ray will be turned through a further angle $2\,\theta$.

The principal adjustments of a sextant are the following:

Adjustments of the sextant

1. The movable reflector or index glass M (fig. 66) should be perpendicular to the plane of the instrument.
2. The fixed reflector or horizon glass F should be perpendicular to the plane of the instrument.
3. The line of collimation, or the imaginary line joining the centre of the object-glass with the centre of the space enclosed by the wires in the eyepiece, should be parallel to the plane of the instrument.
4. When the fixed and movable reflectors are parallel the reading on the arc should be zero.

Adjustment of movable reflector

First adjustment.—To see if the movable reflector is perpendicular to the plane of the instrument.

Place the radius bar M C near the middle of the arc, as at C (fig. 66); turn the face of the instrument upwards, and look obliquely into the reflector M, the eye being in the plane of the sextant and near M. Then the image of the arc A B will be seen in the reflector M; and if this image appears in one unbroken line with the arc A B itself, M is perpendicular to the plane of the instrument.

If the reflection of the line A B appears above or below the line A B itself, then the reflector M is out of adjustment, and must be corrected by certain screws or studs at the back. This adjustment, in good instruments, seldom requires to be made; and when it is requisite, it is best when practicable to send it to the maker to be rectified.

Adjustment of fixed reflector

Second adjustment.—To see if the fixed reflector is perpendicular to the plane of the instrument (technically to test for 'side' error).

Look through the telescope and the fixed reflector F (fig. 66) at a star, the sun, or any well-defined distant terrestrial object, holding the instrument in any position; bring the index towards the commencement of the divisions, and move it gently backwards and forwards until the image of the object is placed as nearly as possible upon the object itself; then, if the image is found to coincide exactly with the object, the fixed reflector F is perpendicular to the plane of the instrument.

If any portion of the object is seen not coinciding with its image, then F is not perpendicular to the plane of the instrument; and the adjustment is made by means of a screw, which in some instruments is under the glass, in others behind it, and in others at the side. The screw must be turned gradually till the image is made to coincide with the object. This adjustment, which is frequently required to be made, may be also tested by holding the sextant with its plane nearly vertical and bringing the direct and reflected images of the sea horizon into contact; on inclining the sextant until its plane is nearly *horizontal* the contact should still be maintained.

Adjustment of line of collimation

Third adjustment.—To see if the line of collimation is parallel to the plane of the instrument.

The telescope being placed in the collar of the sextant, turn the eyepiece round till two wires are parallel to the plane of the instrument.[1] Bring the darkened image of the sun (when at a considerable distance from the moon—*i.e.* from 90° to 120°) to touch the limb of the moon at the middle point of the upper of these wires, and then immediately, before any perceptible change in the distance of the two bodies can take place from their own proper motion, bring the point of contact of the two bodies to the lower wire, when, if they still appear in contact, the axis of the telescope may be considered to be parallel to the plane of the instrument; if otherwise, the adjustment is made by means of two screws in the collar—slackening one and tightening the other. In some instruments, however, these screws are wanting, the adjustment of the parallelism of the tube being supposed to be carefully made before the instrument leaves the maker's hands.

If the two objects appear to *separate* on the wire farthest from the plane of the instrument, the object end of the telescope *droops* towards the sextant, and *vice versâ.*

The adjustment for collimation may also be conveniently tested with an artificial horizon by contacts of the two suns made in a similar way when the sun is moving slowly in altitude, or by the moon and a bright star or two bright stars.

With a properly adjusted telescope two images seen in contact on the wires will *overlap* in the middle of the field.

The adjustment for collimation is seldom required to be made for ordinary observations at sea, but it should be carefully examined before taking lunars or before observing on shore with the artificial horizon.

Let M the centre of the movable reflector be considered the

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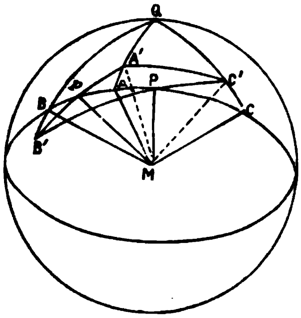
[1] This is effected by looking at a distant object or star and slightly separating the real and reflected images, keeping both in the field of view, and turning the eyepiece until both images are on the same wire.

Theory of error of collimation

centre of an imaginary sphere, and B A C a great circle, the plane of which is that of the face of the sextant and of which Q is the pole.[1]

FIG. 69.



Let A and C be two objects between which an angle is measured, the direction of the perpendicular to the index glass being M P, and that of the perpendicular to the horizon glass M *p*, the line of collimation M A being (if in adjustment) in the same plane as M P and M *p*.

In this case a ray of light from C is reflected from the index glass in the direction M B (P B = P C), and this ray is again reflected from the horizon glass in the direction A M (*p* B = *p* A), so that C and A appear in coincidence, and C M A = 2 *p* P is the true angle between C and A and also the angle measured by the sextant.

If, however, the line of collimation is inclined to the plane of the sextant so that it meets the surface of the sphere at A′ instead of A (A′ A measuring that inclination), then on tracing *backwards* the ray A′ M the plane of reflection from the horizon glass is A′ M B′, and B′ M (*p* A′ = *p* B′) is the direction of the ray as reflected from the index glass; therefore B′ P C′ represents the plane of reflection from the index glass, and C′ M (B′ P = P C′) the original direction of the ray which is seen reflected in the direction M A′.

Hence in this case the *true* angle between two objects A′ and C′ seen in coincidence is the angle A′ M C′, and the *sextant* angle is as before A M C = 2 *p* P.

Let the sextant angle A C = a
" true angle A′C′ = a'
" inclination of line of collimation A A′ = i.

Then since from the laws of reflection A A′ = B B′ = C C′, Q A′ C′ is an isosceles triangle in which Q A′ = Q C′ = 90° − i, and the angle Q = a and the side A′ C′ = a'.

Hence $\cos a' = \sin^2 i + \cos^2 i . \cos a$.

Let $a' = a - x$, so that x is the correction to be applied to the instrumental reading to obtain the true angle.

Then on making this substitution and expanding $\cos (a - x)$

$$\cos a . \cos x + \sin a . \sin x = \sin^2 i\,(1 - \cos a) + \cos a.$$

---

[1] This section may be omitted on first reading.

Since x is always a small quantity, we may consider $\cos x =$ unity, and the above expression becomes

$$\sin a \,.\, \sin x = \sin^2 i \, (1 - \cos a)$$
$$\therefore \sin x = \sin^2 i \,.\, \tan \tfrac{1}{2} a.$$

Examples

Required the error in the observed angles 90° and 150°, when the inclination of the line of collimation is 1°.

On substituting in the above formula values of $i = 1°$, and $a =$ first 90° and then 150°, it is seen that the error in one case is 1′ 3″, in the other 3′ 54″.

From this it appears that a slight inclination of the line of collimation to the plane of the instrument produces a considerable error in determining the true angle between two objects; that this error increases as the angle increases; and that the observer should always take care, in accurate observations, to make the contact as near the middle point of the field of view as possible.

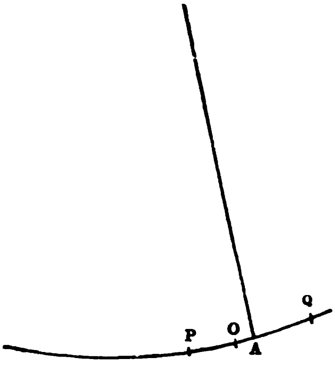
When this error exists *observed* angles are always too large.

The index error of the sextant

Fourth adjustment.—We have supposed above that the graduations of the arc commenced at A (fig. 67), the point in the arc cut by a line M A, parallel to the fixed reflector F; but this is seldom the case, the zero point of the arc being often a little to the right or left of A.

Let us suppose the graduation to commence at O (fig. 70), to the left of A, then the angle D (fig. 67) would be equal to the reading on O C + the small arc O A. The arc O A is called the *index error,* the value of which is usually determined by measuring a small angle, such as the sun's horizontal diameter,[1] off and on the arc, that is, to the right and left of O; to enable us to do which, the divisions of the arc are continued a little to the right of the zero point O (arc of excess, usually measuring 5°).

FIG. 70.



The index error may also be defined as twice the angle between the planes of the horizon and index glasses when the angle read on the arc is zero.

Let P be the point on the arc through which the line M C passes (figs. 67 and 70) when there is a contact of the direct and reflected limbs of the sun *on* the arc, and Q the point through which M C passes when there is a contact of the two limbs to the right of O, and therefore called *off* the arc; then, since the direct and reflected suns must coincide when the line M C is at A, the arc A P = arc A Q.

---

[1] The reason for measuring the *horizontal* diameter (especially at low altitudes) will be found on page 150. *Horizontal* diameter does not of course mean the diameter when *on* the horizon.

Let a = O P, the measure of the sun's horizontal diameter on the arc;
b = O Q, „ „ „ off the arc;
e = O A, the index error required.

Then, since A P = A Q,

$$\therefore \quad e + a = b - e;$$

$$\text{or} \quad 2e = b - a,$$

$$\therefore \quad e = \tfrac{1}{2}(b - a);$$

or the index error is equal to half the difference of the measures of the sun's diameter off and on the arc.

In this case, the index error O A, or e, must be added to the arc O C, to obtain the angle between the two bodies x and y (fig. 67); this is evident from the figure: hence the index error is said to be additive when the reading on the arc is less than the reading off the arc.

In the same manner it may be shown that if the zero point O is to the right of A, the index error $e = \frac{1}{2}(a - b)$, and is subtractive; that is, if the reading on the arc is greater than the reading off the arc, when a contact of the true and reflected limbs of the sun is made.

It is also evident that the arithmetical sum of these readings should, with good observations, equal four times the sun's semi-diameter as given in the *Nautical Almanac*.

The index error may also be found by bringing into coincidence the direct and reflected images of a star or very distant terrestrial object,[1] or the sea horizon when well defined, the reading when the images are in contact being the index error, additive when it is on the right of zero and subtractive when on the left.

The index error can be removed by turning the horizon glass (by its adjusting screws) round an axis perpendicular to the plane of the sextant, but it is not advisable to do this unless the error is very large.

---

[1] With *near* terrestrial objects, since the rays of light from them to the index and horizon glasses are not parallel, the direct and reflected images of the same object will not coincide when these mirrors are parallel, and the index error will differ from that obtained from observation of the sun by the angle subtended at the object by the distance between the mirrors.

This angle (termed the *parallax* of the sextant) is included in the index error when that is determined from the object which is seen through the horizon glass. With the sextants commonly used in the Royal Navy this angle is about 20″ for an object at a distance of half a mile.

In some sextants of French manufacture this object is attained by moving the vernier plate.

Examples

1. On April 19 the sun's diameter *on* the arc measured 29′ 20″, and *off* the arc 34′ 30″. Required the index error.

Reading on arc......	29′ 20″	29′ 20″	
Reading off arc......	34 30	34 30	
	2)5 10	4)63 50	
Index error.........	+2 35	15 57	sun's semi-diameter (agreeing with that in *Nautical Almanac*).

2. On January 1 the mean of five observations of the sun's diameter measured on and off the arc gave the following readings:

On, 34′ 10″; off, 31′ 5″. Required the index error.—*Ans.*, 1′ 32″ subtractive.

NOTE.—In the rare cases of *both* readings being either on or off the arc, half their sum is the index error, and a quarter their difference should be the sun's semi-diameter.

It is to be remarked that, owing to the 'spring' or elasticity of the radius bar, there is a small but perceptible difference between the index errors obtained by the onward and backward movements of the tangent screw.

The stability with which the index error of a sextant is *maintained* is an excellent test of the quality of the instrument.

The vernier

The vernier[1] is a small instrument applied to most graduated nautical instruments to enable readings to be accurately obtained when the index lies *between* the graduations on the arc, or, in other words, to subdivide the spaces between these graduations.

It takes the general form of a small arc whose graduations are closely applied to those of the instrument in question, and the principles of its construction may be illustrated by the following general considerations:

Suppose that the limb of the instrument considered is graduated so that the length of each division is d, and it is required to subdivide each of these into m equal parts; then a space equal in length to $m-1$ divisions of the limb is on the vernier divided into m equal parts, so that the length of m divisions on the vernier $=(m-1)\,d$, and therefore that of *one* division is $\left(\frac{m-1}{m}\right)d$.

---

[1] The vernier was invented by Pierre Vernier in 1631.

Hence the *difference* in length between one division on the instrument and one on the vernier $=d-\left(\frac{m-1}{m}\right)d=\frac{d}{m}$.

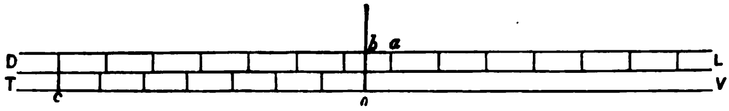
Let this difference be represented by δ.[1]

If in fig. 71 D L represent the limb of an instrument and T V a vernier divided as above; then if $b\,o$ represents the position of the index, the angle to be read off is in excess of a, the nearest integral division on the right by the arc $b\,a$.[2]

On looking to the left, it is seen that the graduations of the limb and vernier are in one line at c, supposed to be n divisions of the vernier from o.

The value of $b\,a$ is therefore $n\,\delta$ within a limit of error of δ, since on looking to the right from c the divisions of the vernier fall short of those on the limb by successive values δ, $2\,\delta$, $3\,\delta$, $n\,\delta$, and therefore from a position in which $b\,o$ was in line with a, the index would have to be moved over an arc $n\,\delta$ in order to occupy its present position.

FIG. 71.



In good sextants it is usual to make each division of the limb equal to 10′, and to construct the vernier so that, if completely divided, 120 of its divisions would equal in length 119 divisions of the limb; this would give the value of δ equal to 5″; but as for most nautical purposes such accuracy is unnecessary, only every *alternate* division of the vernier is given, and the reading can be obtained to 10″. Such an arrangement is generally termed an extended vernier.

Where angles are required to be read off on the arc of excess, as in determining index errors by observations of the sun, the vernier is read from left to right; that is to say, the number of divisions are counted from the left of the vernier until a division on that is seen to be in line with one on the arc.

This arises from the graduations in this case increasing in value from left to right.

The vernier attached to a station pointer and to the azimuth circle of an Admiralty compass usually enables angles to be read off to the nearest minute.

---

[1] δ is termed the *least count* of the vernier, and is equal to the value of the smallest division on the arc divided by the number of divisions on the vernier.

[2] That is on the supposition that the value of the graduations increases from right to left, which, in the sextant, is the case when angles are read off *on* the arc.

The vernier of a barometer is constructed so that twenty-five divisions upon it equal in length twenty-four divisions on the scale, so that, as each of the latter is ·05 inch in length, the graduations can be read to a degree of accuracy of ·002 inch $\left(\frac{\cdot 05}{25}\right)$.

The accompanying figures represent the position of the vernier T V, with regard to the limb of a sextant D L (1) when the angle read off is 3° 30′ *on* the arc, and (2) when the angle is 30′ *off* the arc, the least count of the vernier being 1′.

Fig. 72.

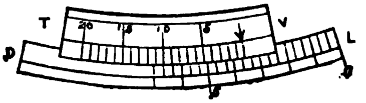
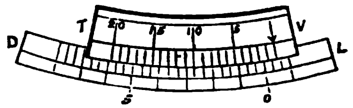


Fig. 73.



Notes on the sextant

In order to equalise the apparent brilliancy of two objects between which an angle is being measured, the coloured shades should be turned down so as to give as little contrast of *colour* as possible, and the telescope, by means of the up and down piece, screwed in or out from the plane of the instrument until the intensities of light from the two objects are equal.[1]

The two faces of each of the coloured shades should be parallel to one another; this may be tested by measuring the sun's diameter with a suitable combination of shades: invert one of the shades by removing the pin upon which they turn, and again measure the diameter. Half the difference of the two readings will be the error due to that shade. The error may also be tested by comparing the index error determined with the coloured glass over the eye-glass of the telescope with those formed by using the ordinary shades.

The front and back surfaces of the index glass should be perfectly parallel to each other; this may be tested by examining through a telescope the distinctness of the image of a star reflected at an angle of about 120°. Any such defect in the horizon glass will equally affect *all* angles (including the index error), and therefore produce no effect.

A source of error to which all circular graduated instruments are liable arises from the centre of the arc of *graduation* not exactly coinciding with the centre of rotation; thus in a sextant, if A is the centre on which the index glass turns, and A_1 the centre of the graduated arc B_1 C_1, on observing an angle B A C the magnitude is incorrectly read off on B_1 C_1 instead of on B C.

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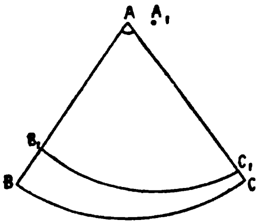
[1] The necessity for doing this arises partly from the fact that of two objects of exactly the same size the more brilliant always appears to be the larger; this effect is termed irradiation.

FIG. 74.



On instruments such as the theodolite and azimuth circle of the standard compass, where the graduation extends round a complete circle, the effect of this error is obviated by taking a mean of two readings 180° apart.

In a sextant this error (which differs for different angles) may be found by comparing *observed* distances between stars with those calculated from their astronomical positions [1]; these observed distances must be 'cleared' for refraction similarly to lunar distances.

Where the latitude is *very exactly known*, this error is also found by comparing observed and calculated meridian altitudes of various stars, such observations being made with the artificial horizon (*see* page 337).

NOTE.—What is commonly known as centering error practically includes errors arising from (1) eccentricity of the centre of the axis of the radius bar and the centre of the arc, or centering error proper; (2) faulty graduation; and (3) flexure of the frame of the instrument caused by varying temperature or by accidental blows.

The combined effect may be tested by observing stars at nearly equal altitudes north and south of the zenith in the artificial horizon; half the difference of the latitudes resulting from each star will be the centering error for that altitude, the correction being *minus* if the latitude from the star on the polar side of the observer is the greater, and *plus* if the contrary is the case; or the angular distances of a number of terrestrial points in the same horizontal plane may be accurately determined with a theodolite and also measured with the sextant.

CORRECTIONS OF ALTITUDES.

Observed altitudes of heavenly bodies and observed angles betweeen them require the application of some or all of the following corrections: Index error of sextant, dip, refraction, semi- Corrections diameter, and parallax. The necessity for applying these corrections arises from the fact that to combine observations made at different places on or above the earth's *surface* with the elements taken from the *Nautical Almanac*, they must all be reduced to a common supposed point of observation, which is considered to be the centre of the earth. The index error has already been described; the other corrections will now be considered in their order of application.

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[1] Sextants are tested at the Observatory, Kew, for all sources of error on payment of a small fee; a description of the instrument employed in doing this is given in the Proc. Royal Society for 1868 and 1883.

A good sextant should not have an error of centering greater than 30″, and coloured shades having an error of more than 40″ (arising from the two sides not being parallel) are rejected.

With sextants which obtain A certificates the error of centering may be considered to vary from 10″ to 50″: those obtaining B certificates have sometimes 1′ 30″.

In 1884 a sextant which had been in use at sea was found to have an error of centering increasing from zero at 0° to + 7′ at 60°, and + 10′ at 90°.

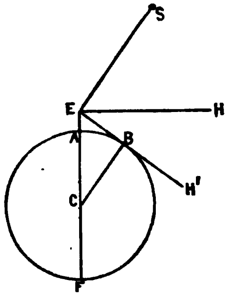
Dip

Let E be the place of the observer's eye, E S the direction of the object the altitude of which is to be found in angular measure, *i.e.* the angle S E H, E H being the horizontal line.[1]

Then, the observer's visible horizon being the tangent to the earth from E, the altitude given by the instrument will be the angle S E H′.

The difference between these two, namely, the angle H E H′, is the dip, or depression of the visible horizon, and is that which must be *subtracted* from the *observed* in order to obtain the apparent altitude of S.

FIG. 75.



To prove that tan dip $= \sqrt{\dfrac{2a}{r}}$.

With the same construction as above,

Let $\text{EA} = a$, $\text{CB} = r$.

Then $\text{HEH}' = 90^\circ - \text{CEH}' = \text{ECB}$.

Thus, $\tan \text{dip} = \tan \text{HEH}' = \tan \text{ECB} = \dfrac{\text{EB}}{\text{CB}}$.

Now (Euc. III. 36) $\text{EB}^2 = \text{EA} \,.\, \text{EF}$

$$= a\ (a + 2r) = a^2 + 2\,a\,r$$

$$\therefore\ \text{EB} = \sqrt{a^2 + 2\,a\,r}$$

$$\therefore\ \tan \text{dip} = \sqrt{\frac{a^2 + 2\,a\,r}{r^2}}$$

$$= \sqrt{\frac{2\,a\,r}{r^2}}$$ nearly (since a^2 is small compared with r)

$$= \sqrt{\frac{2a}{r}}.$$

---

[1] **NOTE.**—The sensible horizon is in strictness drawn from A, but even the nearest of the heavenly bodies is so distant that the length of A E may be considered as nothing in comparison with it; that is, the angle at S subtended by A E is immeasurably small.

Since the dip is a very small angle, tan dip = circular measure of dip.

$$=\frac{\text{dip in minutes}}{57{\cdot}29577^\circ \times 60} \therefore \frac{\text{dip in minutes}}{57{\cdot}29577^\circ \times 60}=\sqrt{\frac{2a}{r}}=\sqrt{\frac{2a}{3960 \times 1760 \times 3}},$$

$$\text{or dip in minutes}=\frac{57{\cdot}29577^\circ \times 60 \times \sqrt{2}}{\sqrt{3960 \times 1760 \times 3}}.\sqrt{a}=1{\cdot}063 \times \sqrt{a}$$

This value of the dip (true dip) is, however, affected by refraction, which raises the apparent horizon above the true (*see* page 148); the amount of this cannot be very accurately determined on account of the variable nature of refraction at low altitudes. Experiments seem to show that refraction diminishes the amount of dip by about $\frac{1}{15}$th of itself; according to Inman, its value is about $\frac{1}{13}$th of itself; others make it about $\frac{1}{14}$th. If we take it at $\frac{3}{40}$ths of itself, the results will correspond very nearly with those found in most tables.

$$\text{Hence tabular dip}=\frac{37}{40} \times 1{\cdot}063 \times \sqrt{a}={\cdot}984\sqrt{a}.$$

Example

Calculate the tabular dip for the height of the eye above the sea = 20 feet.

$$\text{dip}={\cdot}984\sqrt{20}.$$

log. ·984 .... $\bar{1}{\cdot}992995$

„ $\sqrt{20}$ .... 0·650515

„ dip .... 0·643510

$\therefore$ dip = 4′·4 = 4′ 24″

Calculate the dip for the height of the eye above the sea = 110 feet, and for height = 30 feet. *Ans.* 10′ 19″, 5′ 24″

NOTE.—It will be noticed on reference to the Dip table (No. 13 of Inman's Tables) that the dip *changes* most rapidly when the observer is near the surface of the water; hence in a rough sea, where the height is necessarily uncertain, it is customary to observe from a position as high as convenient.

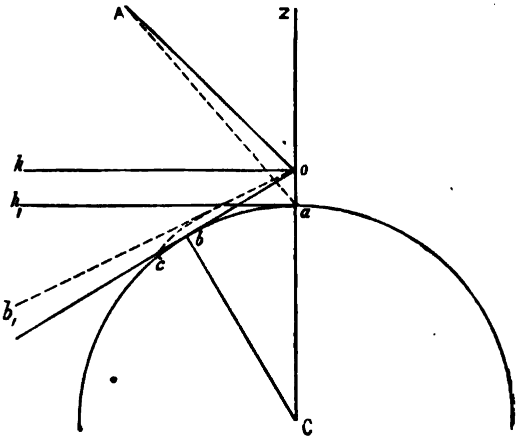
In the event of the horizon from the bridge of a vessel being obscured by fog, a new horizon may often be obtained by observing from a position nearer the level of the sea.

It will be remarked that since in fig. 75 the angle A C B is equal to the true dip or angle H E H′ (since they are both equal to the complement of the angle B E C) the *distance* of the true sea horizon (of which A C B is the angular measure) expressed in minutes of arc or *miles* is approximately equal to the square root of the observer's height in *feet*.

The distance of the *visible* horizon from the observer is measured by the angle *a* C *c* (fig. 76), and is practically equal to the tabular dip increased by one-sixth of itself; thus from an elevation of 20 feet above the sea an object first becomes visible at a distance of 4′ 24″ + 44″ = 5′ 8″ = 5·13 nautical miles.

The adjoining figure will further illustrate the effect of dip on an observed altitude.

FIG. 76.



An observer at *o*, elevated *o a* feet above the surface of the earth, in observing the altitude of a body A above the horizon really measures the angle A *o* b_1, since the true horizon is *raised* by refraction, and instead of being seen in the direction *o b* a more distant point *c* is seen in the direction *o* b_1, a tangent to a curve passing through *c* and *o*.

The angle A *a* h_1 is the *apparent* altitude, and is practically the same as the angle A *o h*, which is equal to A *o* b_1 (the *observed* altitude) less *h o* b_1, which is the *apparent* dip given in Inman's Tables. The *true* dip is the angle *h o b*.

When a heavenly body is so situated with regard to an observer that it appears to be over the *land*, the altitude is measured to the waterline of the shore, and the dip to be applied is taken from the table of dip for a shore horizon (No. 14); this, represented by the angle *h o c* in fig. 77, is greater than the dip for the true horizon *h o b*, and depends upon the distance of the observer from the shore.

Refraction

But for the existence of the atmosphere surrounding the earth, a ray of light emanating from a heavenly body S′ (fig. 78) would proceed in a straight line to the eye of an observer A on the surface of the earth, and he would see it in the direction of A S′. The atmosphere, however, causes every ray from S on passing through it to be deflected from a straight line to a curve concave to the surface of the earth, so that the ray which renders S visible to A has really pursued a curved track such as S *a b c* A (the limit of the atmosphere being represented by the outer circle in the figure), and the observer sees S at X in the direction of the tangent to that curve at A; hence the altitude is augmented by the angle X A S¹. This angle is termed the Refraction.

Astronomical refraction is only a special instance of the universal law by which every ray of light passing from a less dense to a more dense medium is bent (usually in the same plane) so that it assumes a more perpendicular direction towards that latter; the atmosphere at the surface of the earth is more dense than that of the upper regions, and similarly that is more dense than space, hence the path of a ray is curved as in the figure.

FIG. 77.

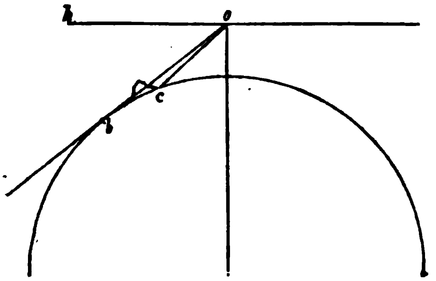
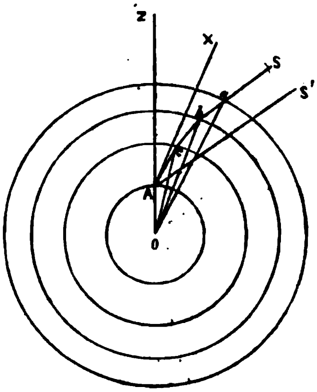


FIG. 78.



To trace the path of a particular ray.

Let O be the centre of the earth, A the place of observation, Z its zenith.

If we imagine the atmosphere to be formed of concentric strata of air diminishing in density as they are further from the earth's surface.

Then a ray S *a* proceeding from a heavenly body and falling upon the surface of the highest stratum of sensible density at *a*, is bent towards the perpendicular *a* O into the direction *a b*; again at *b* it is bent into the direction *b c*; and so on, thus following the course *a b c* A.

If we now pass to the case of nature by supposing the thickness of each stratum to be infinitely small, and the density to change by insensible degrees, the path of the ray will become a continuous curve, and the direction in which the object is seen will be the tangent to the curve at the point nearest the observer's eye.

Let A X be this tangent, and draw A S¹ parallel to *a* S: then if there were no atmosphere A S¹ would be the direction of the body.

Hence by refraction the apparent place X is elevated above the true place by the angle S′ A X, and therefore the correction for refraction is subtractive.

The rays which proceed from a body in the zenith suffer no refraction, because they enter the strata perpendicularly. For inferior altitudes, the refraction increases as the altitude diminishes, and attains its maximum, about 34′, when the body is on the horizon.

The refraction varies approximately as the tangent of the zenith distance of a heavenly body when that zenith distance does not exceed 80°, but it is very irregular for altitudes of less than 5°; it is advisable from this consideration not to observe at sea if possible at a lower altitude than about 10°; at an altitude of 45° the refraction is nearly 1′, and for any given altitude it has the same value for *any* of the heavenly bodies, since they are all situated beyond the limits of the atmosphere. Refraction does not alter the bearing of a heavenly body, but only its altitude.[1]

Refraction tables are usually given for a temperature of 50° F. and barometric pressure of thirty inches; a rise of temperature or fall of the barometer indicates a decrease in the density of the atmosphere and hence diminishes the refraction. The converse to this is also true. These corrections are given in Inman's Tables, No. 17, but need not be considered in taking observations at sea.

With regard to the effect on refraction of the interposition of thin clouds which partially obscure the heavenly body observed, it is to be remarked that though a ray of light is more bent from a straight line on passing through aqueous vapour than through dry air, yet an opposite and equal effect is produced by the fact that air saturated with aqueous vapour is less dense than dry air, and therefore no correction is required to the tabular refraction.

The refraction to be applied to the altitude of the lower limb of the sun or moon differs from that applicable to the upper limb; this difference becomes visible to the eye in the oval form of those bodies when rising or setting, at which time their vertical diameters are one-sixth less than their horizontal diameters.

Semi-diameters

The positions of the various heavenly bodies as given in the *Nautical Almanac* are those of their *centres*; almost all observations at sea of those not appearing actually as points (such as the fixed stars) are necessarily made of one or other of their *limbs*—

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[1] The effect of refraction on the altitudes of heavenly bodies, and therefore on the *distances* between them at different times, was first pointed out by Ptolemy in A.D. 140

a term applied to the upper, lower, or any other edge of a circular disc.

Hence almost every observation of a body having a sensible disc requires the semidiameter to be either added to or subtracted from it, in order to reduce it to what it would have been if the *centre* had been observed, unless the upper and lower limbs have been *both* observed.

For purposes of navigation, telescopes of only weak magnifying power are used in sextants, and hence the small semidiameters of the planets may be disregarded, and only those of the sun and moon require consideration.

Sun

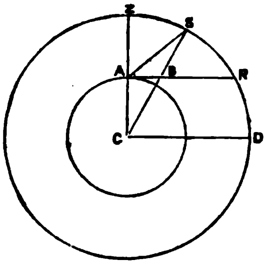
The semidiameter of the sun varies from 16′ 18″ in January, when the earth is nearest, to 15′ 46″ in July, when it is most distant; it requires no correction depending either upon the sun's altitude or the position of the observer on the earth, and is taken directly from the *Nautical Almanac.*

Moon

The semidiameter of the moon is also greatest when in that part of her orbit nearest the earth (perigee), and least when she is most remote (apogee), varying from 16′ 45″ to 14′ 44″; the periods of the recurrence of perigee and apogee average 27 days 13 hours.

Apart from these changes (which are recorded in the *Nautical Almanac* at intervals of twelve hours), the semidiameter of the moon varies with the altitude, being least when on the horizon and greatest when in the zenith (*see* page 153).

Fig. 79.



The semidiameters of all the heavenly bodies in the *Nautical Almanac* are the angles under which their semidiameters would appear if seen from the centre of the earth.

Parallax

Before the altitude of any celestial object can be employed for any practical purpose, it must be reduced to what it would have been if taken not from the surface, but from the centre of the earth, and measured not from the sensible, but from the rational horizon of the place of observation.

Let S be the object observed, A the place of observation, A R the sensible horizon, C D the rational horizon (fig. 79).

The observed altitude when corrected for dip, semidiameter, and refraction, will be measured by the angle S A R, which is the true altitude of the centre above the sensible horizon, and S C D will be the true altitude of the centre above the rational horizon.

The difference between these two angles, since $SCD = SBR$, is

$$SBR - SAR = ASC, \text{ the parallax in altitude.}$$

And the true altitude S C D, of the centre above the rational horizon C D, is

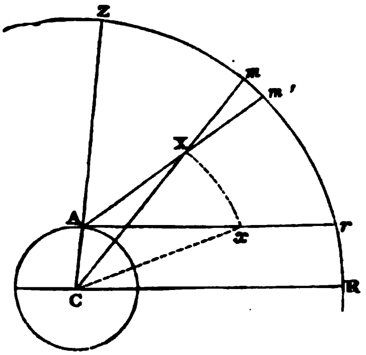
$$SCD = SBR = SAR + ASC.$$

Hence the correction for parallax is *additive*.

Parallax in altitude

From another point of view let A be the place of the spectator on the surface of the earth, C the centre, Z the zenith, X a heavenly body, and Z *m r* the celestial concave.

Fig. 80.



Through X draw the two straight lines A X *m'* and C X *m* to the celestial concave. Then *m'* is the apparent place (corrected for refraction), and *m* the true place of the heavenly body X.

Draw A *r*, a tangent to the earth's surface, at A; draw also C R through the centre parallel to A *r*; then considering the infinite distance of the points *r* or R from the earth, the earth's semidiameter A C will subtend no angle at *r* or R, and A *r* may be considered to coincide with C R, and therefore the arc $rR = 0$. The apparent altitude of X (X A *r*), corrected for refraction, is measured by the arc *m' r* (or *m'* R), and its true altitude (X C R) by *m* R or *m r*. The difference *m' m* between the true and apparent altitudes (or zenith distances) equal to the angle A X C, is called the *parallax in altitude*.

It appears from the figure that the effect of parallax is to *depress* bodies, so that the true altitude *m* R is greater on this account than the apparent altitude *m'* R, and that the true altitude may be obtained by *adding* the parallax in altitude to the apparent altitude (corrected for refraction).

If *x* be the same body when on the horizon, the angle A *x* C is termed its *horizontal parallax*.

The horizontal parallax is seen from the figure to be greater than the parallax at *any* altitude; and it is also evident that parallax in altitude decreases as the altitude increases, being 0 at an elevation of 90° ; that is, the position of a heavenly body *in the zenith* is the same whether seen from the surface or centre of the earth.

Horizontal parallax

Horizontal parallax being defined as the greatest angle at the centre of a heavenly body subtended by the earth's radius, will decrease with the increased distance of any heavenly body[1]; for the fixed stars it is 0, and it has a very small value for all bodies except the moon.

Parallax in altitude can be deduced from horizontal parallax as follows:

In fig. 80,

$$\text{C}x = \text{CX}$$

$$\therefore \frac{\text{CA}}{\text{C}x} = \frac{\text{CA}}{\text{CX}}$$

$$\text{or, } \sin \text{C}x\text{A} = \frac{\sin \text{CXA}}{\sin \text{CAX}};$$

$$\therefore \sin \text{CXA} = \sin \text{C}x\text{A} \,.\, \cos x\text{AX}.$$

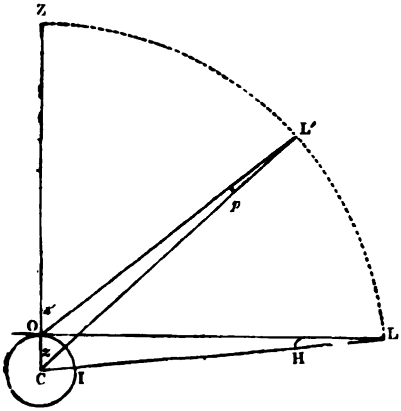
Since C X A and C x A are always small angles (even in the case of the moon), in the above equation we may substitute the arcs for their sines ;

$$\therefore \text{CXA} = \text{C}x\text{A} \,.\, \cos x\text{AX},$$

or, par. in alt. = hor. par. × cos apparent altitude (corrected for refraction).

Parallax affects the positions of heavenly bodies both in right ascension and declination, but in nautical astronomy we need not consider this, since by applying it as a correction to *altitude* the observation is at once reduced to the centre of the earth.

FIG. 81.



In this article the earth is considered to be a *sphere*, The error involved by this consideration, which is of no practical importance in nautical astronomy, will be considered later on (page 377).

Augmentation of moon's semi-diameter

When the moon is above the horizon, as at L′ (fig. 81), its distance O L′ from a spectator at O is less than its distance O L when on the horizon at L. For since the horizontal parallax (O L C) is small, O L is nearly equal to C L, and therefore L I is less than L O by nearly the earth's radius. Hence if two observers were placed at O and I, one would see the moon when at L on his horizon, and the other in his zenith; but from the spectator at O the moon would be more distant than it is from the spectator at I by about 4,000 miles, or the radius of the earth, and the diameter would appear to the former about 30″ less than the latter.[1] It is evident that at any intermediate altitude, as at L′, the distance O L′ is less than O L, and therefore the moon's diameter at L′ would appear to be greater than the true or horizontal diameter at L; that is, the diameter at L′ would be *augmented.* To find this correction, or, as it is termed, augmentation, of the moon's semidiameter, we must proceed as in the following problem.

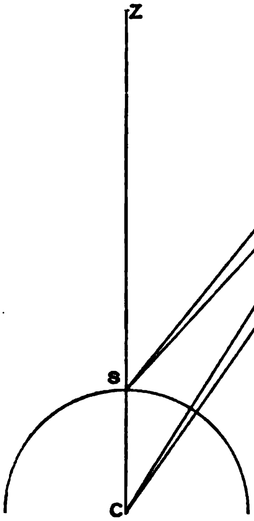
---

[1] Thus, for the moon the mean horizontal parallax is 57′, and for the sun 8·8″.

Given the semidiameter of the moon, to calculate its augmentation for a given altitude.

In Fig. 82 S is place of observer, C centre of earth; M the centre of moon, whose radius $=r$; S b, C a tangents to moon's surface from S and C.

FIG. 82.



Let M C $a=s''$, M S $b=(s+d)''$. Then d'' is the augmentation of moon's semidiameter.

Let app. zen. dist. of moon's centre Z S M $=z$.

Let the corresponding parallax in alt. S M C $=p$.

---

[1] The fact of the moon (and sun) *appearing* to be so much larger when near the horizon than at any other time is an optical delusion probably caused by the apparent vicinity of terrestrial objects with which they can be compared.

$$\text{Then } \frac{r}{\text{S M}} = \sin\,(s+d)'' = (s+d)''\ \sin 1''.$$

$$\frac{r}{\text{C M}} = \sin s'' = s''\sin 1''$$

$$\therefore\ \frac{(s+d)''}{s''} = \frac{\text{C M}}{\text{S M}} = \frac{\sin \text{M S C}}{\sin \text{S C M}} = \frac{\sin z}{\sin(z-p)}$$

$$\therefore\ \frac{d''}{s''} = \frac{\sin z - \sin\,(z-p)}{\sin\,(z-p)}$$

$$\therefore\ d'' = 2\,s''\cos\,(z-\tfrac{p}{2})\ \sin\tfrac{p}{2}\ \text{cosec}\,(z-p).$$

when $p = \text{H} \sin z$, H being the horizontal parallax of the moon.

Examples Calculate the augmentation of the moon's semidiameter, when the apparent altitude of the centre (corrected for refraction) is 60° 10′, the horizontal parallax being 56′ 1″, and semidiameter (as given in the *Nautical Almanac*) 15′ 16″.

$$\text{Aug.} = 2s''\ .\ \text{cosec}\,(z-p)\ \cos\,(z-\tfrac{1}{2}p)\ \sin\tfrac{1}{2}p,$$

$$\text{and } p = \text{hor. par.} \times \text{cos app. alt.}$$

56′ 1″	log hor. par..........3·526468
60	L cos alt.............9·696774
3361″ = hor. par.	log p ....3·223242

log 2 semi..........3·262925	∴ p = 1672″ =	27′ 52″
L sin $\frac{1}{2}p$ ....7·607780	z =	29° 50 0
L cos $(z-\frac{1}{2}p)$...9·939262	$z-p$ =	29 22 8
log cosec $(z-p)$...0·309422	$\frac{1}{2}p$ =	13 56
„ aug................1·119389	z =	29 50 0
∴ aug. = 13·16″	$z-\frac{1}{2}p$ =	29 36 4

Calculate the augmentation of the moon's semidiameter, when the apparent altitude of the centre (corrected for refraction) is 32° 42′, the horizontal parallax being 54′ 42·5″, and semi. (in *Nautical Almanac*) 14′ 56″. *Ans.* 7·86″.

Note The great distance of any heavenly body other than the moon as compared with the radius of the earth renders any augmentation of its semidiameter due to altitude too minute a quantity to be considered.

Summary An *observed* altitude of a star or of a limb of the sun or moon is that actually measured with a sextant and uncorrected.

An *apparent* altitude is an observed altitude corrected for index error and any other instrumental error and also for dip.

A *true* altitude is an apparent altitude further corrected for refraction, parallax, and in certain cases for semidiameter.

Parallax in altitude is sometimes spoken of as 'diurnal' parallax, since it passes through all its changes in a day.

The position of a heavenly body as seen from the centre of the earth and free from the effect of refraction is termed its *geocentric* position.

CHAPTER VII.

Time—Sidereal day—Apparent solar day—Mean solar day—Equation of time—Comparisons of sidereal, mean, and apparent intervals of time—Sidereal year—Tropical year—The calendar—Problems on time—*Nautical Almanac*—Greenwich and local times—Correction of declinations, right ascensions, &c.—Correction of altitudes.

TIME, or duration, is measured by the intervals between the recurrence of various celestial phenomena, such as that of successive arrivals of the sun at the meridian of an observer, the interval between such arrivals being in this case termed a solar day, which is subdivided into twenty-four equal hours. **Time** **Solar time**

This simple and apparently natural standard for estimating duration cannot, however, be applied to astronomy, since it is not of *invariable* length, observation showing that any two successive days estimated in this manner are of unequal durations. **Standards**

This fact has led to the adoption of other standards of measurement, which, though differing from one another in length, each possess the requisite property of being *invariable*.

All of these depend upon the recurrence of real or imaginary celestial phenomena, caused by either the rotation or revolution of the earth.

The rotation of the earth on its axis is always performed in the same time, and but for the apparent and real movements of the heavenly bodies this period would appear the same, whether measured by the interval between two successive passages across the meridian of a star or of the sun or moon.

Similarly, though the period of a complete revolution of the earth round the sun is invariable, yet it will apparently vary if measured by the return of the earth to the same position with regard to different heavenly bodies.

A day may therefore be defined as the interval of time between the departure of any meridian from a heavenly body, or from some point in the heavens, and its succeeding return to it, deriving its name from the body or point with which the motion of the meridian is compared; thus we employ the terms sidereal, solar, and lunar days.[1] **Day**

[1] A mean lunar day contains $24^h\ 48^m\ 29^s$ mean solar time.

Year

Similarly, a year may be defined as the interval of time between the earth in its revolution round the sun occupying in succession the same place with regard to the heavenly bodies or to some point in the heavens, deriving its name from the heavenly body or point thus considered; hence we employ the terms sidereal year and tropical year.

Sidereal day

The sidereal day is considered to begin when the first point of Aries is on the meridian, and is measured by the interval elapsing between two successive meridian passages of that point.

This interval is subdivided into twenty-four sidereal hours, each being again subdivided into minutes and seconds.

In the conduct of the business of an Astronomical Observatory, certain stars, whose positions have been very accurately determined, are employed for the purpose of determining, or correcting, the sidereal time, and for finding the error and rate of the sidereal clock. The results obtained from the observations of such stars can then be referred to the imaginary star situated in the first point of Aries, by the application of their known right ascensions. Such stars are technically known as 'Clock Stars.'

Apparent solar day

An apparent solar day is the interval between two successive transits of the sun's centre over the same meridian; it begins when that point is on the meridian (apparent noon). Observation shows that this interval is never *exactly* the same for two days in succession; thus on December 23 it is about half a minute *longer*, and on September 17 about half a minute *shorter* than its *average* length.[1]

The length of an apparent solar day is variable from two causes:

1st. From the apparent variable motion of the sun in the ecliptic.

2nd. From the apparent motion of the sun being in a circle inclined to the equinoctial.

The velocity of the earth in its orbit varies with its distance from the sun, and hence the sun *appears* to travel with a variable velocity.

When the earth is near E_4 (January 1), fig. 83, it is nearest the sun (perihelion), which apparently occupying the position S_4, fig. 84, will then appear to travel over an arc $S_4\, a$ of about 61′ each day.

When the earth is near E_2 (July 1), and farthest from the sun

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[1] More accurately, the interval from apparent noon on December 22 to apparent noon on December 23 is $24^h\ 0^m\ 30^s$ expressed in mean solar time; the similar interval from September 17 to September 18 similarly expressed is $23^h\ 59^m\ 39^s$.

(aphelion), it is moving most slowly, and the sun seen at S_2 will then move only over an arc $S_2\,b$ of about 57′ each day.[1]

Evidently, then, a meridian $P\,A\,P_1$, when the sun is at S_2 must rotate through 360° + nearly 61′ between two successive passages of the sun's centre across it, and only through 360° + nearly 57′ during a similar interval when the sun is at S_4.

The second cause of the irregularity in the lengths of apparent solar days arises from the sun's apparent path not being on the equinoctial, but on the ecliptic, portions of which latter great circle are variously inclined to the corresponding arcs of the former at different seasons of the year.

Fig. 83.

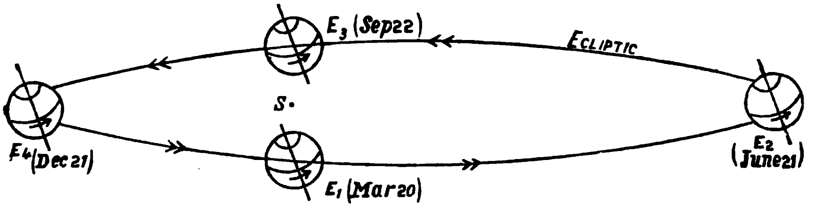
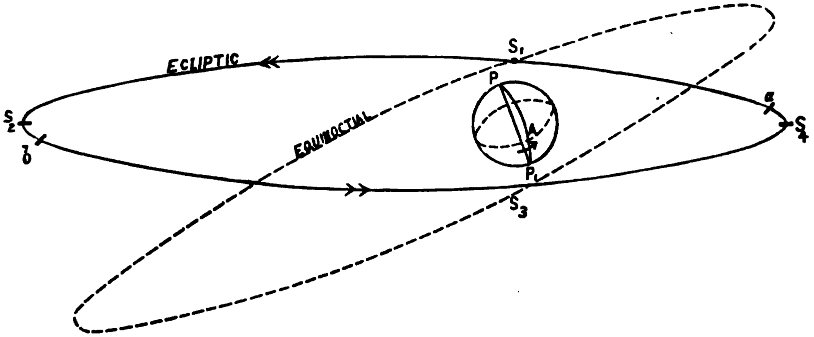


Fig. 84.



Thus at the equinoxes (March 20 and September 22) the small arc of the ecliptic then traversed by the sun and the corresponding arc of the equinoctial are inclined to one another, as in fig. 85, and at the solstices (June 21 and December 21) they are nearly parallel, as in fig. 86; evidently, then, even if the arcs of the ecliptic S S_1 apparently traversed by the sun in one day were *equal* to one another, they would not be represented by equal arcs on the equinoctial, and the equality in length of solar days depends upon that latter condition, since solar time is measured as an angle at the pole of the equinoctial (pole of the heavens), and therefore during equal intervals of time equal arcs of the equinoctial must pass the meridian.

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[1] Accurately, when nearest the sun the earth moves over an arc of the ecliptic of 61′ 12″ daily, and when farthest away 57′ 12″; when at a *mean* distance (March 3 and October 2), it moves over an arc of 59′ 8″ daily. This may be otherwise expressed by saying that the maximum velocity of the earth in its orbit is 67,780 miles per hour, the minimum 63,350, and the mean velocity 65,530 miles per hour.

Since the solar year does not contain an *exact* number of days the phenomena described in the text will not occur on the same day of *every* year; the dates given are those for leap years (civil time); the times of recurrence will be about six hours later each year until the next leap year (*see* Table of Phenomena in *Nautical Almanac*).

If the ecliptic and equinoctial were identical (that is, if the line of the earth's poles were perpendicular to its orbit), apparent solar days would only differ in length from the effect of the earth moving more rapidly round the sun in the northern winter than in the northern summer.

In order to obtain a *uniform* measure of solar time, an imaginary day is employed, which is equal in length to the mean or average of all the apparent solar days in a year.

Mean solar day

Such a day is termed a *mean solar day*, and is regulated by an imaginary or mean sun, commencing when the centre of that mean sun is on the meridian (mean noon), and being divided into twenty-four equal hours, which are again subdivided into minutes and seconds.

FIG. 85.

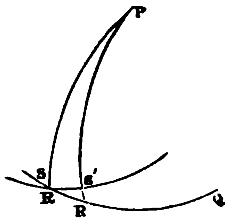
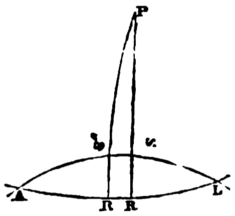


FIG. 86.



The supposed movement of this imaginary sun which regulates the mean solar day will be best understood by supposing a star to start with the real sun, from the point where the earth and sun are nearest to one another (January 1 nearly at S_4, fig. 84, page 159),[1] and to travel on the ecliptic at an *uniform* rate, so as to arrive at the same point again at the same time as the real sun.

The *mean* sun, as it is termed, is supposed to traverse the *equinoctial* at the same uniform rate as that with which the imaginary star traverses the *ecliptic*, starting with it at the time of the vernal equinox.

[1] It will be remarked that though the winter solstice, or time of the sun's having its greatest south declination, occurs now on December 21, the earth is at present *nearest* the sun on January 1.

Equation of time

The difference between time measured by the mean sun and that measured by the apparent sun at any instant, is termed the equation of time; this may be defined as the angle at the pole of the equinoctial contained between circles of declination passing through the centres of the true and mean suns respectively—that is, the difference between their right ascensions; that portion due to the obliquity of the ecliptic has a maximum value of about ten minutes, and that due to the unequal motion of the earth in its orbit is at its greatest about eight minutes.[1]

In considering the relative positions of the true sun and the imaginary star, they are together when the true sun is nearest the earth (nearly at s_4, fig. 84, on January 1); the apparent speed of the true sun is then at a maximum, so that as they both travel round the ecliptic the true sun will pass the first point of Aries about seven minutes before the star. The apparent speed of the true sun is, however, slackening, and when nearly at s_2, on July 1, and farthest from the earth, the true sun and star are together again; from this position the speed of the star is greater at first than that of the true sun, but they come together again nearly at s_4 on January 1. Hence, from s_4 to s_2 (January 1 to July 1) the true sun is in advance of the star, and from s_2 to s_4 (July 1 to January 1) the star is in advance of the true sun.

As our imaginary star passes the first point of Aries, the *mean* sun starts on the *equinoctial* with a speed exactly equal to that of the star on the ecliptic. At first, since their paths are greatly inclined, when they have both travelled any equal arcs the mean sun will be in advance of the star, as measured at the pole of the equinoctial, and will arrive on the same circle of declination as the true sun on April 15, when the equation of time will be zero, and both suns will be in advance of the star.

From this position the mean sun will gain on the true sun until May 14, when it will be about four minutes in advance; but as the arc of the ecliptic traversed by the sun and the corresponding arc of the equinoctial are becoming more nearly parallel, the true sun now gains on the mean sun, both arriving on the same circle of declination on June 14, when there is no equation of time.

A few days afterwards, on July 1, the true sun and the star are together, and both are in advance of the mean sun. The true sun from this point apparently increases its speed, and on July 26 is about six minutes in advance of the mean sun. The inclination of their respective paths, which has been increasing since June 21, now enables the mean sun to gain on the true, and on August 31 there is again no equation of time.

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[1] The remainder of this section may be omitted on first reading this chapter.

From August 31 to November 3 the mean sun continues to gain on the true sun, and is then about sixteen minutes in advance of it, the star and the mean sun having passed through the first point of Libra together on September 22, and being then as much in advance of the true sun as on March 20 at A they were behind it.

From November 3 to December 23 the increasing velocity of the true sun and the diminishing inclination of their paths enable it to gain so much on the mean sun, that at the latter date they are again on the same declination circle, and there is no equation of time. After this, for similar reasons, the true sun continues gaining on the mean sun, until February 11, when they are relatively stationary; the mean sun then gains on the true, and on passing again through the first point of Aries is, as before, about seven minutes behind it.

The necessity for introducing an imaginary star arises from the fact that the first point of Aries, during the period of a revolution of the earth, will have altered its position 50·22″ in the direction from S_1 towards S_4 (*see* fig. 56, page 121).

A summary of the preceding remarks shows that—

From December 23 to April 15 the true sun is in advance of the mean sun;

From April 15 to June 14 the true sun is behind the mean sun;

From June 14 to August 31 the true sun is in advance of the mean sun;

From August 31 to December 23 the true sun is behind the mean sun.

Clocks on shore are regulated to *mean* time, and therefore the clock time will be before the sun from December 23 to April 15; behind from April 15 to June 14; before from June 14 to August 31; and again behind from August 31 to December 23.

The following diagrams graphically illustrate (1) the two parts of the equation of time *separately*, shown on curves B and C, that on B illustrating the part due to the obliquity of the ecliptic, and that on C the part due to the unequal velocity of the earth in its orbit; and (2) their *combined* effect as shown on curve D, the vertical scale being that of minutes of time.[1]

That part of the equation of time caused by the obliquity of the ecliptic (curve B, fig. 87) attains its greatest value ($9^m\ 53^s$) midway between the solstices and equinoxes, being zero at each of those periods. That part caused by the unequal velocity of the earth (curve C) attains its maximum value ($8^m\ 24^s$) midway between perigee and apogee, being zero at those periods.

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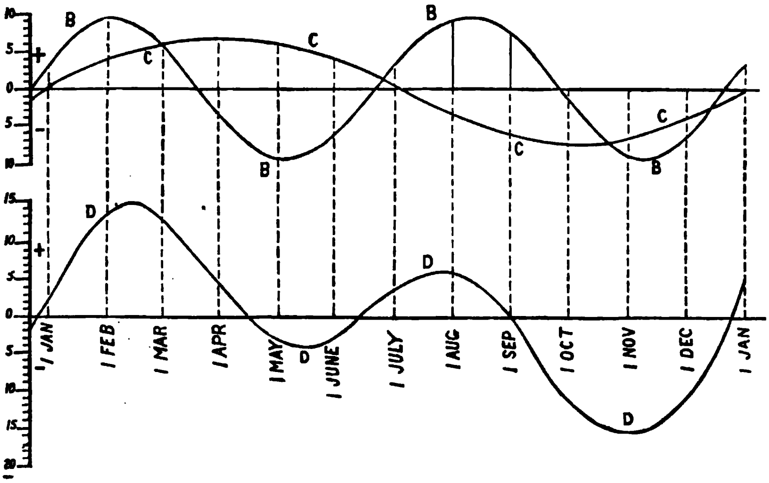
[1] Taken from an interesting article by J. Norman Lockyer on 'The Movements of the Earth,' in *Nature*, September 1884.

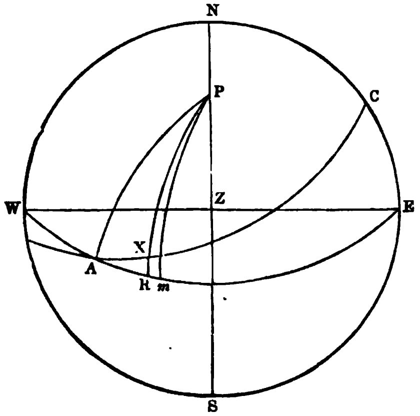
Fig. 87.



Graphical representations of time

The preceding illustrations of the various measurements of time have been drawn on the true supposition of the earth turning round on its axis, and thereby bringing every meridian in succession under the heavenly bodies considered.

Fig. 88.



The same is more usually illustrated on the supposition that the earth, and therefore the celestial meridian of an observer, is *fixed*, and the heavenly bodies move from east to west, the point of view being *outside* the celestial sphere.

Construct a figure, and show what is meant by sidereal time, apparent solar time, mean solar time, and the equation of time.

Let N W S E represent the horizon, P the pole, W Q E the equinoctial, A the first point of Aries, and A C the ecliptic. Let X be the place of the true sun on the ecliptic, and *m* the mean sun; through X and *m* draw the circles of declination P R and P *m*. Then *sidereal time* is the angle Q P A, or arc Q A; *apparent solar time* is the angle Q P R, or arc Q R; and *mean solar time* is the angle Q P *m*, or arc Q *m*—these angles or arcs being always measured from the meridian N Z S westward. Also the angle *m* P R, or arc *m* R, is the *equation of time*.

Definitions

Hence we may give the following definitions:

Sidereal time is the angle[1] at the pole of the equinoctial between the celestial meridian and a circle of declination passing through the first point of Aries, measuring from the meridian westward.

Mean solar time is the angle at the pole between the celestial meridian and a circle of declination passing through the centre of the mean sun, measuring from the meridian westward.

Apparent solar time is the angle at the pole between the celestial meridian and a circle of declination passing through the place of the true sun's centre, measuring from the meridian westward.

The *equation of time* is the angle at the pole between the circles of declination passing through the centres of the true and mean suns.

A *sidereal clock* is a clock adjusted so as to go twenty-four hours during one complete rotation of the earth—that is, during the interval between two successive transits of a fixed star; or, supposing the first point of Aries to be fixed, between two successive transits of the first point of Aries.

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[1] These angles are expressed in *hours*, &c.; on the connection between time and angular measure in *degrees*, *see* page 180.

The sidereal time in common use among astronomers expresses the actual hour angle from the meridian, westward, of the true equinoctial point at the moment of observation. It is therefore affected with the equation of the equinoxes; and is not, strictly speaking, a *mean* or uniformly increasing quantity. It ought, therefore, to be termed *apparent sidereal time* in the same manner as apparent solar time reckons from the actual arrival of the sun's centre on the meridian; and in like manner, as mean solar time is reckoned from the arrival of an imaginary sun, moving uniformly with its mean velocity, so *mean sidereal* time or $\frac{\odot\text{'s mean longitude}}{15}$ would be reckoned from the transit of, not the *true*, but the *mean* equinoctial point. The smallness of the fluctuations to which a clock regulated to *apparent* sidereal time, compared with one regulated to *mean* sidereal time, is subject (being at the utmost only 2·3^s in a period of nineteen years), has prevented the practical inconvenience of this from being felt: no clock being sufficiently perfect to go during so long a period without frequent re-adjustment; and as the corrections applied by astronomers to the observed right ascensions of all objects are adapted to this supposed irregularity in the rate of the clock, the mean right ascensions thence deduced will be correct.

A *mean solar clock* is a clock adjusted to go twenty-four hours between two successive transits of the mean sun.

A clock could not be constructed to always show apparent time, owing to the inequalities in the lengths of successive apparent solar days, and hence in the lengths of apparent solar hours. Apparent solar time is that shown by a sundial.

To find the daily motion of the mean sun in the equinoctial.

Daily motion of mean sun

The mean solar year, or the time the true sun takes to return again to the first point of Aries, has been found to be equal to 365·2422 mean solar days.[1] Let us suppose the mean sun to describe the *equinoctial* in this time, then we shall find its daily motion in the equinoctial[2] as follows:

Let x=daily motion,

$$365{\cdot}2422^{d} : 1^{d} :: 360^{\circ} : x = 0{\cdot}9856472^{\circ} = 59'\ 8{\cdot}33''$$

or, the mean sun's motion in the equinoctial from west to east is 59′ 8·33″ during the mean solar day.[3]

To find the arc described by a meridian of the earth in a mean solar day.

In fig. 89 let E_1 E_2 represent two positions of the earth in its orbit, separated by an interval of one mean solar day; if M represents the mean sun and P A the meridian of a place at a time when the mean sun and a fixed star are crossing that meridian at the same instant, it is evident that on arriving at E_2, the meridian having turned through 360°, the star (situated at an *infinite* distance) will again cross it[4]; but a further arc, A B, must be described before M, the mean sun, crosses; this arc will be that through which the mean sun apparently moves during each mean solar day, or 59′ 8·33″.

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[1] This result is attained from actual observation at astronomical observatories, where the instant of the true sun arriving at the first point of Aries—or, in other words, having no declination—can be ascertained with great accuracy.

The *mean* result is found by comparing observations of this character separated by very long intervals of years.

[2] It will be noticed that the movement of the first point of Aries is that of the equinoctial on the fixed ecliptic, and that the mean sun traverses each mean solar year the *whole* circumference of the equinoctial, but the true sun during that interval does not traverse the whole circumference of the ecliptic.

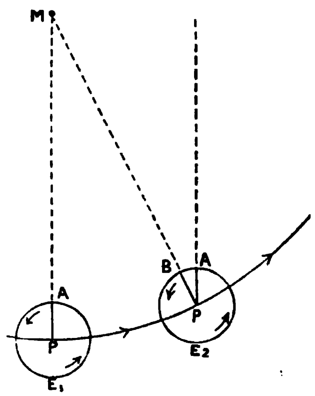
[3] The arc described in a *sidereal* day is 58′ 58·64″.

[4] It has already been remarked (page 117) that almost all the fixed stars are at such a vast distance from the earth that their positions are the same even when seen from opposite points of the earth's *orbit* round the sun.

Conversion of time

Given the length of a mean solar day = 24 mean solar hours; to express its length in sidereal hours, and the converse.

FIG. 89.



In a mean solar day, a meridian of the earth revolves through 360° 59′ 8·33″; in a sidereal day it revolves through 360°. A mean solar day is therefore longer than a sidereal day (or twenty-four sidereal hours) by the portion of sideral time occupied in describing 59′ 8·33″. This interval may be found by the following proportion:

$$360^\circ : 59' \ 8{\cdot}33'' :: 24^{h} : x \text{ the quantity required,}$$

$$\therefore x = 3^{m} \ 56{\cdot}555^{s},$$

$$\therefore 24 \text{ mean solar hours} = 24^{h} \ 3^{m} \ 56{\cdot}555^{s} \text{ sidereal hours.}$$

Acceleration and retardation

The interval 3^{m} $56{\cdot}555^{s}$, being the excess of a mean solar day over a sidereal day expressed in sidereal time, is called the *acceleration of sidereal on mean solar time.*

Given the length of a sidereal day = 24 sidereal hours; to express its length in mean solar time.

In a sidereal day the meridian of any place revolves through 360°: in a mean solar day the same meridian revolves through 360° 59′ 8·33″.

Therefore the length of a sidereal day may be found in mean solar time by this proportion:

$$24 \text{ sideral hours} : 24 \text{ mean solar hours} :: 360^\circ : 360^\circ \ 59' \ 8{\cdot}33'',$$

$$\therefore 24 \text{ sidereal hours} = \frac{24 \times 360^\circ}{360^\circ \ 59' \ 8{\cdot}33''} = 23^{h} \ 56^{m} \ 4{\cdot}0922^{s} \text{ mean solar hours.}$$

The interval 3^{m} $55{\cdot}9078^{s}$, being the defect of a sidereal day from a mean solar day expressed in mean solar time, is called the *retardation of mean solar on sidereal time.*

Since the sidereal day is nearly 3^{m} 56^{s} shorter than the mean solar day, it follows that at any one place a star will cross the meridian earlier each night by that interval, so that a heavenly body on the meridian at 8 P.M. (mean time) on January 1 will be on the meridian at about 6 P.M. on February 1. The times of meridian passage of the principal stars at different months of the year is given in Jeans' *Handbook for the Stars.*

Formulæ for converting any portion of sidereal time into mean solar time, and the converse.

Let S = any interval of absolute time or duration expressed in sidereal time.

M = the same interval expressed in mean solar time.

Then since a sidereal hour is *shorter* than a mean solar hour, the *number* of sidereal hours in a given interval will be *greater* than the number of mean solar hours in the same ratio.

$$\therefore \frac{M}{S}=\frac{360^\circ}{360^\circ\ 59'\ 8{\cdot}33''}={\cdot}9972695,$$

$$\text{and } \frac{S}{M}=\frac{360^\circ\ 59'\ 8{\cdot}33''}{360^\circ}=1{\cdot}0027379,$$

$$\therefore M={\cdot}9972695\times S=S-{\cdot}0027305\,S \ldots\ldots\ldots\ldots(1)$$

$$\text{and } S=1{\cdot}0027379\times M=M+{\cdot}0027379\,M \ldots\ldots\ldots\ldots(2)$$

By means of these formulæ we may convert any portion of sidereal time into mean time, and the converse, as in the following examples:

Express 12 sidereal hours in mean solar time.

By formula (1), $M=S-{\cdot}0027305\,S$, and $S=12$

$$\therefore M=12-{\cdot}0027305\times 12=11^h\ 58^m\ 2{\cdot}04^s,$$

or 12 sidereal hours $=11^h\ 58^m\ 2{\cdot}04^s$ mean solar hours.

Express $11^h\ 58^m\ 2{\cdot}04^s$ mean solar time in sidereal time.

By formula (2), $S=M+{\cdot}0027379\,M$,

$$M=11^h\ 58^m\ 2{\cdot}04^s=11{\cdot}967228+({\cdot}0027379\times 11{\cdot}967228)$$

$$=11{\cdot}999999=12 \text{ hours nearly;}$$

$$\therefore 11^h\ 58^m\ 2{\cdot}04^s \text{ mean solar hours}=12 \text{ sidereal hours.}$$

Examples for practice

Express 16 sidereal hours in mean solar time.

Ans. $15^h\ 57^m\ 22{\cdot}73^s$ mean solar time.

Express 14 mean solar hours in sidereal time.

Ans. $14^h\ 2^m\ 17{\cdot}99^s$ sidereal time.

Time equivalents

By means of formulæ (1) and (2) the tables of time equivalents given in the *Nautical Almanac*, and also those of acceleration and retardation in Inman's Tables (Nos. 40 and 41), are computed.

These tables are used for readily converting by inspection intervals of sidereal time into equivalent intervals of mean solar time, and the converse; a few examples will show this.

Convert by table $8^{h}\ 43^{m}\ 51{\cdot}42^{s}$ sidereal time into mean solar time.

8^{h} (sidereal time)	=	7^{h}	58^{m}	$41{\cdot}36^{s}$	mean solar time
43^{m}	,,	……	42	52·96	,,
51^{s}	,,	……		50·86	,,
$0{\cdot}42^{s}$	,,	……		·42	,,
∴ $8^{h}\ 43^{m}\ 51{\cdot}42^{s}$ sidereal time	=	8	42	25·6	,,

Convert by table $8^{h}\ 42^{m}\ 25{\cdot}6^{s}$ mean solar time into sidereal time.

8^{h} (mean time)	=	8^{h}	1^{m}	$18{\cdot}85^{s}$	sidereal time
42^{m}	,,	……	42	6·90	,,
25^{s}	,,	……		25·07	,,
$0{\cdot}6^{s}$	,,	……		·60	,,
∴ $8^{h}\ 42^{m}\ 25{\cdot}6^{s}$ mean time	=	8	43	51·42	,,

Convert $10^{h}\ 10^{m}\ 30^{s}$ sidereal time into mean time.
Ans. $10^{h}\ 8^{m}\ 49{\cdot}98^{s}$.

Convert $10^{h}\ 10^{m}\ 30^{s}$ mean solar time into sidereal time.
Ans. $10^{h}\ 12^{m}\ 10{\cdot}29^{s}$.

Summary

The following is a summary of the preceding results :

One sidereal day equals $23^{h}\ 56^{m}\ 4{\cdot}09^{s}$ of mean solar time, and during its continuance a meridian rotates through 360°.

One mean solar day equals $24^{h}\ 3^{m}\ 56{\cdot}55^{s}$ of sidereal time, and during this interval a meridian rotates through 360° 59′ 8·33″.

An apparent solar day is variable in length, and a meridian rotates through nearly 361° 1′ in January, when the earth is nearest the sun, and 360° 57′ in July, when it is farthest away.

Twenty-four apparent solar hours equal twenty-four mean solar hours ± the *change* in the equation of time during the day considered, hence the relation between them is variable.

Sidereal year

A sidereal year is the interval between the sun's apparently leaving a fixed point or star and returning to it.

This interval of time is invariable in length, and during its continuance the sun apparently moves over an arc of the ecliptic of 360°.

Tropical year

A tropical or mean solar year is the mean interval between the sun apparently leaving the first point of Aries and returning to it.

This interval of time is also almost invariable,[1] but is shorter by about twenty minutes than a sidereal year, since the first point of Aries moves over 50·22″ each year to meet the sun, which therefore apparently traverses on the ecliptic an arc of 360° −50·22″, or 359° 59′ 9·78″ during that interval.

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[1] Successive tropical years are not *exactly* equal, owing to a very gradual displace-

The tropical year is that on which the changes of the sun's declination, and therefore the recurrence of the seasons, depend, and as the period of its commencement is that most easily observed at astronomical observatories, it forms the basis of reckoning time by years.

Astronomers recognise another year, comprising the period between two successive returns of the earth to its point of nearest approach to the sun (perihelion); this is termed the *anomalistic* year, and consists of $365^{d}\ 6^{h}\ 13^{m}\ 44{\cdot}6^{s}$, expressed in mean solar time.

To find the length of the sidereal year in mean solar time.

Conversion of time

Since the first point of Aries moves with a slow annual motion of about 50·22″ from east to west to meet the sun, the arc of the ecliptic described by the sun from the first point of Aries to the first point of Aries again must be 360° − 50·22″ = 359° 59′ 9·78″, and this is the arc described by the sun in a tropical year; but in a sidereal year the sun describes 360°; hence a sidereal is longer than a tropical year by the time the sun takes to move over an arc of 50·22″. Hence the proportion,

$$\text{sidereal year} : \text{tropical year} :: 360^\circ : 360^\circ - 50{\cdot}22''$$
$$\text{or, sidereal year} : 365{\cdot}242264^{d} :: 360^\circ : 359^\circ\ 59'\ 9{\cdot}78''.$$

Therefore, sidereal year $= 365^{d}\ 6^{h}\ 9^{m}\ 10{\cdot}71^{s}$ (expressed in mean solar time).

The first *point* of Aries (denoted as explained by the symbol ♈) was some 2,000 years ago situated in the *constellation* of Aries; it is now 30° westward of that position, and in the constellation of Pisces; this is caused by the slow annual movement spoken of above.

Summary

The following is a summary of the preceding results:

In one sidereal year the sun apparently describes an arc of 360° in the heavens, and occupies in doing so $365^{d}\ 6^{h}\ 9^{m}\ 10{\cdot}71^{s}$ expressed in *mean solar time*, or $366^{d}\ 6^{h}\ 9^{m}\ 14{\cdot}09^{s}$ expressed in *sidereal time*.

In one tropical year the sun apparently describes an arc of

---

ment of the plane of the ecliptic occasioned by the attraction of the planets (planetary precession); at present the tropical year is about four seconds shorter than in B.C. 160, and the accepted length is the average of as long a period as possible.

359° 59′ 9·78″ in the heavens, and occupies in doing so $365^d\ 5^h\ 48^m\ 47{\cdot}808^s$ expressed in *mean solar time*, or $366^d\ 5^h\ 48^m\ 47{\cdot}808^s$ expressed in *sidereal time*.

The calendar

From the preceding investigation we see that the tropical year does not contain an exact number of mean solar days, nor does the sidereal year contain an exact number of sidereal days; the number of sidereal days in a tropical year must be *one more* than the number of mean solar days in that interval, since, owing to its apparent movement (*see* fig. 84), the mean sun must in one year have fallen behind the stars by a whole circumference of the heavens, or 360° = 24 hours; in other words, when the sun has risen and set 365 times, a star will have done so 366 times.

The seasons being dependent on the changes of declination of the sun, it follows that the civil or ordinary year must be the *tropical* year, and it is the object of a well-adjusted calendar to take account of and properly allow for the fraction of a mean solar day ($\cdot 2422^d$ or $5^h\ 48^m\ 47{\cdot}808^s$) by which this tropical year exceeds 365 whole mean solar days.

Our own method of doing this originates with the Roman calendar, which, until the time of Julius Cæsar, measured the year by 13 *lunar* months, equivalent to 355 mean solar days,[1] the additional days required to make this equal to the solar year being inserted in the most arbitrary and irregular way.

Reformation of calendar

In 45 B.C. the calendar was reformed by Julius Cæsar, and the fraction of the day previously alluded to was partially allowed for by considering three successive years to each consist of 365 days, followed by *one* year of 366 days, termed *bissextile* (leap year).

The term 'bissextile' arose from the additional day being inserted before February 24, which, being the sixth of the calends, and being thus reckoned twice, gave occasion to the term 'bissextile,' *i.e.* twice the sixth.

The year was also ordered to commence on January 1, which date was selected as it was in that year the day of new moon immediately following the winter solstice of the year before.[2]

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[1] Lunar time is connected with solar time by the fact that 235 lunar months correspond almost exactly in duration with 19 solar years of 365¼ days each, so that if new moon occurs on January 1, it will occur on the same date again after a lapse of 19 years.

[2] Some idea of the confusion into which the reckoning of time had fallen may be gained by the fact that he enacted that the preceding year (B.C. 46) should consist of 445 days; this was known as the 'year of confusion.' It should be remarked that there is no year A.D. 0, that preceding A.D. 1 being B.C. 1.—Herschel's *Outlines of Astronomy*.

The correction thus made was equivalent to considering the mean solar year as consisting of 365·25 mean solar days, which was ·0078 day too much, being the difference between 365·25 days and 365·2422 days.

Gregorian correction

In A.D. 1582 this error had accumulated to several days, when Pope Gregory XIII. ordered that October 5 of that year should be considered to be the 15th,[1] and that in future, instead of *every* fourth year being reckoned as one of 366 days, those years which completed *centuries* should be considered as having 365 days, unless the *number of the century* was divisible by four; thus, 1700, 1800, and 1900 are common years, but the year 2000 will consist of 366 days.

This is equivalent to introducing 97 days instead of 100 in every 400 years, so that the *average* length becomes $365\frac{97}{400}$ or 365·2425 days; the *true* length being 365·2422 days, the error now will only amount to rather more than one day in 4,000 years.

This correction (termed the Gregorian correction) was not adopted in England until 1752,[2] but at present the only country in Europe using the uncorrected Julian year is Russia, the difference between what is known as the Old Style (O.S.) and New Style (N.S.) amounting to twelve days; thus, January 15 (O.S.) is January 27 (N.S.).

Problems on Time.

The conversion of time as measured by the apparent or mean sun (solar time) into time as measured by the stars (sidereal time), and *vice versâ,* is of such frequent occurrence in practice that the following problems should now be carefully studied.

As time, however measured, is an angle at the *pole* of the equinoctial or pole of the heavens, these problems are most clearly illustrated by figures drawn on the plane of the equinoctial, the arrow in each case (*see* figs. 91, 92, 93) representing the apparent movement of the heavens caused by the *real* movement of the earth in the opposite direction.

Problem I.

Given mean time and the equation of time at the same instant, to find apparent time; or,

Given apparent time and the equation of time, to find mean time.

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[1] The error had *really* accumulated to twelve days, instead of the ten which were omitted.

[2] September 3 of that year was considered to be September 14.

Figure 90 is a construction on the plane of the horizon, fig. 91 on the plane of the equinoctial. The sun's declination is supposed to be about 10° N. The declination is given to enable the student to insert the sun's place in the figure.

Construction

Let P represent the north pole, W Q E in fig. 90, or Q R Q′ in fig. 91, the equinoctial, N S or Q′ P Q the celestial meridian, X the place of the sun 10° N. of the equinoctial, and *m* the mean sun on the equinoctial. Through X and *m* draw the circles of declination P R and P *m*; then the angle Q P R, or arc Q R, is apparent time; the angle *m* P R, or arc *m* R, is the equation of time; and the angle Q P *m*, or arc Q *m*, is mean time.

Fig. 90.

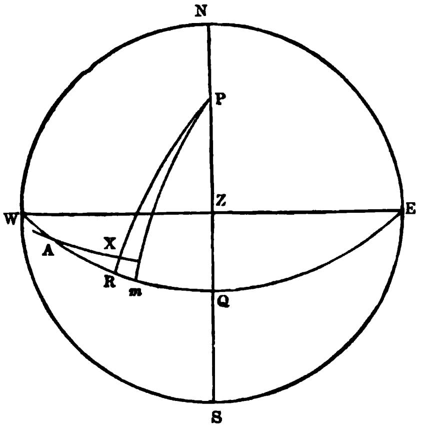
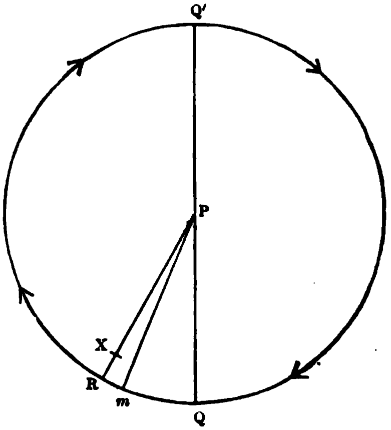


Fig. 91.



By the fig., $QR = Qm + mR$,

or apparent time = mean time + equation of time.

From which formula, apparent time or mean time may be found when the other two quantities are given.

When the equation of time is subtractive from mean time to get apparent time, *m* must be on the other side of R.

In the following easy examples (inserted chiefly for the sake of affording the student useful exercises in constructing diagrams), the positions of the true and mean suns are to be determined as nearly as possible by the eye and without the assistance of mathematical instruments.

Examples

1. Given apparent time $=5^h\ 10^m$ and the equation of time $=15^m\ 10^s$ subtractive from apparent time, the sun's declination being about 20° S.; construct a figure, and find by calculation mean time. *Ans.* Mean time $=4^h\ 54^m\ 50^s$.

2. Given mean time $=10^h\ 14^m\ 15^s$ and the equation of time $=2^m\ 30^s$ additive to mean time; construct a figure, and find by calculation apparent time (declination 10° S.).

Ans. Apparent time $=10^h\ 16^m\ 45^s$.

3. Given apparent time $=4^h\ 30^m$ and equation of time $=3^m\ 0^s$ additive to apparent time; construct a figure, and find by calculation mean time (declination 20° N.).

Ans. Mean time $=4^h\ 33^m\ 0^s$.

4. Given mean time $=14^h\ 30^m$ and equation of time $=2^m\ 0^s$ additive to mean time; construct a figure, and find by calculation apparent time (declination 20° N.).

Ans. Apparent time $=14^h\ 32^m$.

5. Given mean time $=22^h\ 10^m$ and equation of time $=10^m\ 0^s$ subtractive from mean time; construct a figure, and find by calculation apparent time (declination 20° S.).

Ans. Apparent time $=22^h\ 0^m$.

Problem II. Given mean time and the right ascension of the mean sun, to find sidereal time or the right ascension of the meridian.

Let Q A Q′ represent the equinoctial, A the first point of Aries, Q P Q′ the celestial meridian, and P the pole. Then, if the mean sun is west of meridian, let *m* (fig. 92) be the mean sun.

If east of the meridian, let *m′* (fig. 93) be the mean sun.

In the figs. 92 and 93, Q A = sidereal time, since it measures the time elapsed from the first point of Aries being on the meridian.

In fig. 92, Q *m* = mean time.

In fig. 93, *m′*Q = arc that must be described before the mean sun *m′* arrives at the meridian.

$$\therefore\ 24^h - m'Q = \text{mean time},\ \therefore\ m'Q = 24^h - \text{mean time}.$$

Also, A *m*, or A Q *m′* = right ascension of mean sun.

Now (fig. 92) $QA = Am + mQ$,

or sidereal time = R A mean sun + mean time.

In fig. 93, $QA = Am' - m'Q$,

or sidereal time = R A mean sun − (24^h − mean time)

= R A mean sun + mean time − 24^h.

The latter expression for sidereal time differs from the former by subtracting 24 hours; and, assuming that we can always subtract 24 hours from the right-hand side of the equation where it exceeds that number, the first equation may be considered true in both cases; as we shall get the same result for all other positions of *m* and A with respect to the meridian, therefore this rule is always true; viz.

Sidereal time = R A mean sun + mean time.

The right ascension of the mean sun at the instant required is found from the following considerations.

On page ii. of each month in the *Nautical Almanac* is given the sidereal time at mean noon at Greenwich or the angular distance at that instant between the first point of Aries and the mean sun, that is the right ascension of the mean sun *at that instant.*

FIG. 92.

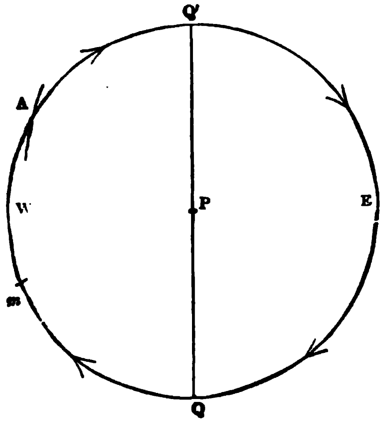
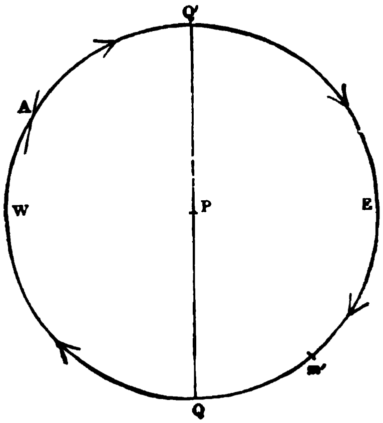


FIG. 93.



This quantity, we have seen (pp. 124, 125), is constantly and regularly increasing, and therefore to find the position of *m* with regard to A for any other place or time the amount of the movement of the mean sun in right ascension during the interval since noon at Greenwich must be added to the sidereal time taken from the *Almanac.*

This interval is always very approximately known, and the required correction (always additive) is taken from Inman's Tables or the *Nautical Almanac.*

Sidereal time may also be found by adding together the apparent solar time and the true sun's right ascension at that instant (page 1 of each month in *Nautical Almanac*).

Examples

6. Given mean time = $6^h\ 10^m$, and the right ascension of the mean sun at the same instant = $2^h\ 22^m\ 41^s$. Construct a figure, and find by calculation sidereal time.

Ans. Sidereal time = $8^h\ 32^m\ 41^s$.

7. Given mean time = $17^h\ 42^m\ 10^s$, and the right ascension of

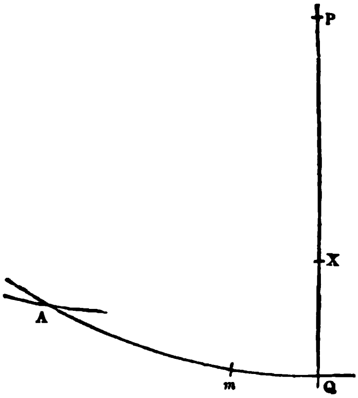
the mean sun at the same instant$=18^h\ 47^m\ 14^s$. Construct a figure, and find by calculation sidereal time.

Ans. Sidereal time$=12^h\ 29^m\ 24^s$.

The *Mean Time of Transit of the First Point of Aries* (given on page iii. of each month in the *Nautical Almanac*) is the distance of the *mean* sun from the meridian, at the instant when the first point of Aries is on the meridian of Greenwich; and as the distance of the first point of Aries from the meridian, at the instant the mean sun is on the meridian, is denominated sidereal time at mean noon, this may, by analogy, be termed the *mean time* at *sidereal noon.* It is the time by a mean time clock adjusted to the Greenwich meridian at the moment that the sidereal clock indicates exactly $0^h\ 0^m\ 0^s$. The use of this column is to facilitate the reduction of sidereal to mean solar time, with the help of the Table of Time Equivalents or Inman's Tables, No. 40.

As the sidereal day is shorter than the solar day, there will always be an occasion (near the time of the vernal equinox) on which two consecutive transits of the first point of Aries occur on the same solar day; the first just after mean noon of one day, and the second just before mean noon of the following day. In the year 1898 this happened on March 21.

Fig. 94.



Problem III.

Given sidereal time and the right ascension of the mean sun at any instant, to find mean time at the same instant.

Let P Q (fig. 94) represent the celestial meridian, A Q the equinoctial, A the first point of Aries, and m the mean sun.

Then Q A$=$sidereal time, Q $m$$=$mean time;

and A $m$$=$right ascension of the mean sun.

By the fig., Q $m$$=$Q A$-$A m,

or mean time$=$sidereal time (increased if necessary by 24 hours)$-$right ascension of mean sun.

Hence it appears, that to find mean time we have only to subtract the right ascension of the mean sun at that instant from the given sidereal time, and the result will be mean time at that instant.

The right ascension of the sun is found as in the preceding problem, but for many nautical purposes the mean time is found with sufficient accuracy by subtracting from the sidereal time the sidereal time at the previous Greenwich mean noon.[1]

Examples

8. Given sidereal time $=12^h\ 10^m\ 10^s$, and the right ascension of the mean sun at the time $=1^h\ 42^m\ 14{\cdot}5^s$; required mean time.
Ans. $10^h\ 27^m\ 55{\cdot}5^s$.

9. Given sidereal time $=6^h\ 32^m\ 40{\cdot}5^s$, and the right ascension of the mean sun at the same time $=7^h\ 37^m\ 42{\cdot}4^s$; required mean time.
Ans. $22^h\ 54^m\ 58{\cdot}1^s$.

[**Note.—The practical rules on the conversion of time involving the correction of elements taken from the *Nautical Almanac* will be deferred until the methods of doing this have been explained.**

Civil and Astronomical Days.

The civil day begins at midnight and ends at the next midnight. The astronomical day begins at noon and ends at noon, and is later than the civil day by 12 hours. Again, in the astronomical day the hours are reckoned throughout from 0^h to 24^h; in the civil day there are twice 12 hours, the first 12 hours being before noon, or before the commencement of the astronomical day (denoted by A.M., *ante meridiem*); the latter are after noon, and distinguished by the letters P.M. (*post meridiem*).[2]

Rule I. *Given civil time, to reduce it to astronomical time.*

1. If the given civil time be P.M., it will be also astronomical time; P.M. being omitted.

2. If the given civil time be A.M., add 12^h thereto, and put the day one back, omitting the letters A.M.; thus:

(1) April 27, at $4^h\ 10^m$ P.M. (civil) is April 27, at $4^h\ 10^m$ (astro.).
(2) April 27, at 4 10 A.M. (civil) is April 26, at 16 10 (astro.).

---

[1] Either of these methods can only be considered as *approximations*, since in one case the right ascension of the sun is not corrected at all, and in the other it is only corrected for an *approximate* mean time.

The required mean time is accurately found as follows:

orrect the mean time of transit of first point of Aries (page iii. of each month) for the change of right ascension of the sun due to the longitude of the required place, thus finding the mean time of transit of ♈ for mean noon *of the place*, and to this add the equivalent in mean time of the given sidereal time.

[2] At an International Conference assembled at Washington, U.S., in October 1884, it was unanimously agreed that great advantage would follow from arranging that the astronomical and civil days should everywhere begin at mean midnight. On the question of the adoption of a *universal* day to begin for all the world at mean midnight of Greenwich, and to be counted from zero up to 24 hours, there was not such unanimity.

Examples

Reduce the following civil times to astronomical times:

	Civil times.		Astronomical times.
10.	June 3, $2^h\ 42^m$ A.M.	*Ans.*	June 2, $14^h\ 42^m$
11.	July 1, 6 18 A.M.	„	June 30, 18 18
12.	Dec. 10, 3 42 P.M.	„	Dec. 10, 3 42

Rule 2. *Given astronomical time, to reduce it to civil time.*

If the astronomical time is less than 12 hours, it will also be civil time P.M.

2. If the astronomical time be greater than 12 hours, reject 12 and put the day one forward; the result will be civil time A.M.; thus:

(1) April 27, at $4^h\ 10^m$ (astro.) is April 27, at $4^h\ 10^m$ P.M. (civil).
(2) April 27, at 16 10 (astro.) is April 28, at 4 10 A.M. (civil).

Examples

Reduce the following astronomical times to civil times:

	Astronomical times.		Civil times.
13.	Sept. 10, $4^h\ 32^m$	*Ans.*	Sept. 10, $4^h\ 32^m$ P.M.
14.	July 5, 16 32	„	July 6, 4 32 A.M.
15.	July 10, 18 42	„	July 11, 6 42 A.M.
16.	Dec. 21, 23 59	„	Dec. 22, 11 59 A.M.

Conversion of arc into time

Rule 3. *To convert arc into time.*

1. Divide the degrees of arc by 15, the quotient is hours.

2. Multiply the remaining degrees, if any, by 4; the result is minutes in time.[1]

3. Divide the minutes in arc by 15; the quotient is minutes in time.

4. Multiply the remaining minutes of arc, if any, by 4; the result is seconds of time.

5. Divide the seconds in arc by 15; the quotient is seconds in time, carried to decimals if necessary.

The sum of all the above will be the arc expressed in time.

NOTE.—The reason for this and the following rule will be found on page 180.

Examples

Reduce 34° 44′ 34″ into time:

$$\begin{array}{rl} 34^\circ = & 2^h\ 16^m\ \ 0^s \\ 44' = & \quad\ \ 2\ \ 56 \\ 34'' = & \qquad\ \ 2{\cdot}26 \\ \hline \therefore\ 34^\circ\ 44'\ 34'' = & 2\ \ 18\ \ 58{\cdot}26. \end{array}$$

---

[1] This is equivalent to multiplying by 60 and dividing by 15; *see* also page 180.

Reduce the following arcs into time:

17.	84°	42′	30″	*Ans.*	5^h	38^m	50^s
18.	96	10	45	„	6	24	43
19.	108	24	22	„	7	13	37·4
20.	178	48	45	„	11	55	15
21.	140	32	10	„	9	22	8·66
22.	240	32	10	„	16	2	8·66

Conversion of time into arc

Rule 4. *To convert time into arc.*

1. Multiply the hours by 15: the result is degrees.

2. Divide the minutes in time by 4; the quotient is degrees.[1]

3. Multiply the minutes remaining, if any, by 15; the result is minutes of arc.

4. Divide the seconds of time by 4; the quotient is minutes of arc.

5. Multiply the seconds (and parts of seconds) remaining, if any, by 15; the result is seconds of arc.

The sum of all the above will be the time expressed in arc.

Examples

Reduce $2^h\ 18^m\ 58{\cdot}26^s$ into degrees.

$$\begin{aligned} 2^h &= 30^\circ\ 0'\ 0'' \\ 18^m &= 4\ \ 30\ \ 0 \\ 58{\cdot}26^s &= \ \ 14\ \ 33{\cdot}9 \\ \therefore 2^h\ 18^m\ 58{\cdot}26^s &= 34\ \ 44\ \ 33{\cdot}9. \end{aligned}$$

Find the arcs corresponding to the following times:

23.	3^h	52^m	4^s	*Ans.*	58°	1′	0″
24.	17	8	22	„	257	5	30
25.	8	17	15·5	„	124	18	52·5
26.	12	14	16·75	„	183	34	11·25
27.	9	13	8	„	138	17	0
28.	15	17	18·4	„	229	19	36

NOTE.—In Inman's Tables, the angles in the log, sine and log, haversine tables are given both in arc and in time. The reduction from degrees to hours, and the converse, is by means of such a table (or Table 36) readily made, simply by inspection.

'Nautical Almanac'

From daily observations of the positions of the principal heavenly bodies made at astronomical observatories for several centuries, their movements are so well understood that very accurate *predictions* of their places at future dates can be made. Such predictions made for certain definite times at Greenwich are found in the *Nautical Almanac*, which is published several years in advance[1]; the daily or hourly *changes* of the elements of these positions is also given, so that an observer at any part of the world and at any time can readily ascertain the true position of a heavenly body at Greenwich at the time of his making an observation of it, provided that he knows the time which has elapsed since the last predicted position given in the *Almanac*, or, in other words, provided he knows the time at *Greenwich* when his observation was made. The positions given are such as would be seen by an observer at the *centre* of the earth at the given Greenwich times.

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[1] This is equivalent to multiplying by 15 and dividing by 60.

The reader should now study the general arrangement of the *Nautical Almanac* as a work which has constantly to be referred to; those portions in most frequent use are the eighteen pages devoted each month to the positions and apparent magnitudes of the sun and moon, and also the list of the positions of the principal stars.

THE GREENWICH DATE.

Ship time and Greenwich time

In every practical problem in nautical astronomy it is necessary to ascertain the apparent or mean time at Greenwich (usually to the nearest minute) at which the observation was taken.

The connection between local or ship time estimated as *civil* time and Greenwich astronomical time will be understood from the following considerations.

The longitude of a place has been defined as an arc of the equator intercepted between the meridian of Greenwich and the meridian passing through the place (page 2). The connexion between this mode of estimating longitudes and that in which, instead of degrees of arc (360 of which equal the circumference of the equator), we employ hours and minutes, will be seen on referring to fig. 95.

Let P represent the north pole of the earth; P *g* the meridian of Greenwich; P E a meridian 15° east of Greenwich; P W a meridian 15° west of Greenwich. The arrows indicate the direction of rotation of the earth.

---

[1] The *Nautical Almanac* was originally published in 1767, being preceded in France by a similar work, the *Connaissance des Temps*, which was published in 1696. Originally intended exclusively for nautical purposes, it did not attain its present bulk until 1834, when large additions were made to it in the interests of astronomers.

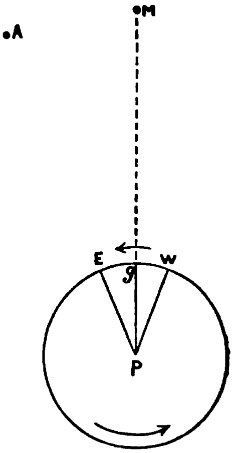
Accompanying the first *Nautical Almanac* was published the *Tables requisite to be used with the 'Nautical Ephemeris,'* which form the basis of our modern nautical tables; both of these works were compiled by Maskelyne, the Astronomer Royal. Prior to 1832 the elements of position in the *Nautical Almanac* were all given for *apparent* time.

The excellent 'Explanation' at the end of the *Nautical Almanac* (originally drawn up in 1832 by a committee of the Royal Astronomical Society, should be carefully studied by the reader.

With regard to the mean sun M, 24 mean solar hours will elapse between two successive passages across the *same* meridian, and therefore if we suppose 24 meridians to be drawn at equal distances apart (that is, $\frac{360°}{24}=15°$) they will, by the rotation of the earth, pass under the mean sun at intervals of one mean solar hour.

If one of these 24 meridians P g be that of Greenwich, the first of those eastward of that meridian P E will evidently pass under the mean sun one mean solar hour *before*, and the first westward of Greenwich P W one hour *after* the mean sun has crossed that meridian.

FIG. 95.



In other words, at Greenwich mean noon the local mean time (ship mean time) at every place on the meridian P E will be 1 P.M., and similarly at the same instant the ship mean time at every place on the meridian P W will be 11 A.M.

In precisely the same way if on the same figure A represents the first point of Aries, this will by the rotation of the earth apparently cross the meridians of places separated by 15° of longitude at equal intervals of one *sidereal* hour, and evidently when on the meridian of P g it will have crossed that of P E one sidereal hour *before*, and will be on the meridian of P W at one sidereal hour *later*.

The same remarks apply to the measurement of time by *any* heavenly body, whether fixed or in motion, *provided that motion is uniform in right ascension;* during an interval of 24 hours the movement of the apparent sun is sensibly uniform in this respect, and hence, since when either the mean or apparent sun or the first point of Aries is on the meridian of P g they will be 15° of arc from the meridians of P E and P W, the mean, apparent, or sidereal times to observers on those meridians will differ from that on the meridian of P g by one mean, apparent, or sidereal hour.

From this it follows that *all* descriptions of time in common use are correctly converted into arc at the rate of 15° to an hour.

To find the Greenwich date

Rule 1. *First method. By estimated ship time and longitude.*

(1) Express the time at the ship astronomically to the nearest minute.

(2) Reduce the longitude into time to the nearest minute, and put it under ship time.

(3) If in west longitude, add longitude in time to ship time; the sum, if less than 24 hours, will be the time at Greenwich, or the Greenwich date on the same day as at the ship.

But if the sum be greater than 24 hours, reject 24 hours; the result will be the Greenwich date on the day following the ship date.

(4) If in east longitude, subtract longitude in time from ship time; the remainder will be the Greenwich date. If the longitude in time is greater than the ship time, 24 hours must be added to the ship time before subtraction is made, and the remainder will be the Greenwich date on the day preceding the ship date.[1]

Examples

June 10, at $6^h\,10^m$ P.M. mean time nearly, in longitude by account 32° 45′ W.: required the Greenwich date, to the nearest minute.

Ship, June 10......$6^h\,10^m$
long. in time.........2 11 W.
∴Green. June 10.....8 21

July 12, at $4^h\,5^m$ A.M. in long. 63° 45′ E.: required the Greenwich date.

Ship, July 11.......$16^h\;\;5^m$
long. in time....... 4 15 E.
∴Green. July 11....11 50

Required the Greenwich date in each of the following examples:

	Ship Times.					Answers, Greenwich Dates.
29.	Mar. 7, at	$3^h\,15^m$	A.M.	Long.	15° 45′ E.	Mar. 6, at $14^h\,12^m$
30.	Mar. 15 „	10 35	P.M.	„	43 5 E.	Mar. 15 „ 7 43
31.	May 12 „	4 30	A.M.	„	45 50 W.	May 11 „ 19 33
32.	May 9 „	5 16	P.M.	„	90 35 E.	May 8 „ 23 14
33.	May 5 „	11 30	P.M.	„	55 47 W.	May 5 „ 15 13
34.	May 20 „	10 25	A.M.	„	150 15 W.	May 20 „ 8 26

Rule 2. *Second method. By chronometer and its error on Greenwich mean time.*[2]

The time at Greenwich, and therefore the Greenwich date, is more correctly found by means of a chronometer whose error on Greenwich mean time is known at the moment of observation.

---

[1] This may be remembered from the following rhymes:

In longitude west, the Greenwich time's best.
In longitude east, the Greenwich time's least.

[2] The methods of finding this error will be explained in Chapter XII.

To the time shown by chronometer, apply its error on Greenwich mean time, adding if the error is slow, and subtracting if it is fast; the result is the Greenwich date in mean time. Sometimes 12 hours must be added to this result, and the day put one back.[1] To determine when this must be done, get an approximate Greenwich date by Rule 1 from the ship mean time and the estimated longitude; if the difference between the Greenwich dates found by the two methods is nearly 12 hours, then the Greenwich date *by chronometer* must be increased by 12 hours, and the day put one back, if necessary, so as to make the two dates more nearly agree both in the day and hour.

The following examples will remove any doubt as to putting the day one back or not, when the 12 hours are added.

Examples

July 10, at 6^h 34^m P.M. mean time nearly, in longitude 60° W., a chronometer showed 10^h 42^m 3^s, its error on Greenwich mean time being 2^m 10^s fast. Required mean time at Greenwich, or the Greenwich date.

Greenwich Time by Chronometer.		Greenwich Date.	
July 10, chron.	10^h 42^m 3^s	Ship, July 10......	6^h 34^m
Error on G. M. T....	2 10 fast	long. in time......	4 0 W.
Gr. July 10............	10 39 53	Gr. July 10.........	10 34

As these two results come out near to each other, the correct Greenwich time is July 10, 10^h 39^m 53^s, and the Greenwich date to the nearest minute is therefore July 10, 10^h 40^m.

Aug. 3, at 5^h 42^m P.M. mean time nearly, in long. by account 150° 30′ W., a chronometer showed 3^h 23^m 15^s, and its error on Greenwich mean time was 10^m $10{\cdot}4^s$ slow. Required the Greenwich date.

Greenwich Time by Chronometer.		Greenwich Date.	
Aug. 3, at.....	3^h 23^m 15^s	Ship, Aug. 3...	5^h 42^m
Error..........	10 10·4 slow	long. in time...	10 2 W.
Aug. 3.........	3 33 25·4	Gr. Aug. 3......	15 44
Add	12		
Gr. Aug. 3....	15 33 25·4		

In this example 12 hours must be added to the Greenwich date by chronometer to bring the Greenwich times more nearly alike.

---

[1] This is owing to the fact that the dial-plates of most chronometers resemble clocks in only indicating to 12 hours; some, however, are now made to show the hours from 0 to 24; such a timepiece with a supposed rate of 4 seconds daily would, if set to Greenwich mean time on leaving England, only differ from that time by about 1¼ hours at the expiration of three years.

March 10, at $2^h\ 10^m$ A.M. mean time nearly, in longitude 20° 42′ E., a chronometer showed $0^h\ 2^m\ 50^s$, and its error on Greenwich mean time was $45^m\ 16^s$ slow. Required the Greenwich date.

Greenwich Time by Chronometer.		Greenwich Date.	
March 10......	$0^h\ 2^m\ 50^s$	Ship, Mar. 9...	$14^h\ 10^m$
Error...........	45 16 slow	long. in time...	1 23 E.
Mar. 10.........	0 48 6	Gr. Mar. 9......	12 47
Add.............	12		
Gr. Mar. 9.....	12 48 6		

In this example 12 hours must be added, and the day put one back, to bring the chronometer Greenwich time more nearly alike to the Greenwich date.

Find accurately the Greenwich times in the following examples:

	M.T. nearly.	Long.	Chron. shows.	Err. of Chron.	Answers.
35.	Aug. 10 $1^h\ 20^m$ P.M.	35° 42′ W.	$4^h\ 2^m\ 10^s$	$18^m\ 45{\cdot}4^s$ fast	10 $3^h\ 43^m\ 24{\cdot}6^s$
36.	July 13 3 42 A.M.	150 50 W.	1 30 0	10 50·6 slow	13 1 40 50·6
37.	June 10 10 42 P.M.	42 0 E.	7 44 10	8 12·0 slow	10 7 52 22
38.	June 19 6 42 A.M.	50 50 W.	10 14 15	12 3·7 fast	18 22 2 11·3
39.	Sept. 3 10 42 A.M.	19 15 E.	9 10 45	12 15·3 slow	2 21 23 0·3
40.	Dec. 30 11 45 P.M.	110 35 W.	7 10 30	9 5·0 fast	30 19 1 25

Having thus found the Greenwich date at the time of taking an observation, the various elements involved, such as declination, &c., are corrected for any changes which may have occurred in them since the nearest time for which they are given in the *Nautical Almanac.* The Greenwich date is usually understood to be required in *mean* time, but on applying the longitude in time to ship *apparent* time the Greenwich date is found, if required, in *apparent* time, and similarly with regard to sidereal time.

NOTE.—For all practical purposes in correcting elements taken from the *Nautical Almanac*, the mode of doing so by using the Greenwich date found by Method 1, page 181, is sufficiently correct, and will be generally used in the Examples in this book; but since it is advisable to take the time at almost all observations by a chronometer whose error on Greenwich is known, the observer would be able, if desirous, to use the accurate Greenwich mean time.

CORRECTION OF ELEMENTS IN 'NAUTICAL ALMANAC.'

To take out the sun's declination for any date

(1) Find the Greenwich date.

(2) Take out of the *Nautical Almanac* the declination *at the nearest mean noon* to the Greenwich date, and put down a little to the right thereof the difference for one hour (var. in 1 hour)

found in the next column to the *apparent* declination.[1] Multiply this quantity by the hours from the *nearest* noon found from the Greenwich date, and the fractional or decimal parts of the hour if necessary; the product will be the change of declination in the time from noon; apply this, according to sign reduced to minutes and seconds of arc, to the declination taken out. The result is the declination of the sun at the time required.[2]

Examples Mar. 2, at $4^h\ 23^m$ P.M. meantime nearly, in long. 32° 42′ W.: required the sun's declination.

Ship, Mar. 2............ $4^h\ 23^m$
long. in time............ 2 11 W.
Gr. Mar. 2.............. 6 34

Sun's Decl.
Mar. 2...7° 7′ 0″ S. ...var. in 1 hour 57·4″ decreasing
corr.... 6 17 − 6
∴ decl. = 7 0 43 S. 344·4 change in 6^h
30^m....½ 28·7
3 ...$\frac{1}{10}$ 2·9 change in 34^m
1 ....⅓ 0·9
60 376·9
∴ corr. for $6^h\ 34^m$...... 6′ 16·9″ −

March 21, at $4^h\ 23^m$ A.M. in long. 100° 10′ E.: required the sun's declination.

Ship, Mar. 20............$16^h\ 23^m$
long. in time .... 6 41 E.
Gr. Mar. 20 .... 9 42 or 9·7 hrs.

Sun's Decl.
Mar. 20...0° 5′ 32″ S. Var. in 1 hour...... 59·2″ decreasing
corr. ......— 9 34 hours from noon... 9·7
∴ decl. = 0 4 2 N. 414·4
5328
574·24″ = 9′ 35″ −

41. July 30, at $3^h\ 20^m$ P.M. mean time, in long. 9° 0′ W.: required the sun's decl. *Ans.* 18° 28′ 14″ N.

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[1] The hourly difference is greatest at the equinoxes (March and September) when it is about 59″ and the sun has no declination or 'crosses the line,' and it is zero at the solstices (June and December) when the declination is at a maximum.

[2] The corrections for Greenwich date of the quantities taken out of the *Nautical Almanac*, when small, are frequently made *by inspection;* the results thus obtained are generally found sufficiently correct for purposes of navigation. The sun's declination is given in the *Nautical Almanac* for both apparent and mean noon at Greenwich; in some problems one is required and in some the other.

42. Dec. 10, at $6^h\ 32^m$ A.M. mean time, in long. 32° 30′ W.: required the sun's decl. *Ans.* 22° 54′ 40″ S.

43. Sep. 22, at $4^h\ 52^m$ P.M. mean time, in long. 57° 15′ W., when a chronometer showed $8^h\ 38^m\ 12^s$, error on Greenwich mean time (G.M.T.) $3^m\ 36^s$ slow: required the sun's decl.
Ans. 0° 1′ 44″ S.

Decl. and Var. in 1 *hour from 'Nautical Almanac.'*

		Var. in 1 hour.
July 30............	18° 30′ 39″ N.	36·7″ decreasing
Dec. 10............	22 55 22 S.	12·7 increasing
Sep. 22......	0 6 38 N.	58·5 decreasing

The sun has north declination from March 20 to Sep. 22, attaining its maximum value (23° 27′) on June 21; and it has south declination from Sep. 22 to March 20, attaining its maximum value (23° 27′) on Dec. 21.

NOTE.—The var. in 1 hour of both declination and equation of time is the variation at the instant of *apparent* noon, and, strictly speaking, is not applicable to any other *apparent* time, nor to any *mean* time whatever; for purposes of navigation no practical error is involved by neglecting this distinction.

Prior to 1863 the var. in 1 hour represented the whole change in 24 hours divided by 24.

The sun's approximate declination and equation of time are given for a period of four years in the Time Azimuth Tables; they repeat their values almost exactly at the end of this interval, so that these elements for any day in 1899 may be looked out for the same date in 1891 or 1895 (*see* Inman's Tables, Nos. 22 and 39).

Great care should be taken in correcting the declination of a heavenly body at those times when it is changing from north to south, and *vice versâ*; similarly with the equation of time on those days when it changes from additive to subtractive, and *vice versâ*.

To take out the equation of time for any date

(1) Find the Greenwich date.

(2) Take out the equation of time for the nearest noon to the Greenwich date and the hourly difference (var. in 1 hour).[1]

(3) Multiply the hourly difference by the hours from noon found from the Greenwich date, and, if great accuracy is required, by the fractional or decimal parts of an hour in the Greenwich date; the result will be the correction to be applied with its proper sign to the equation of time taken out.

Example

July 12, at $5^h\ 5^m$ A.M. mean time nearly, in long. 160° W.: required the equation of time.

---

[1] The hourly difference is zero when the equation of time is greatest, and attains its maximum value near the times when the equation vanishes; thus on December 22 it changes half a minute a day. The greatest value of the equation of time is $16^m\ 19^s$.

Ship, July 11..................17^{h} 5^{m}
long. in time ....10 40 W.
Greenwich, July 11 ....27 45
Greenwich, July 12 .... 3 45

Equation of Time.

July 12 ....5^{m} 15·7^{s}		Var. in 1 hour......0·308^{s} incr.	
Corr. ....+ 1·2			3
∴ Equation =5 16·9			0·924
		30^{m} ½	0·154
		15 ½	0·077
		Corr.	1·155 or 1·2 +

Find the equation of time in each of the following examples:

44. March 2, at 6^{h} 10^{m} P.M. mean time, in long. 38° 42′ W.
45. June 13, „ 10 42 P.M. „ „ 152 45 W.
46. March 29, „ 10 41 A.M. „ „ 87 8 E.

when a chronometer showed 4^{h} 52^{m} 28^{s}, error 1^{m} 0^{s} slow on G.M.T.

Equation of Time from 'Nautical Almanac.'

		Var. in 1 hour.
Eq. of time, March 2,	12^{m} 22·1^{s}...............	0·53^{s} decreasing
„ June 13,	0 9·4	0·51 „
„ March 29,	4 50·5	0·77 „

Ans. to 44, 45, 46: 12^{m} 17·5^{s}, 0^{m} 1·3^{s}, 4^{m} 55·9^{s}.

To take out the right ascension of the mean sun for any date

The right ascension of the mean sun (R. A. M. S.), or the sidereal time at Greenwich, is given in the *Nautical Almanac* for every day at mean noon. As the motion of the mean sun is uniform throughout the year (the motion in every 24 mean solar hours being 3^{m} 56·555^{s} in sidereal time), the change in any given number of hours, minutes, and seconds is easily found by means of a table. This is given in the *Nautical Almanac*, under the title of 'Time Equivalents'; it may be also found, arranged in a very convenient form, in Inman's Tables, No. 40.

As the right ascension is continually increasing, the correction is always additive to the sid. time at the G. M. noon preceding.[1]

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[1] As the beginner is sometimes confused by the similarity of terms employed in speaking of the right ascension of the sun, we give the following explanations:

1. The apparent right ascension of the sun at *apparent* noon is the same as the sidereal time at apparent noon, or the time shown by a sidereal clock when the true sun is on the meridian.
2. The apparent right ascension of the sun at *mean* noon is the right ascension of

Example

July 23, at $2^{h}\ 32^{m}$ P.M., in long. 80° 42′ E.: required the right ascension of the mean sun.

Ship, July 23	$2^{h}\ 32^{m}$
long. in time	5 23 E.
Green., July 22 ...	21 9

Right Asc. Mean Sun.

July 22 sid. time at Gr. mean noon	$8^{h}\ 0^{m}\ 35^{s}$
corr. for 21^{h}	3 27
„ 9^{m}.........	1·5
R. A. M. S............	8 4 3·5

Find the right ascension of the mean sun in the following examples:

47. March 2, at $10^{h}\ 42^{m}$ P.M. mean time, in long. 48° 10′ W.
48. „ 15 „ 6 6 A.M. „ „ 100 0 W.
49. „ 21 „ 10 9 P.M. „ „ 100 0 E.

when a chronometer showed $3^{h}\ 28^{m}\ 30^{s}$, error $1^{m}\ 30^{s}$ slow on G.M.T.

Elements from 'Nautical Almanac,' and Answers.

March 2,	Sidereal time at Greenwich mean noon,	$22^{h}\ 40^{m}\ 44{\cdot}9^{s}$...	*Ans.* $22^{h}\ 43^{m}\ 2{\cdot}0^{s}$
„ 15,	„	23 32 0·1 ...	„ 23 32 7·6
„ 21,	„	23 55 39·4 ...	„ 23 56 13·9

To take out the moon's declination and right ascension for any date

The moon's declination and right ascension are recorded in the *Nautical Almanac* for the beginning of every hour of mean time at Greenwich, together with the variation or change in 10 minutes. To find them for any intermediate time we may proceed as follows:

(1) Find the Greenwich date.

(2) Take out of the *Nautical Almanac* the moon's declination and right ascension (R. A.) for the hour preceding that of the Greenwich date.

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the true sun at the instant of mean noon, and differs from the preceding by the movement of the sun in right ascension during the interval between apparent and mean noon—that is, during the interval of the equation of time.

3. The sidereal time at mean noon is the right ascension of the *mean* sun at mean noon, or the time shown by a sidereal clock when the *mean* sun is on the meridian; it differs in value from (2) by the amount of the equation of time.

4. The mean time of transit of the first point of Aries (given on page iii. of each month in the *Nautical Almanac*) is the distance of the *mean* sun from the meridian when the first point of Aries is *on* the meridian, or, in other words, is the mean time at sidereal noon.

In each of the above cases, by the first point of Aries is understood the *true* position of the equinoctial point at the instant considered.

Throughout this work the term 'Right ascension of mean sun' (R. A. M. S.) will be applied to the sun at the *instant considered*, and 'Sidereal time at mean noon' will mean the right ascension at the *instant of Greenwich mean noon.*

(3) Take out the difference of decl. and right ascension for 10^{m} (var. in 10^{m}) opposite the declination and right ascension taken out.

(4) Multiply the 10^{m} diff. by the number of minutes in Greenwich date, and remove the decimal point one place to the left: the result is the correction in decl. and right ascension for Greenwich date in seconds, which turn into minutes and seconds if necessary, and apply to the decl. and right ascension originally looked out.[1]

The correction for declination is additive or subtractive according as the declination is increasing or decreasing.

The correction for right ascension is always additive.

Examples January 24, at 5^{h} 40^{m} P.M. mean time, in long. 60° 15′ W.: find the moon's declination and right ascension.

Ship, Jan. 24	5^{h} 40^{m}
long. in time	4 1 W.
Gr. Jan. 29	9 41

	Moon's Decl.	Var. in 10^{m}.		Moon's R. A.	Var. in 10^{m}.
Jan. 24, 9^{h}...	10° 9′ 7″ N.	116·9″ incr.		0^{h} 54^{m} 49^{s}	22·27^{s} incr.
corr. for 41^{m}	7 59	41	corr.	1 31	41
	10 17 6	1169		0 56 20	2227
		4676			8908
		479·29			91·307
		7′ 59″ +			

50. June 2, at 10^{h} 30^{m} A.M. mean time, in long. 53° 15′ W.: find the moon's right ascension and declination.

Ans. R. A. 2^{h} 47^{m} 29·3^{s}, decl. 17° 12′ 45″ N.

51. Sept. 7, at 4^{h} 15^{m} A.M. mean time, in long. 56° 30′ E.: find the moon's right ascension and declination.

Ans. R. A. 14^{h} 53^{m} 51·9^{s}, decl. 16° 51′ 39″ S.

52. July 10, at 9^{h} 31^{m} A.M. mean time, in long. 44° 20′ W.,

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[1] The var. in 10^{m} given in the *Nautical Almanac* is the rate of change of the elements at the *commencement* of each hour, but no practical error is involved by considering it as the rate of change *during* the hour.

The declination and right ascension of a planet is taken from the Planetary Ephemerides at mean noon given in the *Nautical Almanac* and corrected for the Greenwich date in a similar way to the method employed in correcting the sun's declination, using the proportional part of the differences in 24 hours; or the *hourly* variations may be taken from the Planetary Ephemerides at transit at Greenwich. The greatest daily changes in the R. A. and decl. of Venus are 6 minutes and 35″, and the corresponding changes for Mars 4 minutes and 25″.

when a chronometer showed 1^h 2^m 10^s, error 34^m 50^s fast on G.M.T.: find the moon's right ascension and declination.

Ans. R. A. 11^h 27^m $47{\cdot}8^s$, decl. 1° 23′ 0″ S.

Moon's R. A. and Decl. from 'Nautical Almanac.'

	Moon's Right Ascension.	Var. in 10^m.	Moon's Decl.	Var. in 10^m.
June 2, at 2^h...	2^h 47^m $23{\cdot}1^s$	$20{\cdot}83^s$ incr....	17° 12′ 26″ N.	63·4″ incr.
Sept. 6, at 12 ...	14 52 44·7	$23{\cdot}16^s$ incr....	16 48 19 S.	68·9″ incr.
July 10, at 0 ...	11 26 55·2	$19{\cdot}49^s$ incr....	1 17 58 S.	111·7″ incr.

The moon's declination varies from 28° 35′ N. to 28° 35′ S. (*see* page 221); the var. for 10^m has a maximum value of about 3′.

The moon's right ascension increases from 0^h to 24^h during every month; its maximum change in 1 hour is 2^m 55^s and minimum 1^m 45^s.

To take out the moon's semi-diameter and horizontal parallax for any date

The moon's semidiameter and horizontal parallax are given in the *Nautical Almanac* for every mean noon and mean midnight at Greenwich; to find these quantities for any other time we may proceed as follows:

(1) Find the Greenwich date.

(2) Take out of the *Nautical Almanac* the semidiameter and horizontal parallax for the epochs between which the hour of the Greenwich date lies and take their differences.

(3) Divide the differences by 12, and multiply by the hours and decimals of the Greenwich date, which will give the corrections in seconds to be applied according to sign to the semidiameter and horizontal parallax at the earlier epoch.

Examples

Aug. 3, at 4^h 10^m P.M. mean time nearly, in long. 56° 15′ W.: required the moon's semidiameter and horizontal parallax.

Ship, Aug. 3 ....4^h 10^m
long. in time ....3 45W.
Greenwich, Aug. 3...7 55 = 7·9 hrs.

Moon's Semidiameter.

August 3, at noon...15′ 6·6″
" mid. ...15 10·6
4·0
7·9
12)31·6
Corr. 2·6 +
Semi. at noon 15 6·6
Corrected semi. 15 9·2

Moon's Horizontal Parallax.

August 3, at noon...55′ 20·6″
" mid. ...55 35·3
14·7
7·9
1323
1029
12)116·13
Corr. 9·6 +
Hor. par. at noon 55 20·6
Corrected hor. par. 55 30·2

Note.—These corrections are always so small that for most practical problems except Lunars they can be taken out by inspection or entirely omitted.

Find the moon's semidiameter and horizontal parallax in the following examples:

53. March 2......at 6^h 42^m P.M.......in long. 100° 0′ W.
54. „ 14......at 3 30 A.M....... „ 120 0 W.
55. „ 24......at 10 9 P.M....... „ 60 42 E.

when a chronometer showed 6^h 8^m 20^s, error 1^m 8^s fast on G.M.T.

Elements from 'Nautical Almanac.'

Moon's Semidiameter.	Moon's Hor. Par.		Answers.
March 2, mid. 16′ 5·1″	Mid. 58′ 54·7″	53	Semi. 16′ 4·7″
„ 3, noon 16 2·1	Noon 58 43·6		H. P. 58 53·4
„ 13, mid. 14 48·9	Mid. 54 15·7	54	Semi. 14 47·9
„ 14, noon 14 47·8	Noon 54 11·5		H. P. 54 11·7
„ 24, noon 16 18·7	Noon 59 44·6	55	Semi. 16 21·2
„ 24, mid. 16 23·7	Mid. 60 1·8		H. P. 59 53·3

The moon's equatorial horizontal parallax varies from 61′ 24″ to 53′ 48″ according to her distance from the earth.

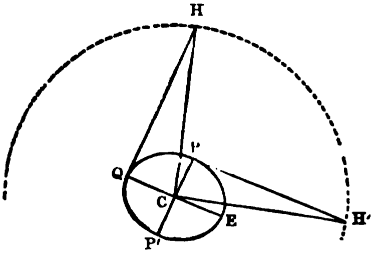
The moon's semidiameter is always nearly $\frac{3}{11}$ of the horizontal parallax, and varies from 16′ 45″ to 14′ 44″.

The change of semidiameter in 12 hours never exceeds 8″, and that of horizontal parallax in the same interval never exceeds 27″.

Equatorial horizontal parallax

The horizontal parallax given in the *Nautical Almanac* is the *equatorial* horizontal parallax, or the *greatest* angle under which the earth's equatorial (and greatest) semidiameter would appear if seen from the centre of the moon.

Fig. 96.



Thus if H represents the moon on the horizon of Q, a place on the equator, the horizontal parallax taken out of the *Almanac* is the angle Q H C, which is *greater* than the horizontal parallax for any other latitude, that at the pole P H¹ C being the *least* of all; the extreme difference being only about 12″ (*see* Inman's Tables, No. 50) it may be neglected in nautical astronomy, and the parallax taken from the *Almanac* considered as the horizontal parallax of *any* place.

PRACTICAL RULES ON CONVERSION OF TIME.

Rule (*a*). *Given mean solar time and the equation of time; to find the apparent solar time: or*

Given apparent solar time and the equation of time; to find mean solar time.

(1) Find the Greenwich date.

(2) Correct for this date the equation of time (taken out of page i. of the required month in the *Nautical Almanac* when apparent solar time is given, and out of page ii. when mean time is given).

(3) Apply the equation of time, with its proper sign (as shown in the *Nautical Almanac*), to the given time; the result is the time required.

Examples

April 27, at $9^h\ 10^m$ P.M. mean time, in long. 16° W.: required apparent solar time.

Mean Time.	Equation of Time (page ii. 'Naut. Alm.').	Ship Mean Time.
Ship, April 27 . $9^h\ 10^m$	27 . . $2^m\ 26{\cdot}9^s$ add to	April 27 . $9^h\ 10^m\ 0^s$
long. in time . 1 4 W.	mean time.	eq. of time . . 2 31 +
Gr., April 27 . 10 14		∴ apparent time 9 12 31
or 10·2 hrs.	Var. in 1 hour, $\cdot 40^s$ incr.	
	Hours from noon 10·2	
	Corr. 4·08 +	
∴ corrected equation of time = 2 30·98 +		

June 22, at $5^h\ 42^m$ P.M. apparent solar time, in long. 100° 30′ E.: required mean solar time.

Apparent Time.	Equation of Time (page i. 'Naut. Alm.').	Ship App. Time.
Ship, June 22 . $5^h\ 42^m$ P.M.	22nd . $1^m\ 40{\cdot}2^s$ add	June 22 $5^h\ 42^m\ 0^s$
long. in time . 6 42 E.	to app. time.	eq. of time . . 1 39·6 +
Gr., June 21 . 23 0	Var. in 1 hour 0·54 incr.	∴ mean time 5 43 39·6
or 1 hour from noon	Corr. for do. 0·54 —	
∴ corrected equation of time = 1 39·66 +		

56. July 4, at $6^h\ 10^m$ P.M. mean time, in long. 100° W.: required apparent solar time. *Ans.* $6^h\ 5^m\ 53{\cdot}2^s$.

57. Dec. 10, at $4^h\ 42^m$ P.M. apparent solar time, in long. 80° 45′ W.: required mean solar time. *Ans.* $4^h\ 35^m\ 18{\cdot}1^s$.

58. Feb. 23, at $10^h\ 40^m$ A.M. apparent solar time, in long. 1° 6′ W.: required mean solar time. *Ans.* $10^h\ 53^m\ 43{\cdot}5^s$ A.M.

Elements from 'Nautical Amanac.'

Equation of time, July 4	$4^m\ 1{\cdot}1^s$ sub.	Var. in 1 hour	$\cdot 444^s$	incr.
" Dec. 10	6 53·5 sub.	"	1·148	decr.
" Feb. 23	13 43·1 add.	"	·331	decr.

Rule (*b*). *Given mean solar time; to find sidereal time.*

(1) Find the Greenwich date.

(2) Increase the right ascension of the mean sun (taken out of the *Nautical Almanac* as the *sidereal time* at the mean noon preceding the Greenwich date), by the table of Time Equivalents (Inman's Tables, No. 40), for the change due to the Greenwich date.

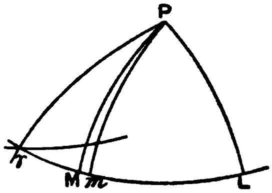
(3) Add together the corrected right ascension of the mean sun and mean time at the ship.

(4) The sum (rejecting 24 hours if greater than 24 hours) will be sidereal time.

The reason for this may be seen from the following figure (fig. 97):

Let M be the position of the mean sun in the equator at the moment of its transit over the meridian of Greenwich and ♈ the first point of Aries; then will the angle ♈ P M represent the sidereal time at Greenwich mean noon (which is recorded in the *Nautical Almanac*). Let *m* be its position at any subsequent time, represented by the angle *m* P L, when referred to ship mean time under the meridian P L; then will M *m* represent the increase of right ascension of the mean sun, or the portion of the acceleration due to the time elapsed since the preceding Greenwich mean noon.

FIG. 97.



Then $\Upsilon PL = \Upsilon PM + MPm + LPm$

$$\text{Or ship sidereal time} = \left\{\begin{matrix}\text{Sidereal time at pre-}\\ \text{ceding Greenwich}\\ \text{mean noon}\end{matrix}\right\} + \left\{\begin{matrix}\text{Acceleration for}\\ \text{Green. date}\end{matrix}\right\} + \text{S M T}$$

Examples

Feb. 24, at $10^h\ 40^m\ 30^s$ A.M. mean time, in long. 1° 6′ W.: required sidereal time.

Ship, Feb. 23	22^h	40^m	30^s	
long. in time		4	24	W.
Gr., Feb. 23 .	22	44	54	

Right Ascension Mean Sun.

Feb. 23 sid. time at Gr. noon	22^h	10^m	$3{\cdot}52^s$
cor. for 22^h		3	36·84
" 44^m			7·23
" 54^s			·15
right asc. mean sun ...	22	13	47·74
ship mean time	22	40	30·00
sidereal time	20	54	17·74

(rejecting 24 hours).

59. July 10, at 0ʰ 42ᵐ 10ˢ P.M. mean time, in long. 84° 42′ W.: required sidereal time. *Ans.* 7ʰ 56ᵐ 29·8ˢ.

60. Sept. 30, at 6ʰ 42ᵐ 10ˢ A.M. mean time, in long. 100° 42′ W.: required sidereal time. *Ans.* 7ʰ 18ᵐ 58·59ˢ.

61. Dec. 8, at 10ʰ 10ᵐ 42ˢ P.M. mean time, in long. 18° 32′ E.: required sidereal time. *Ans.* 3ʰ 20ᵐ 47·1ˢ.

Elements from 'Nautical Almanac.'

Sidereal time at Greenwich mean noon:

July 10	7ʰ	13ᵐ	17·14ˢ
Sept. 30	12	36	34·64
Dec. 8	17	8	36·96

Rule (*c*). *Given apparent solar time; to find sidereal time.*

(1) Find the Greenwich date in apparent time.

(2) Correct the equation of time for this date.

(3) Apply corrected equation of time to ship apparent time, and thus obtain ship mean time and thence Greenwich mean time. Then, as in the last rule,

(4) Correct sidereal time at previous Greenwich mean noon, thus finding R. A. M. S.

(5) Add together ship mean time and right ascension of mean sun.

(6) The sum (rejecting 24 hours if greater than 24 hours) will be sidereal time.

May 24, at 6ʰ 8ᵐ 40ˢ A.M. apparent solar time, in long. 21° 12′ 15″ W.: required sidereal time. **Examples**

Ship A. T., May 23	**18ʰ**	**8ᵐ**	**40ˢ**
long. in time	**1**	**24**	**49 W.**
Gr. A. T., May 23	**19**	**33**	**29**
or 4·5 hrs. from noon, May 24.			
Equation of Time.			
24th 3ᵐ 28·3ˢ subtr. from A. T.			
Var. in 1 hour, decr.			
			·2ˢ
Hours from noon...			**4·5**
Corr.			**·90**
Corr. Eq. of time		**3**	**29·2**
App. time	**18**	**8**	**40**
May 23rd S. M. T.	**18**	**5**	**10·8**
long. in time ...	**1**	**24**	**49 W.**
Gr. M. T., May 23	**19**	**30**	**0**

Right Ascension Mean Sun.

23rd, at noon ...	**4ʰ**	**4ᵐ**	**2·37ˢ**
Corr. for 19ʰ		**3**	**7·27**
„ 30ᵐ			**4·93**
R. A. mean sun	**4**	**7**	**14·57**
ship M. T.	**18**	**5**	**10·80**
sidereal time...	**22**	**12**	**25·37**

62. July 4, at $3^h\ 42^m$ A.M. apparent solar time, in long. 84° 42′ W.: required sidereal time. *Ans.* $22^h\ 34^m\ 52{\cdot}09^s$.

63. Oct. 21, at $8^h\ 48^m$ P.M. apparent solar time, in long. 88° 8′ E.: required sidereal time. *Ans.* $22^h\ 32^m\ 28{\cdot}36^s$.

Eq. of Time, and sidereal time from 'Nautical Almanac.'

Equation of time.					Sidereal time at Greenwich Mean Noon.
July 4, $4^m\ 0{\cdot}8^s$	add	Var. in 1 hour	$0{\cdot}456^s$	incr.	July 3, $6^h\ 45^m\ 41{\cdot}24^s$
Oct. 21, 15 19·1	sub.	„ „	0·393	„	Oct. 21, 18 59 22·26

Rule (*d*). *Given sidereal time, to find mean solar time.*

(1) Find the Greenwich sidereal time by applying the longitude to the given sidereal time.

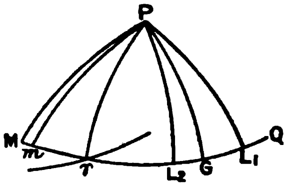
(2) Correct the mean time of transit of the first point of Aries (taken from the *Nautical Almanac*) for the retardation due to the Greenwich sidereal time.

(3) The given sidereal time added to the corrected mean time of transit of ♈ will be the mean time required.

This rule is capable of being explained by means of a figure (fig. 98).

Let M be the position of the mean sun in the equator at the moment of the preceding transit of ♈ over the meridian of Greenwich, when the Greenwich mean time is represented by the angle ♈ P M. Let also *m* be its position at any subsequent time when the sidereal time at the ship is represented by the angle ♈ P L. Let P L_1, P L_2, and P G represent respectively the meridians of Greenwich and the spectator's place; P L_1, exemplifying a case in east, and P L_2 in west longitude—the meridian being supposed to revolve from west to east, while the equator M ♈ Q and the equinoctial colure P ♈ remain stationary.

FIG. 98.



The arc M *m*, the measure of the angle M P *m*, is the change of right ascension, or the amount of the 'retardation' which has accrued since ♈ left the meridian of Greenwich, or that due to the sidereal interval ♈ P G.

Now ♈ P G (or the Gr. sid. date) = ♈ P L_1 − G P L_1, or ♈ P L_2 + G P L_2

$$\text{Or} \qquad \text{Gr. sid. date} = \text{ship sid. time} \mp \text{long. in time} \left\{ \text{if } \begin{matrix}\text{East}\\ \text{West}\end{matrix} \right\}$$

And if L be taken to represent L_1 or L_2, as the case may be—

$$LPm = \Upsilon PM - Mm + LP\Upsilon$$

$$\text{Or ship mean time} = \left\{ \begin{matrix}\text{Greenwich mean time of}\\ \text{preceding transit of } \Upsilon\end{matrix} \right\} - \left\{ \begin{matrix}\text{Retardation}\\ \text{for G. S. D.}\end{matrix} \right\} + \text{L. S. T.}$$

Example.—Convert 15^{h} 9^{m} 43·5^{s} sidereal time, Nov. 10, at Washington, into its corresponding local mean time: long. of Washington, 5^{h} 8^{m} 12^{s} W.

	h	m	s		h	m	s
L. S. T.	15	9	43·5	Mean time transit of ♈ Gr. Nov. 10	8	42	6·38
Long.	5	8	12 W.				
G. S. D.	20	17	55·5				
Retardation for G. S. D.—							
20^{h}		3^{m}	16·59^{s}				
17^{m}			2·78				
55·5^{s}			0·15				
		3	19·52	Correction for retardation −		3	19·52
					8	38	46·86
				L. S. T.	15	9	43·50
				Washington mean time required, Nov. 10	23	48	30·36

In the application of the preceding formula it is necessary, however, to attend to the following caution. In order that the correction for retardation employed may truly represent the algebraic sum of the two parts of which it is composed—that is, the correction of the mean time of transit of ♈ at Greenwich for longitude, and that corresponding to the given sidereal time—it will be necessary to take into account that due to 24^{h} (or 3^{m} 55^{s}·91), in cases where, in obtaining the G. S. date, 24^{h} has been either added or rejected—adding 3^{m} 55·91^{s} in the former case and subtracting in the latter.

Example.—At a place in longitude 7^{h} 30^{m} 20^{s} E. the sidereal time on May 5 was 3^{h} 19^{m} 57^{s}; what was the corresponding local mean time?

	h	m	s		h	m	s
L. S. T.	3	19	57	Mean time preceding transit of ♈, Gr. May 4 . . .	21	7	57·32
Long.	7	30	20 E.				
G. S. D.	19	49	37				
Retardation for G. S. D. —							
19^{h}		3^{m}	6·76^{s}				
49^{m}			8·03				
37^{s}			0·10				
		− 3	14·89				
Add for 24^{h} added		+ 3	55·91				
		+ 0	41·02	Correction for retardation +		0	41·02
					21	8	38·34
				L. S. T.	3	19	57·00
				L. M. T. May 4	24	28	35·34
				or May 5	0	28	35·34

Given the SUN'S *observed altitude; to find the true altitude.*

The true altitude of the sun's centre is found by observing the altitude of either the upper or lower limb,[1] and then subtracting or adding the semidiameter taken from the *Nautical Almanac* after the other corrections—namely, for index error, dip, refraction, and parallax—have been applied. In Inman's Tables, the two corrections for refraction and parallax of the sun are combined in one table, and called the 'correction of the sun's apparent altitude.' Hence this rule.

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[1] The lower limb is usually observed, except when the sun is low; the refraction is then less irregular if the upper limb is taken. L. L. or $\underline{\odot}$ is the symbol employed to

(1) Correct the observed altitude for index correction and dip.

(2) Subtract the refraction and add the parallax taken from Tables Nos. 16, 18; or else take out the 'correction of the sun's apparent altitude' (No. 20), and subtract it.

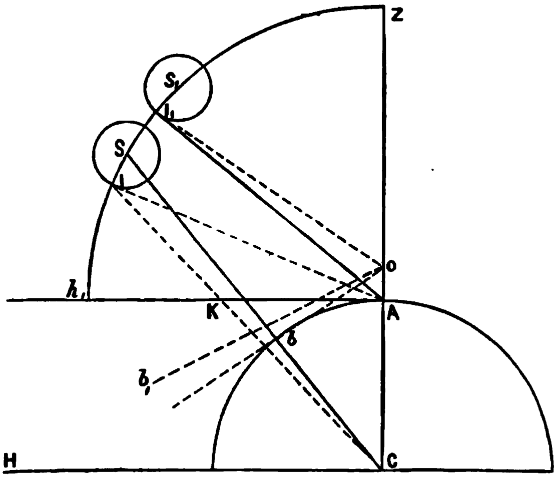
(3) To this add the sun's semidiameter if the altitude of the lower limb is observed, but subtract it if the upper limb is observed; the result is the true altitude of the sun's centre.[1]

---

denote the sun's lower limb, and U. L. or $\overline{\odot}$ that representing the upper limb. Similar abbreviations are used for the moon, but, except when full, only one limb can at any time be observed. At sea the mean of three or five observations taken quickly one after another is that employed.

[1] The accompanying figure represents *all* the corrections applied to an observed altitude of the sun's lower limb. An observer at o measures the altitude $I_1\, o\, b_1$ of the lower limb of the *apparent* sun S_1 above the visible horizon, and from that observation wishes to determine the *true* altitude of the *real* sun's centre above the rational horizon as if observed from C the centre of the earth—that is, the angle SCH.

FIG. 99.



Now $SCH = ICH + ICS = ICH +$ sun's semidiam.

$ICH = IK\,h_1 = IA\,h_1 + CIA = IA\,h_1 +$ sun's parallax

$IA\,h_1 = I_1 A\,h_1 - IAI_1 = I_1 A\,h_1 -$ refraction of sun's lower limb

$I_1 A\,h_1 = I_1\,o\,b_1 -$ apparent dip (*see* fig. 76, page 148).

Hence true alt. of centre = observed alt. of lower limb − apparent dip − refraction + parallax + semidiam.

Examples

The observed altitude of the sun's lower limb (L. L.) was 47° 32′ 15″, the index correction was +2′ 10″, and the height of the eye above the sea 15 feet: required the true altitude of the sun's centre, the semidiameter in *Nautical Almanac* being 15′ 49″.

Observed altitude	47°	32′	15″
index correction.....................		2	10+
	47	34	25
dip		3	49−
	47	30	36
correction in altitude			49−
	47	29	47
semidiameter		15	49+
True altitude	47	45	36

64. The observed altitude of the sun's L. L. was 48° 30′ 15″, index correction −2′ 50″, and height of eye above the sea 15 feet: required the true altitude, the semidiameter being 15′ 55″.
Ans. 48° 38′ 46″

65. The observed altitude of the sun's L. L. was 40° 42′ 15″, index correction +5′ 10″, and height of eye above the sea 20 feet: required the true altitude, the semidiameter being 16′ 4″.
Ans. 40° 58′ 4″.

66. The observed altitude of the sun's upper limb (U. L.) was 55° 57′ 40″, index correction −3′ 40″, height of eye above the sea 19 feet: required the true altitude, the semidiameter being 16′ 6″. *Ans.* 55° 33′ 4″.

67. The observed altitude of the sun's L. L. was 39° 25′ 15″, index correction −3′ 15″, height of eye above the sea 15 feet: required the true altitude, the semidiameter being 16′ 3″.
Ans. 39° 33′ 11″.

Given the MOON'S *observed altitude; to find the true altitude.*

The moon's horizontal parallax and semidiameter change so rapidly that they cannot be considered (as in the corresponding case of the sun) to be constant for twenty-four hours. The parallax and semidiameter taken out of the *Nautical Almanac* must therefore be corrected (usually by inspection) for the Greenwich date in order to find the horizontal parallax and horizontal semidiameter at the time of the observation. Moreover, since the moon is nearer the observer when at *any* altitude than when it is on his horizon, the horizontal semidiameter must also be corrected for augmentation (page 153). The corrections of the moon's apparent altitude for parallax and refraction are combined in one (*see* Inman's Tables, No. 34): it is entered with the corrected horizonal parallax at top, and the apparent altitude of the moon's centre at the side. Hence this rule.

(1) Find the Greenwich date.

(2) Correct the moon's semidiameter and horizontal parallax, taken from the *Nautical Almanac*, for the Greenwich date (page 189).

(3) Add to the semidiameter the augmentation, taken from the table of augmentation (Inman's Tables, No. 32).

(4) Correct the observed altitude for index correction, dip, and augmented semidiameter.

(5) Add the moon's correction in altitude, taken out of Inman's Tables, No. 34.[1] The result is the moon's true altitude.

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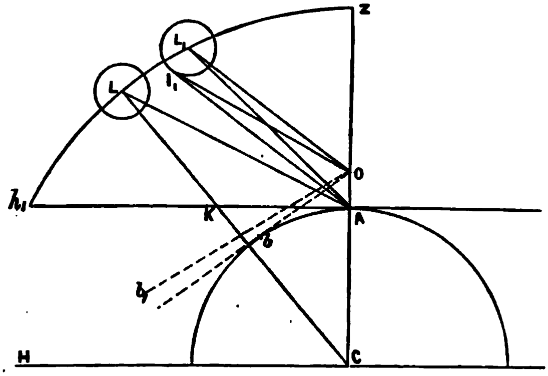
[1] The accompanying figure (fig. 100) represents all the corrections applied to an observed altitude of the moon's lower limb. An observer at o measures the altitude

Fig. 100.



$I_1 o b_1$ of the lower limb of the apparent moon L_1 above the visible horizon, and from that observation wishes to determine the true altitude of the centre of the real moon L above the rational horizon—that is, the angle LCH.

Now $LCH = LKh_1 = LAh_1 + ALC = LAh_1 +$ par. in alt.

$LAh_1 = L_1Ah_1 - L_1AL = L_1Ah_1 -$ refraction of moon's centre

$L_1Ah_1 = I_1Ah_1 + I_1AL_1 = I_1Ah_1 +$ semidiameter augmented for altitude

$I_1Ah_1 = I_1ob_1 -$ apparent dip (*see* fig. 76, page 148).

Hence true alt. of centre = obs. alt. of lower limb − dip + augmented semidiameter − refraction + parallax.

[1] The refraction and parallax combined are given in Inman's Tables, No. 34; these corrections are calculated for the centre of the moon, and therefore (unlike the sun) the semidiameter is applied *before* them.

The moon's altitude might also be corrected by applying the refraction and parallax *for the altitude of the limb observed*, and then the semidiameter *without augmentation*

Example

April 7, at 4^h 47^m P.M. mean time nearly, in long. 10° W., the observed altitude of the moon's L. L. was 72° 15′ 0″, the index correction was −4′ 20″, and height of eye above the sea 15 feet: required the true altitude.

		Moon's Semi.		Moon's Hor. Par.	
Ship, April 7 ………	4^h 47^m	7th, at noon …	15′ 41″	noon……………	57′ 32·0″
long. in time ………	0 40 W.	„ mid. …	15 45·8	mid. …………	57 50·8
Gr., April 7…………	5 27		4·8		18·8
		Corr. for 5^h 27^m by inspection	2	Corr. …	8
			15 43		57 40
		Aug^n. …	15		
			1 58		

Obs. alt. …	72°	15′	0″	
in. corr. …		4	20	−
	72	10	40	
dip ………		3	49	−
	72	6	51	
semi. ……		15	58	+
	72	22	49	
corr. for 57′		16	57	+
„ 40″			12	+
∴ true alt.	72	39	58	

68. July 12, at 9^h 18^m P.M. mean time nearly, in long. 44° 40′ W., the observed altitude of the moon's L. L. was 27° 56′ 40″, the index correction + 2′ 20″, and height of eye above the sea 20 feet: required the true altitude. *Ans.* 28° 56′ 11″.

69. May 15 (10^h 30^m P.M.), at 8^h 6^m 10^s by a chronometer which was 6^h 1^m 30^s slow on G.M.T., the observed altitude of the moon's L. L. was 21° 14′ 10″, the index correction + 2′ 20″, and height of eye above the sea 15 feet: required the true altitude. *Ans.* 22° 16′ 7″.

70. May 16, at 1^h 58^m A.M. mean time nearly, in long. 12° 30′ E., the observed altitude of the moon's U. L. was 45° 20′ 30″, the index correction + 4′ 10″, and height of eye above sea 20 feet: required the true altitude. *Ans.* 45° 43′ 13″.

Elements from 'Nautical Almanac.'

Moon's Semidiameter.		Moon's Horizontal Parallax.	
July 12, mid. ……	14′ 55·9″	………… mid. ……	54′ 47·8″
„ 13, noon ……	14 59·3	………… noon ……	55 0·3
May 15, mid. … ..	14 41·1	………… mid. ……	54 53·0
„ 16, noon ……	14 46·3	………… noon ……	54 57·7

Given a STAR'S *observed altitude; to find its true altitude.*

The stars are at such a vast distance from the spectator that (excepting probably a few) even the diameter of the earth's *orbit* subtends no angle at them: hence a star is considered to have no parallax (page 152); and the only corrections used for reducing the observed altitude to the true are the *index correction, dip,* and *refraction.* Hence this rule:

(1) To the observed altitude apply the index correction with its proper sign.

(2) Subtract the dip and also the refraction; the result is the true altitude of the star.

Examples

The observed altitude of Arcturus (α Böotis) was 36° 10′ 20″, index correction + 2′ 42″, and height of eye above the sea 20 feet: required the true altitude.

Observed altitude	36°	10′	20″	
index correction		2	42	+
	36	13	2	
dip		4	24	−
star's apparent altitude	36	8	38	
refraction		1	20	−
star's true altitude	36	7	18	

71. The observed altitude of Aldebaran (α Tauri) was 13° 4′ 30″, index correction − 10′ 40″, and height of eye above the sea 16 feet: required the true altitude. *Ans.* 12° 45′ 43″.

72. The observed altitude of α Andromedæ was 27° 10′ 20″, index correction + 5′ 40″, and height of eye above the sea 20 feet: required the true altitude. *Ans.* 27° 9′ 43″.

73. The observed altitude of Capella (α Aurigæ) was 56° 4′ 40″, index correction − 3′ 25″, and height of eye above the sea 28 feet: required the true altitude. *Ans.* 55° 55′ 23″.

74. The observed altitude of Procyon (α Canis Minoris) was 87° 4′ 15″, index correction + 7′ 5″, and height of eye above the sea 10 feet: required the true altitude. *Ans.* 87° 8′ 10″.

NOTE.—For the purposes of nautical astronomy an observed altitude of a *planet* is corrected in precisely the same way as that of a star, the small corrections for semidiameter and parallax being neglected, as these never amount to more than a few seconds. In practical work at sea it is as a rule sufficiently accurate to correct all altitudes of heavenly bodies only to the nearest minute of arc (*see* page 360)

CHAPTER VIII.

Hour angles—Sun's hour angle—Star's hour angle—Times of meridian passages—Moon's meridian passage—Corrections for run—Effect of run on altitude and time.

Hour angles

THE hour angle of a heavenly body is the angle at the elevated pole between the celestial meridian and the circle of declination passing through the place of the body.

Hour angles are measured westward from the celestial meridian from 0^h to 24^h.[1]

Thus in figs. 101 and 102 (drawn on the plane of the equinoctial), representing a heavenly body in two positions, one west and the other east of the meridian of an observer in latitude Z Q, the hour angle is the angle Z P X, measured by the arc of the equinoctial Q R in fig. 101, and by Q Q_1 R in fig. 102.

FIG. 101.

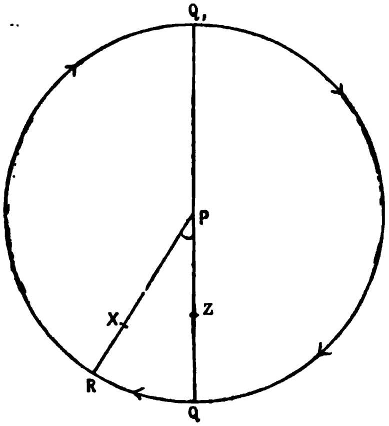
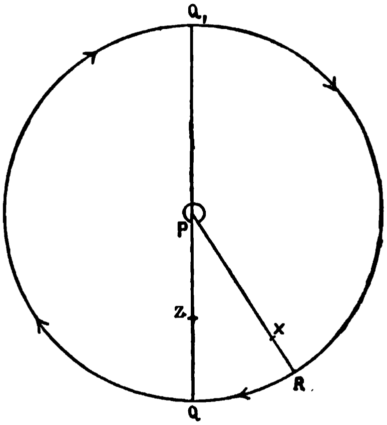


FIG. 102.



Where the heavenly body considered is either the apparent or mean sun, the hour angle is by definition (page 164) the apparent or mean astronomical *time*, and, similarly, the hour angle of the first point of Aries is sidereal *time*.

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[1] Hour angles of *any* heavenly body may be considered as portions of *sidereal* time, since they are differences of the right ascensions of the meridian and the heavenly body considered; if the hour angle is *less* than 12 hours the body must be *west*, and if *more* than 12 hours, *east* of the meridian.

In a large number of problems considered in succeeding chapters the calculation of hour angles will be found of frequent occurrence; thus in every question where finding *time* is involved, the first result obtained by observation is the hour angle of the body observed.

Theory

Given the altitude and declination of a heavenly body, and the latitude of the place of observation, to calculate the hour angle of the heavenly body.

In figs. 103, 104 let P Z Q be the celestial meridian, P the pole, Z the zenith of spectator, and X the place of the heavenly body. Let A Q be the equinoctial; through X draw P X R a circle of declination, and Z X a circle of altitude. Then, in the spherical triangle Z P X the three sides are given, to calculate P the hour angle.

FIG. 103.

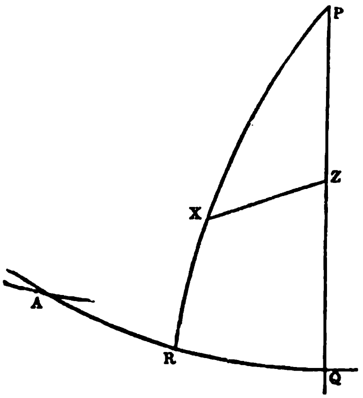
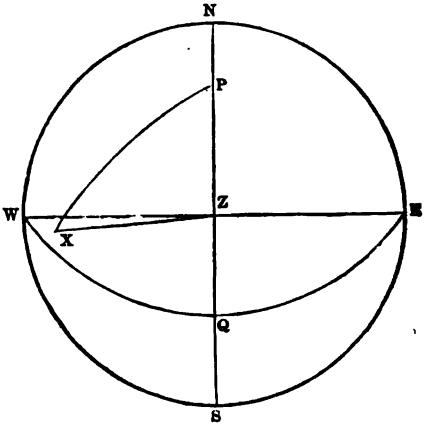


FIG. 104.



For P X = polar distance = 90° − decl.;
Z X = zenith distance = 90° − alt.;
P Z = colatitude = 90° − lat.

Examples

Calculate the hour angle of a heavenly body X, west of meridian, having given the latitude of observer = 48° 40′ 45″ N., the star's declination = 8° 25′ 45″ N., and star's altitude = 13° 57′ 45″; constructing the figure on the plane of the horizon[1] (*see* fig. 104).

---

[1] To obviate the *calculation* of hour angles, tables have at different times been published, from which for a given latitude, altitude, and declination, the hour angle may be taken by *inspection*. The earliest of these appears to have been Lalande's 'Horary Tables,' published in 1793, and calculated for every degree of declination from 24° N. to 24° S., and for every degree of latitude and altitude within the limits of 60° N to 60° S. of latitude and 0° to 48° of altitude.

A more complete set of tables was published in 1827 by Commander Lynn, I.N., embracing declinations from 24° N. to 24° S., latitudes from 60° N. to 60° S., and

Construction

Let N W S E represent the horizon, N Z S the celestial meridian, W Z E the prime vertical. Take N P=latitude; then P will represent the north pole of the heavens, since the altitude of the pole=latitude of spectator (for lat. Z Q=90−P Z and N P=90−P Z, ∴ Z Q=N P, the altitude of pole, *see* page 227), and P Z is the colatitude. Take P Q=90°; then Q is a point in the equinoctial: through E and W, the east and west points, and Q, draw the circle W Q E; this will represent the equinoctial. Let X be the place of the heavenly body at the time of the observation (placed according to its altitude and declination); and through X draw X P, a circle of declination, and X Z, a circle of altitude. Then, in the spherical triangle Z P X are given P X the polar distance (90°−decl.)=81° 34′ 15″, Z X the zenith distance (90°−alt.)=76° 2′ 15″, and P Z the colat. (90°−lat.)=41° 19′ 15″; to calculate Z P X, the hour angle.

P X...	81°	34′	15″	log. cosec...	0·004717
P Z...	41	19	15	„ „ ...	0·180275
	40	15	0		
X Z...	76	2	15		
S...	116	17	15	½ L hav ...	4·929099
D...	35	47	15	„ „ ...	4·487496
				L hav Z P X...	9·601587
				∴ Z P X=	$5^h\ 13^m\ 39^s$ [1]

Sometimes the hour angle is obtained from the formula

$$\sin\frac{h}{2}=\sqrt{\frac{\cos S\sin(S-a)}{\cos l\sin p}}$$

when l = latitude, p = polar distance, and a = altitude, S being

$$\frac{a+l+p}{2}$$

---

altitudes from 0° to 60°, and by an auxiliary table for interpolation enabling the hour angle to be taken out very accurately.

In 1790 Margett published tables expressed graphically in curves, from which hour angles and azimuths could be taken by *measurement*.

In more recent times tables for the same purpose have been published by M. Hommey 1863, and Mr. Davis, of the *Nautical Almanac* office, 1897.

[1] Hour angles are of course *angles*, and might be expressed as *degrees*, &c., but as they are usually employed to determine *time*, it is conveniently arranged in Inman's Tables to convert them from arc to time by inspection; when the heavenly body considered is *west* of the meridian, the hour angle is less than 12 hours, and is taken from the *top* of the page in the Table of Haversines; when *east*, it is greater than 12 hours, and is taken from the *bottom* of the page. It is usual in calculating hour angles to work with the *secants* of the *declination* and *latitude*, instead of the cosecants of the polar distance and colatitude. *See* example on page 284.

This may be proved from the fundamental formula, as follows:

$$\cos h = \frac{\sin a - \sin l \cos p}{\cos l \sin p}$$

$$\therefore 1 - \cos h = \frac{\cos l \sin p - \sin a + \sin l \cos p}{\cos l \sin p}$$

And $$2 \sin^2 \frac{h}{2} = \frac{\sin (l+p) - \sin a}{\cos l \sin p}$$

$$= \frac{2 \cos \frac{1}{2}(l+p+a) \sin \frac{1}{2} (l+p-a)}{\cos l \sin p}$$

Hence $$\sin^2 \frac{h}{2} = \frac{\cos \mathrm{S} \sin (\mathrm{S}-a)}{\cos l \sin p}$$

If $$\mathrm{S} = \frac{a+l+p}{2}$$

And also $$\sin \frac{h}{2} = \sqrt{\left(\frac{\cos \mathrm{S} \sin (\mathrm{S}-a)}{\cos l \sin p}\right)}$$

The above example would then be worked as follows:

Alt.	13° 57′ 45″		
Lat.	48 40 45	………sec	·180275
P.D.	81 34 15	……cosec	·004717
	144 12 45		
$\mathrm{S}=$	72 6 22	………cos	9·487498
$\mathrm{S}-a=$	58 8 37	……… sin	9·929097
			2/19·601587
$\sin \frac{h}{2}=$			9·800794

$$\frac{h}{2} = 2^{\mathrm{h}}\ 36^{\mathrm{m}}\ 49{\cdot}5^{\mathrm{s}}$$

and $h = 5\ 13\ 39$.

Examples

1. Find the hour angle of a heavenly body, west of meridian, having given the latitude = 47° 20′ N., the declination = 11° 24′ 24″ N., and the altitude = 42° 33′ 10″; and construct the figure.

Ans. Hour angle = $2^{\mathrm{h}}\ 27^{\mathrm{m}}\ 51^{\mathrm{s}}$.

2. Find the hour angle of a heavenly body, east of meridian, having given the latitude = 56° 10′ N., the declination = 33° 11′ 44″ N., and the altitude = 59° 4′ 0″; and construct the figure.

Ans. Hour angle = $21^{\mathrm{h}}\ 59^{\mathrm{m}}\ 30^{\mathrm{s}}$.

3. Find the hour angle of a heavenly body, east of meridian, having given the latitude = 50° 48′ 0″ N., the declination = 14° 28′ 47″ S., and the altitude = 3° 13′ 0″; and construct the figure. *Ans.* Hour angle = $19^{\mathrm{h}}\ 36^{\mathrm{m}}\ 24^{\mathrm{s}}$.

Hour angle of sun

Given the hour angle of SUN; to find ship mean time.

The angle hour of the sun is by definition (page 164) the apparent time, and the problem is similar to that on page 191, being a conversion of apparent time into mean time.

First. Let the sun be west of meridian. Let P be the pole, Z the zenith, and X the place of the sun. Then (fig. 105),

Apparent time = hour angle Z P X, or arc Q R;

and if the equation of time be *subtractive* from apparent time (and this is known from the *Nautical Almanac*), let m be the mean sun;

Then Q m = mean time required.

By the figure, Q m = Q R − R m,

or mean time = sun's hour angle − equation of time.

If the equation of time is additive to apparent time, then the mean sun m is in advance of the true sun X, and m should be placed in the figure on the other side of R. In this case mean time = sun's hour angle + equation of time.

Fig. 105.

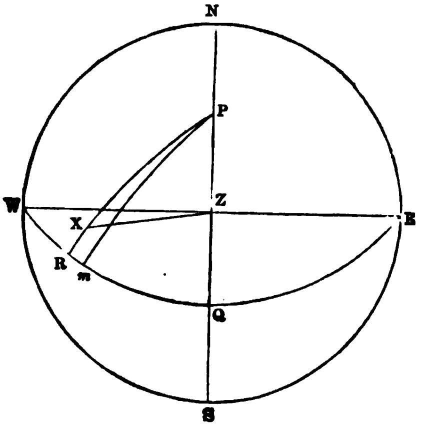
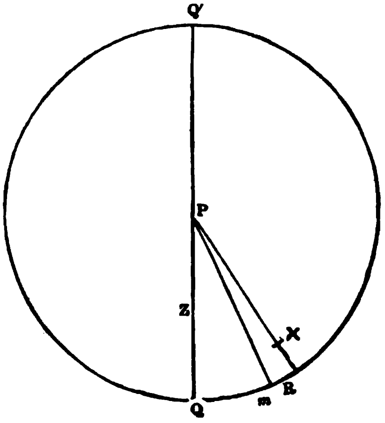


Fig. 106.



The proper sign to be used is always given in the *Nautical Almanac* with the equation of time.

Next. Let the sun be east of meridian. It will be more convenient in this case to project the figure on the plane of the equinoctial. Let, therefore, P be the pole, Z the zenith, and X the place of the sun. Then (fig. 106),

Apparent time = arc Q Q′ R = hour angle of sun;

and if the equation of time is additive to apparent time, let m be the mean sun; then the arc Q Q′ m = mean time required.

And by the figure, Q Q′ m = Q Q′ R + R m,

or mean time = sun's hour angle + eq. of time.

Examples

4. Given the sun's hour angle = $3^h\ 42^m\ 10^s$ and the equation of time = $14^m\ 10^s$ additive to apparent time; required mean time, and construct figure, decl. 0°.

Ans. Mean time = $3^h\ 56^m\ 20^s$.

5. Given the sun's hour angle = $17^h\ 57^m\ 40^s$, and the equation of time = $3^m\ 42^s$ subtractive from apparent time; required mean time and construct figure, decl. 10° N.

Ans. Mean time = $17^h\ 53^m\ 58^s$.

Hour angle of star

Given the hour angle of any other heavenly body, as a star; to find ship mean time.

We have to prove the following rule, viz.:

When the star is either *west* or *east* of meridian,
Mean time = star's hour angle + star's right ascension
− right ascension of mean sun.

To do this we must show that this equation is true for all positions of the star, the mean sun, the first point of Aries, with respect to each other. The figures in this problem are projected on the plane of the equinoctial.

First, suppose the heavenly body to be *west* of meridian.

(*a*) Let P be the pole, and X a heavenly body; and let A, the first point of Aries, and m, the mean sun, be situated with respect to X and to each other as in fig. 107.

Then Q R = star's hour angle, A m = right ascen. of mean sun,
A R = star's right ascen., and Q m = mean time required.

By the figure, Q m = Q R + A R − A m,

or mean time = star's hour angle + star's R A − R A mean sun.

(*b*) Let the relative positions of A, m, and X be as represented in fig. 108.

Then Q R = 24^h − star's hour angle, A m = R A mean sun,
A R = star's right ascen., and 24^h − Q m = mean time required.

By the figure, Q m = Q R + R m = Q R + A m − A R,

or (24^h − mean time) = (24^h − star's hour angle) + R A mean sun − star's R A

∴ mean time = star's hour angle + star's R A − R A mean sun,

the same as before; and the same result may be obtained for any other positions of A, m, and X, with respect to each other. Therefore, when the body is *east* of meridian,

Mean time = star's hour angle + star's R A − R A mean sun.

On drawing figures in a similar way for other cases both east and west of meridian (for which purpose future examples will furnish sufficient exercise) we arrive at the following convenient rule:

Add the star's right ascension to the hour angle taken from the table of haversines (remembering when the heavenly body is west of meridian to take that angle from the top, and when east from the bottom), and from the result (adding 24 hours if necessary) subtract the right ascension of the mean sun; the remainder will be mean time required.

Examples

6. Given the hour angle α Aquilæ $= 2^h\ 37^m\ 23^s$, star's right ascension $= 19^h\ 43^m\ 52^s$, and right ascension of mean sun $= 9^h\ 51^m\ 38^s$; find ship mean time, and construct the figure; star's declination $= 9°$ N. *Ans.* Mean time $= 12^h\ 29^m\ 37^s$.

FIG. 107.

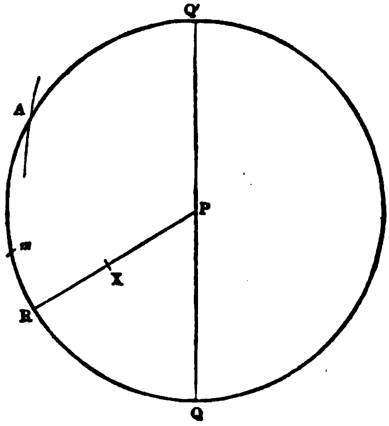
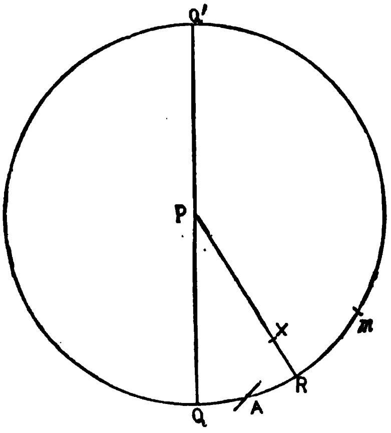


FIG. 108.



7. Given the hour angle of α Cygni $= 19^h\ 58^m\ 25^s$; star's right ascension $= 20^h\ 36^m\ 36^s$, and right ascension of mean sun $= 9^h\ 17^m\ 7^s$; find ship mean time, and construct the figure; star's declination $= 45°$ N. *Ans.* Mean time $= 7^h\ 17^m\ 54^s$.

8. Given the hour angle of β Geminorum $= 22^h\ 44^m\ 3^s$, star's right ascension $= 7^h\ 36^m\ 35^s$, and right ascension of mean sun $= 22^h\ 34^m\ 43^s$; find ship mean time, and construct the figure; star's declination $= 28°$ N. *Ans.* Mean time $= 7^h\ 45^m\ 55^s$.

Practical rule

To find mean time at the ship, having given the hour angle of a star.

(1) Find the Greenwich date.
(2) Take out the star's right ascension.
(3) Take out also the sidereal time at the preceding Greenwich

mean noon, and correct it for Greenwich date, thus finding the right ascension of the mean sun.

(4) To the star's hour angle add star's right ascension, and from the sum (increased if necessary by 24 hours) subtract the right ascension of the mean sun; the result is ship mean time.

Examples Feb. 10, at $9^h\ 22^m$ P.M. mean time nearly, in long. 27° 15′ W., the hour angle of Regulus (α Leonis) was $20^h\ 44^m\ 43^s$ EAST of meridian. Required mean time at the ship.

Ship, Feb. 10	$9^h\ 22^m$		
long. in time	1 49 W.	Star's H. A.	$20^h\ 44^m\ 43{\cdot}0^s$
Gr. Feb. 10	11 11		
R. A. mean sun.		star's R. A.	10 0 15·3
Feb. 10, Sid. time at Gr. mean noon ...	$21^h\ 19^m\ 46{\cdot}0^s$		
cor. for 11^h	1 48·4		30 44 58·3
,, 11^m	1·8	R. A. mean sun	21 21 36·2
R. A. mean sun ...	21 21 36·2	∴ ship mean time	9 23 22·1

9. Sept. 10, at $7^h\ 11^m$ P.M. mean time nearly, in long. 32° E., the hour angle of Arcturus (α Böotis) was $4^h\ 22^m\ 15^s$ WEST of meridian. Required mean time at the place. *Ans.* $7^h\ 14^m\ 10^s$.

10. Nov. 22, at $7^h\ 15^m$ P.M. mean time nearly, in long. 22° 0′ W., the hour angle of Aldebaran (α Tauri) was $18^h\ 49^m\ 40^s$ EAST of meridian. Required mean time at the place. *Ans.* $7^h\ 12^m\ 8^s$.

11. June 23, at $4^h\ 15^m$ A.M. mean time nearly, in long. 100° 40′ E., the hour angle of α Lyræ was $3^h\ 42^m\ 40^s$ WEST of meridian. Required mean time at the place. *Ans.* $16^h\ 11^m\ 50^s$.

Elements from 'Nautical Almanac.'

Sid. Time at Mean Noon.		Star's R. A.	
Sep. 10	$11^h\ 17^m\ 53{\cdot}2^s$	Arcturus	$14^h\ 10^m\ 38^s$
Nov. 22	16 5 41·7	α Tauri	4 29 36
June 22	6 2 28·7	α Lyræ	18 33 13

NOTE.—The Greenwich date as found above is only approximate, and any error in it will produce an error in the correction applied to the R. A. mean sun; but since in practice, at all observations for finding hour angles, the time is noted by a chronometer whose error on Greenwich time is known, the right ascension should be corrected for *this* date, and the ship time only used to determine whether or not to add 12 hours to the time *shown* by chronometer. Or we may proceed as follows:

To obtain mean time accurately, find Greenwich *sidereal* date = ship sid. time (or star's hour angle + star's right ascension) ± long. in time.

$$\text{Then S. M. T.} = \left\{\begin{matrix}\text{G. M. T. of preceding transit} \\ \text{of first point of Aries}\end{matrix}\right\} - \left\{\begin{matrix}\text{Retardation} \\ \text{for G. S. D.}\end{matrix}\right\} + \text{ship sid. time.}$$

The practical rules given above are equally applicable to observations of the moon and planets after having corrected their right ascensions for Greenwich date.

The *practical* examples on finding hour angles from observations of heavenly bodies will be given in the chapter on Longitude.

Notes on hour angles

The formula from which hour angles are calculated is as follows:

$$\text{Cos ZPX or } h=\frac{\cos \text{ZX}-\cos \text{PX} \cos \text{PZ}}{\sin \text{PX} \sin \text{PZ}} (\textit{see} \text{ fig. 104, page 202}).$$

Replacing the right-hand side of this equation by equivalent values of a the altitude, l the latitude, and d the declination,

$$\cos h=\frac{\sin a-\sin l \sin d}{\cos l \cos d}.$$

(1) When the observer is on the equator and $l=0$ this becomes

$$\cos h=\frac{\sin a}{\cos d}.$$

(2) When the declination of the heavenly body is 0 it becomes

$$\cos h=\frac{\sin a}{\cos l}.$$

(3) When both latitude and declination equal 0 it becomes

$$\cos h=\sin a,$$

and the hour angle is the complement of the altitude.

STARS AND MOON PASSING MERIDIAN.

Given mean time or apparent time at a certain place; to find what bright star will pass the meridian the next after that time [1] (fig. 109).

Let A Q represent the equinoctial, P Q the celestial meridian, and X a star passing the meridian. Then the right ascension of the meridian will at that instant be equal to the right ascension of the star, and, knowing the time at the place, we can determine the position of the mean sun with respect to the meridian, and that of the first point of Aries, since we also know the right ascension of the mean sun from the *Nautical Almanac*.

Let, therefore, m be the mean sun, A the first point of Aries.

Then Q m = mean time at the place, A m = right ascension of mean sun, and A Q = right ascension of the star X
= right ascension of the meridian.

By the figure, A Q = A m + m Q,
or star's R A = R A mean sun + mean time = sidereal time;
∴ sidereal time = right ascension of meridian.

It appears from this, that in order to find how soon after a certain time a star passes the meridian, we have only to find sidereal time for the instant required, and this will be the right ascension of the meridian, and therefore of any heavenly body on the meridian. Then the star in the astronomical catalogue in the *Nautical Almanac* whose right ascension is the next greater will be the next to pass the meridian, and is therefore (if of sufficient magnitude to be readily seen) the one required.

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[1] By a *bright* star is to be understood one of the first or second magnitude.

It will be also noticed that all stars whose right ascensions are *greater* than that of the meridian must be eastward of it, and all whose right ascensions are *less*, westward of it; hence the stars *near* the meridian on either side will be easily recognised.

Fig. 109.

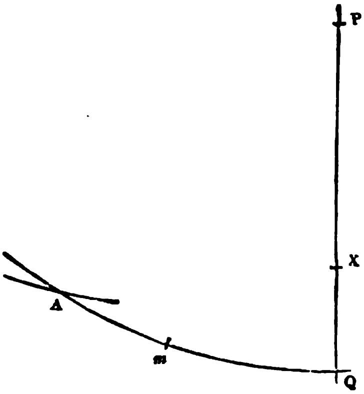
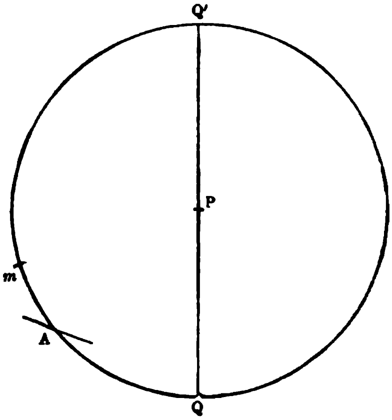


Fig. 110.



Example

12. Feb. 24, at 4^h 42^m P.M. mean time nearly, in long. 100° E.: find what bright star will pass the meridian the next after that date.

Right Ascension Mean Sun.

Ship, Feb. 24	4^h	42^m	Feb. 23, Sid. time				Ship, Feb. 24 .	4^h	42^m	0^s
			at Gr. mean noon	22^h	13^m	$9{\cdot}0^s$				
long. in time	6	40 E.	Cor. for 22^h . .		3	36·8	R. A. mean sun	22	16	46
Gr., Feb. 23	22	2	2^m . .			0·3	R. A. merid.	2	58	46
			R. A. M. S. =	22	16	46·1				

Let Q A Q′ represent the celestial equator, P the pole, and Q P Q′ the celestial meridian (fig. 110). From Q measure Q $m = 4^h$ 42^m; then m is the mean sun. From m measure an arc m Q′ A $= 22^h\ 16^m\ 46^s$; then A is the first point of Aries, and the arc A Q is the right ascension of the meridian or sidereal time, and therefore the right ascension of any star on the meridian.

$$\begin{aligned} \text{R A of meridian} &= \text{sidereal time} = \text{Q}\,m - m\,\text{A} \\ &= \text{mean time} - (24^h - \text{R A mean sun}) \\ &= 2^h\ 58^m \end{aligned}$$

Looking into the 'Catalogue of the Mean Places[1] of Stars'

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[1] The 'Catalogue of *Mean* Places of stars' is the most convenient to refer to for

(*see Nautical Almanac* or Inman's Tables, No. 23), we find the star whose right ascension is next greater than 2^h 58^m and whose magnitude is sufficiently great to render it easily seen is *a* Persei, which is therefore the bright star that will come to the meridian the next after 4^h 42^m P.M. on Feb. 24.

If it is required to know what principal stars will pass a given meridian between any two given dates, we must proceed as above to find sidereal time at each date. Then all the stars in the catalogue whose right ascensions lie between the sidereal times thus determined will pass the meridian above pole between the two given dates, and all whose right ascensions lie between those sidereal times increased by 12 sidereal hours will pass the meridian below pole between the given dates.[2]

Examples

13. Oct. 3, in long. 90° W., find what bright stars given in the *Nautical Almanac* will pass the meridian between the hours of 9 and 12 P.M.

Ship, Oct. 3………	9^h	0^m		Ship, Oct. 3………	12^h	0^m	
long. in time ……	6	0 W.		long. in time ……	6	0 W.	
Greenwich, Oct. 3	15	0		Greenwich, Oct. 3	18	0	
Right Ascension Mean Sun.				**Right Ascension Mean Sun.**			
Oct. 3, Sid. time Gr. noon………	12^h	48^m	24^s	Oct. 3…………………	12^h	48^m	24^s
corr. for 15^h ……		2	27	corr. for 18^h ……		2	57
R. A. M. S.	12	50	51	R. A. M. S.	12	51	21
ship, Oct. 3 ……	9	0	0	ship, Oct. 3 ……	12	0	0
R. A. meridian…	21	50	51	R. A. meridian …	0	51	21

In the Catalogue of Stars those whose right ascensions lie between 21^h 50^m 51^s and 0^h 51^m 21^s are from *a* Aquarii to *γ* Cassiopeiæ inclusive.

14. What bright stars will pass the meridian of a ship in long. 40° E. between 8^h and 10^h P.M. mean time on Nov. 20?

Ans. From *a* Andromedæ to *γ* Andromedæ.

15. What bright star will pass the meridian of a ship in long. 30° W. the first after 10^d 30^m P.M. on Oct. 10?

Ans. a Andromedæ.

16. What bright stars will pass the meridian of a ship in long. 56° W., between the hours of 6 and 10 P.M. on March 10?

Ans. From *γ* Orionis to *β* Argûs.

---

this purpose, for which extreme accuracy is not generally required, the object being to ascertain some time in advance when observations should begin to be made; in both the catalogues of mean and apparent places of stars, + denotes north and − south declination (*see* also Inman's Tables, No. 23).

[2] Meridian passages above and below pole will be further described on page 229.

17. What bright stars will pass the meridian of Greenwich, between the hours of 7 and 9 P.M. mean time, on August 20?

Ans. From η Ophiuchi to α Lyræ.

18. What bright stars will pass the meridian of a ship in long. 86° E., on Oct. 20, between the hours of 10 P.M. and midnight?

Ans. From α Andromedæ to α Eridani.

19. What bright star will pass the meridian of Greenwich the first after 9^h P.M. on Sept. 12? *Ans.* α Cygni.

Elements from 'Nautical Almanac.'

	Sid. Time at Mean Noon.		Sid. time at Mean Noon.
Nov. 20......	$15^h\ 57^m\ 39^s$	Aug. 20......	$9^h\ 54^m\ 56^s$
Oct. 10......	13 16 0	Oct. 20......	13 55 26
Mar. 10......	23 12 17	Sept. 12......	11 25 37

Time of a star's meridian passage

To find at what time any star will pass a given meridian.

This problem may be considered as the converse of the preceding.

Let X be the given star on the meridian P Q, A the first point of Aries, and m the mean sun (fig. 111).

Then $Q\,m$ = mean time required,
$A\,m$ = right ascension of the mean sun at that time,
$A\,Q$ = star's right ascension;
and by the figure, $Q\,m = A\,Q - A\,m$,
or mean time = star's R A − R A mean sun.

From which expression the mean time of the star's transit may be found. This problem is so similar in character to the last that we may at once refer the reader to the following example, observing that for most practical purposes the time of meridian passage found by subtracting the sidereal time at *nearest G. M. noon without correction* from the R. A. of the star is sufficiently accurate, never being more than 2^m in error.

Example

At what time will Sirius pass the meridian of a place in long. 68° 30′ W. on Nov. 21?

		R.A. Mean Sun.			
Star's R.A. + 24^h =	$30^h\ 38^m\ 23^s$	Nov. 20 (noon)	$15^h\ 57^m\ 26^s$	Star's R.A. .	$30^h\ 38^m\ 23^s$
Sid. time at Gr. mean noon . .	15 57 26	corr. for 19^h .	3 7·3	R.A. mean sun	16 0 36
ship M.T. nearly	14 40 57	„ 15^m .	2·5	∴ ship M. T. .	14 37 47
ong. in time . . .	4 34	R.A. mean sun	16 0 35·8		
Gr., Nov. 20	19 15 approximately				

Therefore the transit of Sirius is at $14^h\ 37^m\ 47^s$ on Nov. 20, or at $2^h\ 37^m\ 47^s$ A.M. on Nov. 21.

To find at what time it will pass the meridian on the morning of Nov. 20, we must evidently begin one day back, and take out the right ascension of the mean sun for Nov. 19.

Note.—If greater accuracy is required the R. A. mean sun should be recorrected for the more correct Greenwich mean time found from the more accurate ship mean time $= 14^h\ 38^m$, or it may be obtained directly as follows:

When the star is on the meridian it is evident that the ship sidereal time, or R. A. of the meridian, will be identical with the R. A. of the star; hence on applying the longitude to this the Greenwich sidereal time is found. Then (*see* page 194)

$$\left.\begin{array}{l}\text{Accurate S. M. T.}\\ \text{of star's transit}\end{array}\right\} = \left\{\begin{array}{l}\text{G. M. T. of preceding}\\ \text{transit of first point}\\ \text{of Aries}\end{array}\right\} - \left\{\begin{array}{l}\text{Retardation for}\\ \text{Greenwich side-}\\ \text{real time}\end{array}\right\} + \text{star's R. A.}$$

Example.—Find accurately at what time Regulus will pass the meridian of Madras (long. $5^h\ 21^m\ 3{\cdot}8^s$ E.) on Feb. 5.

	h	m	s		h	m	s
Star's R. A. or local sid. time	10	0	53·86	Gr. M. T. of preceding			
longitude	5	21	3·80 E.	transit of ♈ Feb. 5	2	59	5·18
G. S. D.	4	39	50·06				
Retardation for G. S. D.—							
4ʰ		0ᵐ	39·32ˢ				
39ᵐ			6·39				
50·06ˢ			·14				
		0	45·85	correction for retardation –		0	45·85
					2	58	19·33
				star's R. A.	10	0	53·86
				Local M. T. of star's mer. pass Feb. 5	12	59	13·19

20. At what time will *a* Pegasi pass the meridian of Portsmouth, long. 1° 6′ W., on Nov. 25? *Ans.* $6^h\ 38^m\ 58^s$ P.M.

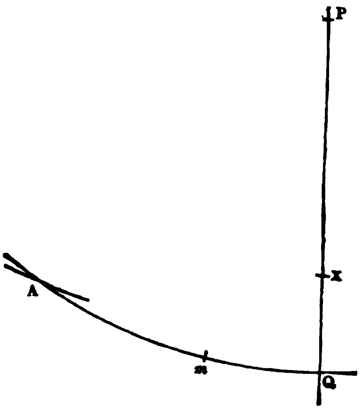
21. At what time will Regulus (*a* Leonis) pass the meridian of Land's End, long. 5° 42′ W., on May 30? *Ans.* $5^h\ 27^m\ 45^s$ P.M.

22. At what time will Antares pass the meridian of Portsmouth, long. 1° 6′ W., on Aug. 20? *Ans.* $6^h\ 24^m\ 11^s$ P.M.

23. At what time will *a* Leonis pass the meridian of Lisbon, long. 9° 8′ W., on Nov. 2? *Ans.* $7^h\ 14^m\ 7^s$ A.M.

24. At what time will Antares pass the meridian of Copenhagen, long. 12° 35′ E., on Feb. 3? *Ans.* $7^h\ 26^m\ 19^s$ A.M.

Fig. 111.



25. At what time will Fomalhaut pass the meridian of Calcutta, long. 88° 26′ E., on Mar. 8? *Ans.* $11^h\ 44^m\ 40^s$ A.M.

26. Find at what time *a* Canis Majoris passed the meridian of

Greenwich; the star's R. A. being $6^{h}\ 38^{m}\ 52{\cdot}2^{s}$, and the right ascension of mean sun at Greenwich mean noon being $11^{h}\ 6^{m}\ 2{\cdot}3^{s}$. Construct the figure, to show the positions of the first point of Aries and the mean sun with respect to the meridian.

Ans. $19^{h}\ 29^{m}\ 38^{s}$ nearly.

27. Find at what time *a* Aquilæ passed the meridian of Greenwich, having given the right ascension of the star $=19^{h}\ 43^{m}\ 51{\cdot}5^{s}$, and the right ascension of the mean sun at Greenwich mean noon $0^{h}\ 6^{m}\ 40{\cdot}4^{s}$; and construct the figure. *Ans.* $19^{h}\ 33^{m}\ 58^{s}$.

28. Find at what time *a* Leonis will pass the meridian of Greenwich, when its right ascension is $10^{h}\ 0^{m}\ 49{\cdot}76^{s}$, and the right ascension of the mean sun at Greenwich mean noon $=$ $4^{h}\ 32^{m}\ 4{\cdot}68^{s}$; and construct the figure. *Ans.* $5^{h}\ 27^{m}\ 51^{s}$.

Elements from 'Nautical Almanac.'

	Sid. Time at Mean Noon.			Right Asc. of Star.		
Nov. 25	16^{h}	17^{m}	22^{s}	22^{h}	57^{m}	26^{s}
May 30	4	31	25	10	0	9
Aug. 20	9	54	43	16	19	58
Nov. 1	14	42	54	10	0	11
Feb. 2	20	50	31	16	20	2
Mar. 7	23	0	37	22	49	11

ALTITUDE OF A HEAVENLY BODY ON THE MERIDIAN.[1]

Practical rule *To find the meridian altitude of a heavenly body, and also whether it will pass north or south of zenith.*

(1) Take out the declination.

(2) Under the latitude of the place put the declination, giving them their proper names N. or S.

(3) If the names are *alike* (both north or both south), take the difference and mark it with the common name of the latitude and declination, if the declination be greater than the latitude, otherwise with the contrary name.

(4) If the names are *unlike* (one north and one south), take the sum and mark it with the name of the declination.

(5) The result will be the meridian distance of the heavenly body from the zenith N. or S., according as the result was marked N. or S., and this subtracted from 90° will give the meridian altitude.

---

[1] This problem is of use when determining the latitude by meridian altitude of a star (*see* p. 238); the sextant being set to the approximate altitude, found from the calculated altitude by applying refraction, dip, and index error with the signs reversed, and the horizon near the north or south point carefully swept when near the time of meridian passage. This greatly facilitates the identification of the star.

Example In latitude 25° N. will the following heavenly bodies pass the meridian north or south of the zenith, and what will then be their altitudes, their declinations being 10° N., 30° N., 10° S., and 50° S. respectively?

(1)	(2)	(3)	(4)
lat. ... 25° N.	lat. ... 25° N.	lat. ... 25° N.	lat. ... 25° N.
decl.... 10 N.	decl.... 30 N.	decl.... 10 S.	decl.... 50 S.
diff. ... 15 S.	diff. ... 5 N.	sum... 35 S.	sum ... 75 S.
90	90	90	90
Alt. 75	Alt. 85	Alt. 55	Alt. 15

29. At what time will *α* Columbæ pass the meridian of a place in lat. 42° 20′ S. and long. 54° 40′ W. on May 10, and will it pass N. or S. of the zenith? also, what will then be its altitude?

Ans. $2^h\ 19^m$; alt. 81° 49′ N. of zenith.

30. At what time will Sirius pass the meridian of a place in latitude 61° N. and long. 10° W. on March 16, and will it pass N. or S. of the zenith? also, what will then be its altitude?

Ans. $7^h\ 0^m$; alt. 12° 28′ S. of zenith.

Elements from 'Nautical Almanac.'

	R. A. Mean Sun.	Star's R. A.	Star's Decl.
May 10	$3^h\ 13^m\ 52^s$	$5^h\ 34^m\ 25^s$	34° 9′ 12″ S.
Mar. 16	23 37 10	6 39 0	16 31 46 S.

Altitude of a heavenly body off the meridian.

Here, in the astronomical triangle lying between the pole, the zenith, and the given star, it is assumed that we know the colatitude (90° − *l*), the polar distance (90° ± *d*) of the star and its hour-angle (*h*); hence, two sides and the included angle being given, it is required to find the third side, or the body's zenith distance (*z*).

By trigonometry $\text{vers}\ z = \text{vers}\ (l \underset{\sim}{+} d)$[1] $+ \text{vers}\ \theta$

Where $\text{hav}\ \theta = \cos l \cos d\ \text{hav}\ h$.

Example.—In latitude 41° 17′ N. what would be the altitude of a star whose declination was 15° 23′ N. when its hour angle was $4^h\ 37^m$?

	l 41° 17′ N.
cos l 9·875904	d 15 23 N.
cos d 9·984155	$(l - d)$ 25 54
hav h 9·509007	
hav θ 9·369066	vers $(l - d)$ 0·100442
θ = 57° 51′ 00″	vers θ 467862
	vers z 0·568304
	z 64° 25′ 29″
	∴ true alt. 25 34 31

---

[1] + when declination and latitude are of opposite names, and ~ when of like names.

Note.—All the preceding rules relating to meridian passages of *stars* are equally applicable to the planets, but, since their right ascensions and declinations are not invariable, these elements should be taken from the *Nautical Almanac* (Planetary Ephemerides at Mean Noon), and corrected for the Greenwich date (usually by inspection) at time of observation.

The rules given apply to meridian passages *above* pole; the lower transit of a *star* occurs 12 sidereal hours or $11^h\ 58^m\ 2^s$ mean solar time after the upper transit, and for *any* heavenly body it may be shown by drawing suitable figures that the mean time of meridian passage *below* pole = (12^h + R. A. of the body) − R. A. mean sun.

The meridian altitude *below* pole is found by subtracting the polar distance from the latitude.

Moon's meridian passage

The time of the transit of *any* heavenly body can be found by means of the preceding rules; but in the case of the moon the following method of finding the time of her passage over a given meridian may be generally used with advantage.

The mean time of the moon's transit for every day at Greenwich, expressed in *astronomical* time, is given in the *Nautical Almanac.*[1] At any place east of Greenwich, the time of transit, owing to the moon's motion to the eastward,[2] must take place sooner (independent of the difference of time due to the difference of longitude), and to a place westward of Greenwich later, than the time recorded in the *Nautical Almanac.* Thus, if we suppose such daily motion to be 60 minutes: to a place 90° east of Greenwich the transit will take place 6 hours earlier than that at Greenwich (on account of the difference of longitude) + $\frac{90}{360}$ of 60^m, or 15 minutes, due to the moon's motion to the eastward in the six hours before she reaches the meridian of Greenwich, such motion being supposed equable during the interval. To a place west of the first meridian, a retardation will take place for the same reason.

Thus in fig. 112 if S represents the mean sun, and M_1, M_2 two positions of the moon in her orbit round the earth, separated by an interval of one day, it is evident that to observers on a meridian of the earth PE, east of PG, the meridian of Greenwich, there will be a *smaller* interval between the sun's meridian passage and that of the moon than there will be at Greenwich, and to observers on a meridian PW, west of Greenwich, this interval will be *greater*, since the moon will have moved in the interval of the earth's rotation; the mean velocity of the moon in her orbit is 2,280 miles per hour, equivalent in arc to about her diameter.

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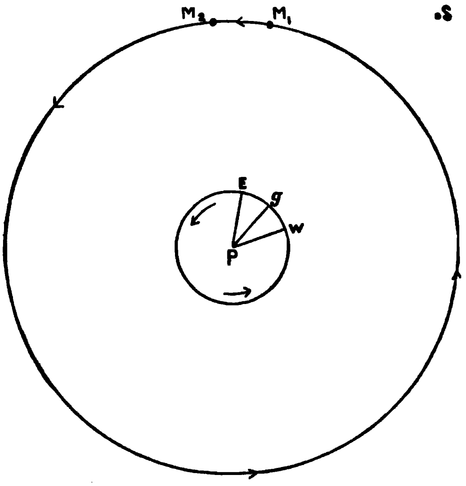
[1] On page iv. of each month *two* meridian passages are given, being the mean times of the upper and lower transits (*see* page 228). In all cases the *upper* transit is that to be employed, the times of lower transits being useful (except in very high latitudes) chiefly for calculations of tides.

[2] By this, which is termed the *daily retardation*, is to be understood the daily motion of the moon in her orbit round the earth compared with that of the mean sun, that is, the daily difference of their right ascensions, which causes the moon to arrive at any meridian *later* on each successive day.

The moon's daily motion in R. A. compared with that of the sun varies between 40^m and 60^m, this variable rate of motion being caused, as in the case of the sun, by the inclination of the moon's orbit to the equinoctial, combined with her variable velocity in that orbit, so that it is not difficult to construct a small table of the correction of the transits given in the *Nautical Almanac* for any given longitude: this has accordingly been done, and may be found in Inman's Tables (No. 33).

The moon's daily motion used should be that found by taking the difference between the two transits (upper) at Greenwich that happen before and after the one at the place: that is, if the place be in west longitude, the difference should be taken between the transit on the given day and the one following; if in east longitude, that on the given day and the one preceding. By observing this rule, the error arising from the unequal motion of the moon in R. A. is diminished.

Fig. 112.



Two examples of finding the time at Greenwich at the transit of the moon over a given meridian will show the use of the table.

On April 27, required the approximate Greenwich mean time at the transit of the moon over the meridian of a place in longitude 50° W.

By *Nautical Almanac*, mer. pass. (upper) on 27th...	11^h	46·3^m
„ „ on 28th...	12	32·0
		45·7
correction from table...		6·3 +
∴ time of transit at place...	11	52·6
long. in time...	3	20·0 W.
∴ Greenwich mean time of transit at place...	15	12·6

On April 27, required the approximate Greenwich mean time at the transit of the moon over the meridian of a place in longitude 50° E.

By *Nautical Almanac*, mer. pass. on 27th...11^h 46·3^m
„ „ on 26th...11 2·7
43·6
correction... 6·0 —
∴ time of transit at place...11 40·3
long. in time... 3 20·0 E.
∴ Greenwich mean time of transit at place... 8 20·3

Required the ship mean time of the moon's meridian passage on July 19 (astronomical day), in longitude 60° W., and on July 27 (astronomical day), in longitude 175° E., having given the following elements from the *Nautical Almanac* :

Gr. mer. pass. on July 19......11^h 24·3^m July 27......17^h 30·1^m
„ „ 20......12 19·2 „ 26......16 49·5

Ans. Mer. pass. at place on July 19 at 11^h 33·3^m
„ „ July 27 at 17 11·1 = July 28 at 5^h 11·1^m A.M.

The construction of the above-mentioned table may be explained as follows :

Let the moon's motion to the eastward in 24 hours $= d$.

A meridian, as that of Greenwich, describes in that time one complete revolution, or 360°.

Let the longitude of some given place $= \text{D}°$, and the motion of the moon for D°, expressed in time, $= x$.

Then $360° : \text{D}° :: d : x$ (supposing the moon's motion equable during the interval considered),

$$\text{or } x = \frac{\text{D}°}{360°} \times d.$$

By assuming different values for D° and d, the correction x may be easily calculated, and a table formed of the results.

Example Given the daily motion of the moon $= 45·7^m$; required the correction of the meridian passage in the *Nautical Almanac* for a place in longitude 50° W.

$$\text{D} = 50° \qquad d = 45·7 \qquad x = \frac{\text{D}°}{360°} \times d = \frac{50}{360} \times 45·7 = 6·3^m.$$

The times of meridian passages of *planets* may be found by a method similar to that given for the moon.

The times of these meridian passages at Greenwich are given in the *Nautical Almanac*, and the ship times of passage over any other meridians will differ from these by the amount of relative change of right ascension as compared with that of the mean sun in the interval corresponding to the longitude.

It will, however, be remarked that the *apparent* movements of the planets are such that their right ascensions sometimes increase and sometimes decrease, while that of the mean sun constantly *increases*, and therefore the correction must sometimes be applied in an opposite way to that given in the examples on the moon.

The moon is not so frequently observed as the other heavenly bodies, partly on account of the numerous corrections which have to be applied, owing to her being the nearest to the earth, but principally on account of the uncertainty with regard to the position of the horizon at night. In all cases, since the declination and right ascension of the moon change very rapidly, the time at which any observation is made is taken by a chronometer whose error on G. M. T. is approximately known.

The preceding method becomes more accurate by substituting for the *average* motion of the moon its *real* hourly motion in right ascension given in the *Nautical Almanac* in the table of Moon Culminating Stars.

As the length of the lunar day exceeds that of the solar day by intervals varying from 40 to 60 minutes (averaging 48 minutes), there must be days in each month when the moon does not cross the meridian at *both* upper and lower transits; thus at new moon an upper transit is missed and at full moon a lower transit.

In the case of the moon the interval between the sun's crossing any meridian and the moon's doing so is always *increasing*, but, since with the planets such an interval is sometimes *increasing* and sometimes *decreasing*, the phenomenon of two upper or two lower transits in one day sometimes occurs.

To find the mean time of the moon's meridian passage, accurately.

The formula which we have employed in determining the time of a star's transit over the meridian is not applicable to the accurate finding of the time of the moon's meridian passage, because a, which represents the right ascension of the heavenly body when ♈ was on the meridian of Greenwich (which is the starting-point of the calculation), does not remain constant. The problem must therefore be attacked in another manner.

Let a be the right ascension of the mean sun at the moment of its preceding transit over the meridian of the observer (that is, at the moment of local mean noon), and let a' be the corresponding right ascension of the moon at the same instant. The longitude of the place being known, both these elements can be ascertained from the tables in the *Nautical Almanac.*

Then $\alpha' - \alpha$ represents the difference of right ascension of the two bodies at the preceding mean noon at the place, and is the angle through which the meridian must revolve from west to east to overtake the moon.

Let $\Delta\alpha$ and $\Delta\alpha'$ represent respectively the change of right ascension of the sun and moon in one mean solar hour; the former being therefore the acceleration for one hour, or $9{\cdot}8565^{s}$.

If the moon remained stationary, at the end of one mean solar hour the meridian would have advanced towards the moon through an angle expressed in time equal to $3600^{s} - \Delta\alpha$; but meanwhile, owing to her motion, the moon would also have advanced to the eastward through an angle equal to $\Delta\alpha'$, and the angle still to be described by the meridian before overtaking the moon would have been increased by that quantity. The effect would be the same as if the original angle to be described by the meridian $(\alpha' - \alpha)$ had been increased by a quantity equal to $(\Delta\alpha' - \Delta\alpha)$; or as if the portion of the angle $(\alpha' - \alpha)$ described by the revolving meridian in one mean solar hour were only $(3600^{s} - (\Delta\alpha' - \Delta\alpha))$, this expression representing the relative hourly velocity of the meridian in overtaking the moon.

This condition holding good throughout the whole interval between the meridian transits of the two bodies, it follows that the whole time (T) that elapses is given by the expression—

$$T = \frac{\alpha' - \alpha}{3600 - (\Delta\alpha' - \Delta\alpha)}$$

Or

$$T = \frac{\alpha' - \alpha}{3609{\cdot}86 - \Delta\alpha'}$$

And the starting-point of the calculation being mean noon at the place, T is therefore the local mean time of the moon's meridian passage.

Example.—Find accurately the time of the moon's meridian passage at a place in long. 40° 45′ W., on May 2.

	h	m
May 2 S. M. T.	0	0
Long. . .	2	43 W.
G. M. T., May 2 at mean noon	2	43
	$3609{\cdot}86^{s}$	
$\Delta\alpha'$	$131{\cdot}88$[1]	
	$3477{\cdot}98$	

Whence we obtain from the *Nautical Almanac*:

	h	m	s
Moon's R. A., α'	6	50	41·36
Sun's α	2	40	44·28
$\alpha' - \alpha$	4	9	57·08
or			$14997{\cdot}08^{s}$

---

[1] $\Delta\alpha'$ not being constant, should be taken out for the middle of the interval between

Then $T = \dfrac{\alpha' - \alpha}{3609{\cdot}86 - \Delta\,\alpha'}$	In logs 4·176008 3·541327
$= \dfrac{14997{\cdot}08}{3477{\cdot}98}$	0·634681
$= 4{\cdot}3120^{h}$	$4{\cdot}3120^{h}$
$= 4^{h}\ 18^{m}\ 43{\cdot}3^{s}$	

The mean time of the moon's meridian passage at the place.

Motions of the moon

Some of the many very complicated movements of the moon with regard to the earth may be understood from the following general description.

The orbit (in relation to the earth) is an ellipse having our globe in one focus, and whose plane is inclined to that of the ecliptic at a mean angle of 5° 8′ in such a manner that to an observer in both hemispheres the full moon is low in summer and high in winter; the distances of the moon from the earth varies from 221,614 to 252,972 miles. The positions of the points where the moon is seen to pass from one side of the ecliptic to the other (nodes) are not fixed, but shift so as to describe a complete circle in about 18⅔ years in a direction the *same* as the movement of the hands of a watch, during which time the moon will have apparently covered every star situated within 5° 8′ north and south of the ecliptic.

If a line be drawn connecting the positions of the moon when nearest to and farthest from the earth (perigee and apogee), such a line makes a complete revolution in about 9½ years in a direction *opposite* to that in which the hands of a watch move.

EFFECTS OF CHANGE OF SHIP'S POSITION ON (1) ALTITUDE AND (2) TIME.

In many practical problems in nautical astronomy the results are obtained by combining observations taken from a ship at two or more positions several miles apart; as the object is generally to determine the result (whether for latitude, longitude, or time) corresponding to the *latest* position occupied by the ship, the usual practice is to apply to the first observation such corrections as would enable the observer to consider *both* observations as taken from the *second* position.[1]

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the meridian passage of the sun and the approximate meridian passage of the moon at the place.

[1] This correction was first considered by Facio Duillier in 1728. By using the angle between the *second* bearing and the course reversed, a correction is found which, applied to the *second* altitude, will reduce both observations to the place of first sights. The sign of this correction is of course opposite to that considered in the text. *See* also page 387.

The zenith of the observer will evidently move with the movement of the ship, and describe an arc on the celestial concave precisely similar to that described on the surface of the sea by the ship.

Correction in altitude for run

A change in the position of the zenith involves generally a change in the zenith distance of the object observed; also, if that object is the sun, it generally involves a change in the ship time, which depends on the angular distance of the sun from the meridian passing through that zenith, and similarly with other heavenly bodies a change in their hour angles.

Fig. 113.

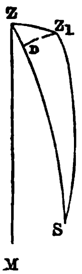
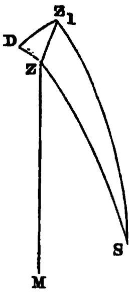


Fig. 114.



To consider first the effect of a movement of the ship on the zenith distance (or altitude) of the same heavenly body, known technically as the 'correction for run,' let Z be the zenith of the ship at the first observation, Z_1 its zenith at the second observation, Z M representing a portion of the celestial meridian; then arc $Z Z_1$ measures the distance run in the interval. Let S be the place of the heavenly body at the first observation: with centre S at distance $S Z_1$ describe an arc cutting S Z (fig. 113) or S Z produced in D (fig. 114); then the triangle $Z Z_1 D$, being small, may be considered as a right-angled plane triangle,[1] and Z D is the correction to be applied to Z S in order to find $Z_1 S$ the distance of S from the zenith at the second observation.

Now $Z D = Z Z_1 \cos Z_1 Z S$ = distance run × cos angle between the direction of the ship's run and the bearing of the body (observed by compass or taken from the azimuth tables) at the first observation.

This correction Z D may be readily found by means of the traverse table, for since diff. lat. = dist. cos course; if therefore in triangle $Z Z_1 D$ the angle $Z Z_1 D$ be considered as the course, and $Z Z_1$ the distance, the correction Z D for run expressed in minutes of arc will correspond in the traverse table to the difference of latitude.

The angle $Z_1 Z D$ is the difference between the course of the ship in the interval and the bearing of the body, when the run of the ship has been *towards* the place of the body, as in fig. 113; and the supplement of this angle when the direction of the ship's run has been *from* the place of the body, as in fig. 114. In the former case it is manifest that the correction Z D for run must be added to the first observed altitude, and in the second subtracted, in order to make the altitude of the body the same as it would have been if it had been also observed at the place of the ship at the second observation.

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[1] The true expression for correction for run, considering the various arcs as forming spherical triangles, and the rhumb line having been turned into the corresponding great circle track, becomes corr. $= z z_1 \cos z_1 z s - \frac{1}{2} z z_1^2 \cot s z \sin^2 z_1 z s \sin 1''$; hence it is seen that practically with a run between the times of observation of even 50 miles, the error involved by considering the triangle $z_1 z D$ *plane* (that is, neglecting the second term of the expression for the corr.) is always very small.

From this it will be seen that if the course of the ship between two observations is directly towards or directly from the object as seen at the first position, the altitude at the second place of observation would have been greater or less than that actually observed at the first place by a number of minutes equal to the number of nautical miles run in the interval, also that if the course is at right angles to the first bearing of the object the altitude would be the same at *both* places.

Practical rule

(1) Enter the traverse table with the distance run as a distance, and the angle (supposed less than 90°, or 8 points) between the bearing of the heavenly body at the first observation and course of the ship, as a course, and take out the corresponding *diff. lat.*, which *add* to the first true altitude (the tenths in the diff. lat. being turned into seconds by multiplying them by 60); the result will be the altitude corrected for run.

(2) But if the above angle be greater than 8 points or 90° subtract the same from 16 points, or 180°, and look out the remainder as a course, *subtracting* the diff. lat. corresponding thereto from the first true latitude; the result will be the altitude corrected for run.

If the vessel changes course between the observations the angle between the *true* or *magnetic* bearing of the heavenly body at the first position and the *true* or *magnetic* course and distance made good should be employed; if the course is unaltered the difference between the *compass* course and *compass* bearing will give the desired angle, since both course and bearing are affected by the same deviation.

Examples

31. The compass course of the ship was N. 50° W. 10 miles, and bearing by compass of the sun S. 79° E. Required the correction for the first altitude for run.

The angle between N. 50° W. and S. 79° E. is 151°; subtract 151° from 180°; and enter traverse table with the remainder, namely, 29° as a course and 10 miles as a distance: the corresponding diff. lat. is $8{\cdot}8' = 8'\ 48''$ to be *subtracted* from the true altitude.

32. The compass course of the ship was E.N.E. 25 miles, and bearing by compass of the sun E. by S. Required the correction of the first altitude for run.

The angle between E.N.E. and E. by S. is 3 points; entering traverse table with 3 points as a course, and 25 miles as a distance, the corresponding diff. lat. = 20·8′ = 20′ 48″ to be *added* to the true altitude.

33. The true course of the ship was S. 50° W. 15 miles, and the true bearing of the sun S. 17° E. Required the correction of the first altitude for run. *Ans.* +5′ 42″.

34. The true course of the ship was W.½N. 19 miles, and the true bearing of the sun was S. by E.½E. Required the correction for run. *Ans.* −7′ 18″.

Correction in time for run

The effect of change of position on ship time will be understood by considering that a vessel steering to the eastward is constantly meeting the sun, and on completely circumnavigating the globe will have *gained* a whole day, while if proceeding westward such a voyage will entail *losing* a whole day.[1] On reference to fig. 95, page 180, it will be seen that the vessel's speed to the *eastward* produces the same effect to an observer on board as if the earth turned on its axis more rapidly—that is, it makes the days shorter—while the opposite effect is produced by a ship steering to the *westward*. The difference in the length of day is equal to the difference of longitude in time made good.

Apparent time is that kept by clocks and watches on board ships at sea, the correction for difference of longitude being usually made at *noon* each day; one watch is usually kept correctly adjusted to Greenwich mean time so as to quickly ascertain the ship apparent or mean time, for convenience in taking observations depending directly on that time.

The following examples illustrate this part of the subject:

Examples

From observations at 8 A.M. a chronometer is found to be $1^h\ 22^m\ 15^s$ fast on ship app. time, and the run to the eastward from then till noon changes the longitude 65′. What is then the error on ship apparent time?

Expressed in time 65′ of longitude = $4^m\ 20^s$, by which amount the local time at the ship's second position is *later* than that at the first position (*see* fig. 95, page 180); hence the chronometer will be *less* fast and at apparent noon will show $1^h\ 17^m\ 55^s$. Had the chronometer been $1^h\ 22^m\ 15^s$ *slow* on apparent time at the first place, it would have been less fast (=more slow) at the second place, where the error would have been $1^h\ 26^m\ 35^s$ slow.[1]

---

[1] That is to say, the sun will have crossed the meridian once *more* or once *less* frequently than if she had remained stationary. It is customary to make the change of date when crossing the 180th meridian; thus a vessel crossing this meridian from the eastward skips a day: if it is Monday forenoon when it is reached, it becomes Tuesday forenoon immediately it is crossed; and similarly if crossing from the westward the same day is counted twice.

From observations at 8·40 A.M. a chronometer was found to be $2^h\ 17^m\ 5^s$ fast on ship apparent time, from which time a run to the westward was made until noon, which changed the longitude 73′. What was then the error on ship apparent time?

Expressed in time 73′ of longitude $=4^m\ 52^s$, by which amount the local time at the second position is *earlier* than that at the first position (*see* fig. 95, page 180); hence the chronometer will be *more* fast, and at apparent noon will show $2^h\ 21^m\ 57^s$.

Had the chronometer been originally $2^h\ 17^m\ 5^s$ *slow* on apparent time, it would have been more fast (=less slow) at the second place, where the error would have been $2^h\ 12^m\ 13^s$ slow.

Hence on proceeding eastward a chronometer which is fast on ship time becomes less fast, and if slow more slow, by the difference of longitude expressed in time.

Also if proceeding westward a chronometer which is fast becomes more fast, and if slow less slow on ship time.[2]

Examples

35. In latitude 50° N. a watch is found from observations at 8 A.M. to be $2^h\ 4^m\ 20^s$ slow on ship apparent time. Required the error at 11 A.M., the run in the interval being S.E. (true) 42 miles.
Ans. $2^h\ 7^m\ 22^s$ slow.

36. In latitude 40° S. a watch at 9 A.M. was found to be $1^h\ 2^m\ 27^s$ fast on ship apparent time. Required the error at 2 P.M., having run N.W. by W. (true) 52 miles during the interval.
Ans. $1^h\ 6^m\ 14^s$ fast.

37. In latitude 35° N. at 8 A.M. the error of a watch on ship apparent time was $1^h\ 19^m\ 34^s$ fast. Required the error at $11^h\ 30^m$ A.M., having run S. 48° E. by compass (var. 12° E.) 35 miles during the interval (for *deviation*, *see* page 21).
Ans. $1^h\ 18^m\ 33^s$ fast.

Error on apparent time

On considering the preceding method of finding the error of a watch at any instant on apparent time (and therefore finding the sun's hour angle), it is to be remarked that this hour angle is often required to be known when the sun is so situated that it cannot be obtained from direct observation of the sun's altitude at the time (*see* page 203); in these cases it is calculated, as in the preceding examples, from the latest good determination by observation, the error involved being that of the dead reckoning during the elapsed time. When, however, circumstances of cloudy weather have prevented any such *recent* determination the hour angle is found from the assumed longitude and known Greenwich time (given by chronometer) as follows:

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[1] Theoretically the proportional part of the *rate* of chronometer due to the absolute interval of time elapsed should also be applied, but unless this rate is very large it is not customary to do so.

[2] The error on *Greenwich* time is not affected by change of longitude.

In longitude by dead reckoning 60° W. at the instant when the Greenwich mean time (found by applying the known error to the time shown by chronometer) is $3^h\ 50^m\ 10^s$ the local or ship mean time is evidently $\frac{60}{15}=4$ hours *earlier* (*see* fig. 95, page 180), and therefore the ship (astronomical) mean time is $23^h\ 50^m\ 10^s$; on applying to this the equation of time, ship apparent time or the sun's hour angle is found.

Similarly in longitude by dead reckoning 60° E., when the Greenwich mean time is known to be $20^h\ 12^m\ 10^s$ the local or ship mean time is 4 hours *later*, and therefore $0^h\ 12^m\ 10^s$; on applying to this the equation of time, the ship apparent time or the sun's hour angle is found as before.

Examples

38. In longitude by dead reckoning 24° 10′ W., when a chronometer showed $1^h\ 19^m\ 28^s$, required the sun's hour angle.

Error of chronometer on Greenwich mean time $1^m\ 24^s$ fast; equation of time $7^m\ 15^s$ subtract from mean time.

Ans. $23^h\ 34^m\ 9^s$.

39. In longitude by dead reckoning 34° 15′ E., when a chronometer showed $4^h\ 17^m\ 10^s$, required the sun's hour angle.

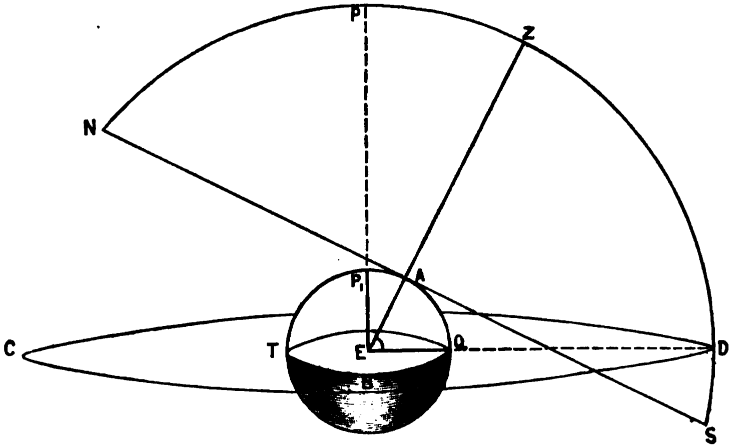
Error of chronometer on Greenwich mean time $1^h\ 2^m\ 7^s$ slow; equation of time $12^m\ 13^s$ add to mean time. *Ans.* $7^h\ 48^m\ 30^s$.

CHAPTER IX.

Latitude—Meridian altitudes of the sun, moon, and stars above and below pole—Altitudes near meridian above pole—Delambre's method—Pole-star—Double altitudes of sun and stars—Douwes's method—Three altitudes at equal intervals of time apart—Combined altitudes.

THE latitude of any place on the earth (considered as a sphere [1]) is measured from the terrestrial equator, either as the length of an arc of a meridian or as the angle subtended by that arc at the centre of the earth. **Definitions**

FIG. 115.



From the latter point of view the latitude of a place is equal to the declination of the zenith.

Thus if A be a place on the earth whose zenith is Z, and if T E Q represent the plane of the equator, the latitude of A is measured by the arc A Q or the angle A E Q subtended by that arc at E, the centre of the earth; this angle is also evidently measured by the arc Z D on the celestial concave, which being the angular distance of the zenith from the equinoctial C B D is by definition its declination.

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[1] The various errors in the problems of nautical astronomy occasioned by considering the earth to be a sphere instead of a spheroid will be dealt with in Chapter XIII.

Latitude equals altitude of pole

That this is equal to the altitude of the celestial pole P is seen by drawing N A S to represent the plane of the horizon of A, and therefore at right angles to Z A, when, since Z N and P D are each equal to 90°, they are equal to one another.

If from these equals the common part P Z be taken away, the remainder P N, or altitude of the pole, equals the remainder Z D, or the latitude.

The fact of the altitude of the pole being equal to the latitude of the observer would, if the position of the pole were marked by a star, suggest a very easy mode of finding the latitude by simply measuring that altitude; although neither pole is thus marked in the heavens, yet *near* the north pole in a conspicuous star from the altitude of which at *any* time of night the altitude of the pole may readily be found. This mode of proceeding has been practised from the earliest times.[1]

Meridian passages

Finding the latitude of a place is therefore ascertaining by astronomical observations either the declination of the zenith or the altitude of the pole. There are several methods of finding these elements, of which we shall first give that most frequently employed, which is to find the distance of the zenith from some heavenly body whose declination is known when that body is on the meridian of the observer, this zenith distance being found by observing the altitude and subtracting it from 90°. By the rotation of the earth every heavenly body apparently describes a circle or parallel of declination round the elevated pole, and therefore crosses each meridian *twice*.

These passages across the meridian are termed culminations or transits, that which occurs below the pole being the lower culmination or inferior transit, and that above the pole the upper culmination or superior transit.

The circumstances which determine whether for any particular place and particular heavenly body it will cross the meridian *above the horizon once* or *twice* will be seen from the following figures:

Let N W S E represent the horizon, P the pole, and Z the zenith; then N Z S is the celestial meridian. Let x_1 *cxd* be a parallel of declination described by the heavenly body about the pole P (fig. 116).

Then if P x_1 (the polar distance of the heavenly body) is less than P N (equal to the latitude of the observer) that body will be on the meridian above the horizon *twice*, once at x, when it is said to pass *above* pole, and again at x_1, termed meridian passage below pole;[1] such a body is always above the horizon, and said to be circumpolar. If the polar distance of another body is greater than P N (fig. 117) during part of its daily path it will be below the horizon and invisible, only the meridian passage above pole being seen.

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[1] It is given in the earliest treatises on navigation.

From the foregoing it will be seen that at either pole (where Z and P are coincident) the stars which are visible neither rise nor set, but describe circles round the zenith; also that at the equator, where both poles are on the horizon, no heavenly body can be observed below pole.

Fig. 116.

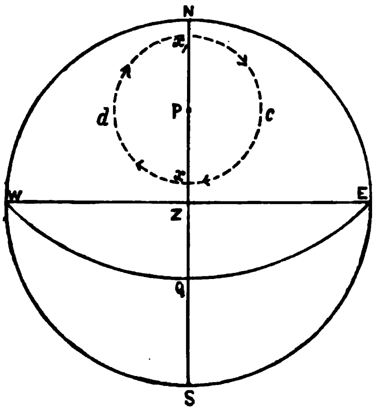
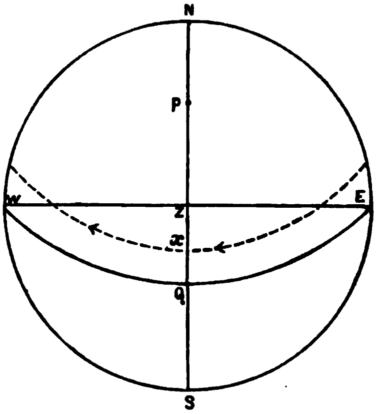


Fig. 117.



In considering those bodies whose declinations change (as, for instance, the sun), since at its nearest approach to the north pole on June 21 its polar distance is 66° 33′, the north latitude of an observer must be in excess of that in order to see the sun cross the meridian below pole, while at its nearest approach to the *south* pole on December 21 the latitude of an observer must be greater than 66° 33′ south in order to observe a meridian passage below the south pole.

The moon's polar distance may be 5° 8′ less than that of the sun, so that the equatorial limits of latitude, where a meridian passage of the moon may be visible below pole, are 61° 25′ north or south.

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[1] If a *star*, the interval between these meridian passages will be 12 sidereal hours or $11^h\ 58^m\ 2^s$ mean solar hours; if the *sun*, it will be 12 apparent solar hours; and if the moon, the interval will average $12^h\ 24^m$ mean solar time.

Latitude by Meridian Altitude above Pole.

Theory

Given the meridian altitude of a heavenly body above the pole, its declination, and the bearing of the zenith from the body, to find the latitude.

There are three cases.

1. Declination and zenith bearing of the same name.
2. Declination and zenith bearing of different names, and the declination greater than the zenith distance.
3. Declination and zenith bearing of different names, and the declination less than the zenith distance.

In each of the following figures 118, 119, 120 :

Let N W S E represent the horizon.
N Z S „ the celestial meridian.
W Z E „ the prime vertical.
Take Z X = zenith distance of the heavenly body, equal to the complement of the altitude.
Q X = the declination of the body.

Then the great circle W Q E is the equinoctial, and Z Q is the latitude required.

(Fig. 118) $ZQ = ZX + QX$. (Fig. 119) $ZQ = QX - ZX$. (Fig. 120) $ZQ = ZX - QX$. (1) Lat. (N) = zen. dist. (N) + decl. (N). (2) Lat. (N) = decl. (N) − zen. dist. (S). Lat. (N) = zen. dist. (N) − decl. (S).

Hence the rule.

FIG. 118.

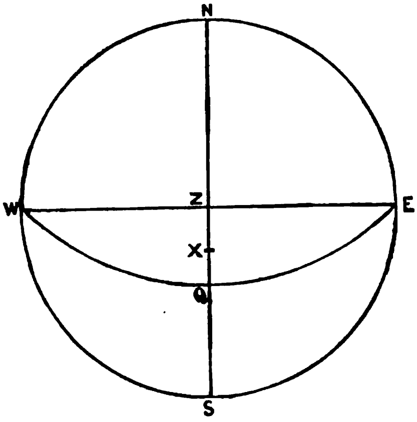
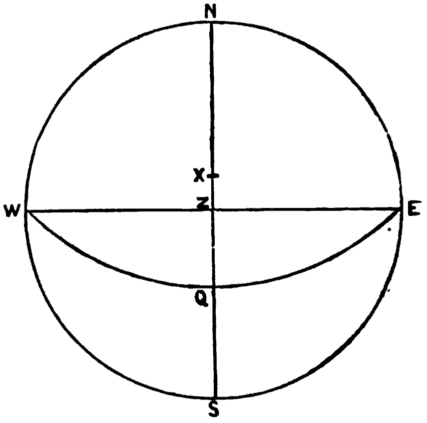


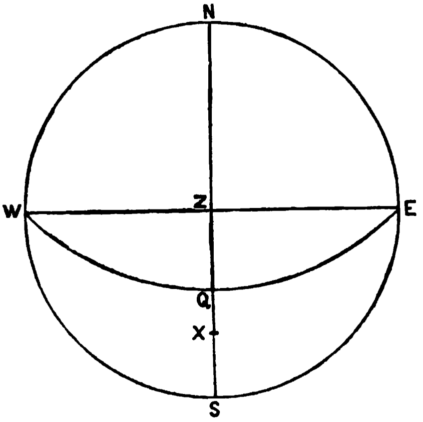
FIG. 119.



When the declination and zenith bearing are of the same name, take the sum—which will be the latitude, and of the same name. When the declination and zenith bearing are of different names take the difference; this will be the latitude, and of the name of the greater.

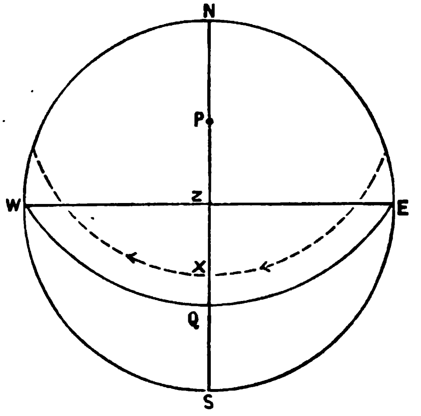
When the declination is 0 the zenith distance equals the latitude, and when the zenith distance is 0 the declination equals the latitude.

Fig. 120.



When the sun, or any heavenly body, is crossing the meridian very nearly in the zenith of the observer, and it is doubtful whether it will cross north or south of him, it is advisable to note by compass the position on the horizon of the true north or south point, and observe either the altitude or its supplement.

Fig. 121.



Given the meridian altitude of a heavenly body = 56° 10′ 15″, **Examples** zenith north of the body, and its declination=15° 22′ 10″ N.; construct a figure, and find by calculation the latitude.

Let N W S E (fig. 121) represent the horizon, N Z S the celestial meridian, and W Z E the prime vertical. Take S X=56° 10′ 15″, and X Q=15° 22′ 10″, and through Q draw the great circle W Q E; then W Q E represents the equinoctial, and Z Q=latitude required.

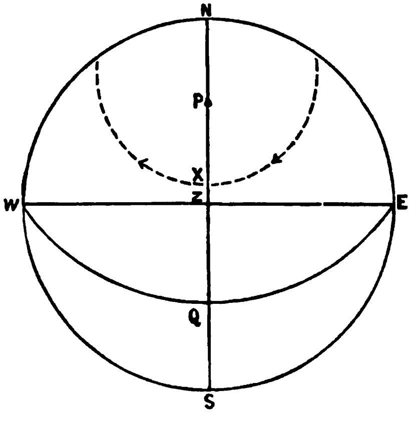
By the figure, Z Q=Z X+X Q=merid. zen. dist.+decl.

Calculation.

	90°		
S X=	56	10′	15″
Z X=	33	49	45
X Q=	15	22	10
∴ Z Q=lat.=	49	11	55 N.

Given the meridian altitude of a heavenly body=72° 42′ 15″, zenith south of the body, and its declination=47° 32′ 14″ N.; construct a figure, and find by calculation the latitude (fig. 122).

Fig. 122.



Let N W S E represent the horizon, N Z S the celestial meridian, and W Z E the prime vertical. Take N X=72° 42′ 15″, and X Q= 47° 32′ 14″, and through Q draw the great circle W Q E; then W Q E represents the equinoctial, and Z Q=latitude required.

By the figure, Z Q=X Q−X Z,
or latitude=mer. zen. dist.−decl.

Calculation.

	90°		
N X =	72	42′	15″
X Z =	17	17	45
X Q =	47	32	14
∴ Z Q = lat. =	30	14	29 N.

Latitude by Meridian Altitude of the Sun.

Examples

1. On April 27, in long. 87° 42′ W., the observed meridian altitude of the sun's lower limb was 48° 42′ 30″ (zenith north), the index correction was +1′ 42″, and the height of the eye above the sea 18 feet. Required the latitude.

Ship app. time, Apr. 27	0^h 0^m	Sun's decl. (at app. noon).				Obs. alt.	48°	42′	30″
long. in time …………	5 51 W.	Apr. 27	13°	43′	53″ N.	index corr.		1	42+
Gr. app. time, Apr. 27	5 51	corr. …		4	36		48	44	12
or $5·8^h$			13	48	29	dip ………		4	11−
		var. in 1^h					48	40	1
		47·64				corr. in alt.			46−
hours from noon		5·8					48	39	15
		38112				semi. ……		15	55+
		23820				true alt. …	48	55	10
		276·312 = 4′ 36″					90		
						zen. dist…	41	4	50 N.
						decl. ……	13	48	29 N.
						lat. ………	54	53	19 N.

NOTE.—When a vessel is steaming at a high speed on courses near North or South true, there is a sensible difference between the meridian altitude and the maximum altitude. In these cases it is advisable to calculate what a watch will show at ship apparent noon, and observe at that instant (*see* page 361).

2. June 10, in long. 23° 40′ E., the observed meridian altitude of the sun's lower limb was 36° 10′ 30″ (Z. N.), index correction +4′ 5″, height of eye 24 feet. Required the latitude.

Ans. 76° 37′ 45″ N.

3. August 7, in long. 62° 11′ E., the observed meridian altitude of sun's upper limb was 41° 49′ 50″ (Z. N.), the index correction −3′ 15″, height of eye 12 feet. Required the latitude.

Ans. 65° 0′ 22″ N.

4. May 3, in long. 14° 20′ W., the observed meridian altitude

of sun's upper limb was 15° 6′ 10″ (Z. S.), index correction −1′ 15″, height of eye 20 feet. Required the latitude.

Ans. 59° 34′ 14″ S.

5. July 17, in long. 72° 30′ E., the observed meridian altitude of sun's upper limb was 26° 18′ 50″ (Z. N.), index correction +2′ 10″, height of eye 25 feet. Required the latitude.

Ans. 85° 15′ 16″ N.

6. March 3, 1884, in long. 38° 15′ W., the observed meridian altitude of the sun's L. L. was 38° 10′ 15″ (zenith N. of the sun), the index correction was −2′ 30″, and the height of the eye above the sea 22 feet. Required the latitude. *Ans.* 45° 12′ N.

7. May 7, 1884, in long. 31° 15′ W., the observed meridian altitude of the sun's L. L. was 67° 40′ 20″ (zenith N. of the sun), the index correction was +1′ 45″, and the height of the eye above the sea 23 feet. Required the latitude. *Ans.* 39° 9′ N.

8. July 11, 1884, in long. 37° W., the observed meridian altitude of the sun's L. L. was 68° 23′ 30″ (zenith N. of the sun), the index correction was +2′ 10″, and the height of the eye above the sea 24 feet. Required the latitude. *Ans.* 43° 25′ N.

9. October 13, 1884, in long. 15° 40′ E., the observed meridian altitude of the sun's L. L. was 47° 5′ 20″ (zenith N. of the sun), the index correction was −1′ 35″, and the height of the eye above the sea 20 feet. Required the latitude. *Ans.* 34° 44′ N.

10. December 13, 1884, in long. 22° W., the observed meridian altitude of the sun's L. L. was 29° 47′ 10″ (zenith N. of the sun), the index correction was −3′ 15″, and the height of the eye above the sea was 23 feet. Required the latitude.

Ans. 36° 53′ N.

11. February 9, 1885, in long. 21° 30′ W., the observed meridian altitude of the sun's L. L. was 35° 11′ 20″ (zenith N. of the sun), the index correction was −2′ 10″, and the height of the eye above the sea was 18 feet. Required the latitude.

Ans. 40° 10′ N.

12. May 17, 1885, in long. 27° 30′ W., the observed meridian altitude of the sun's L. L. was 48° 30′ 20″ (zenith N. of the sun), the index correction was −2′ 40″, and the height of the eye above the sea was 18 feet. Required the latitude.

Ans. 60° 48′ N.

13. July 14, 1885, in long. 14° 50′ E., the observed meridian altitude of the sun's L. L. was 39° 58′ 30″ (zenith S. of the sun), the index correction was −1′ 40″, and the height of the eye above the sea was 18 feet. Required the latitude.

Ans. 28° 14′ S.

14. October 17, 1885, in long. 81° 27′ W., the observed meridian altitude of the sun's L. L. was 52° 11′ 10″ (zenith S. of the sun), the index correction was +1′ 15″, and the height of the eye above the sea was 18 feet. Required the latitude.

Ans. 47° 6′ S.

15. October 9, in long. 19° 20′ W., the observed meridian altitude of the sun's lower limb was 22° 20′ 30″ (Z. S.), index correction −2′ 10″, height of eye 20 feet. Required the latitude.

Ans. 73° 53′ 28″ S.

Elements from 'Nautical Almanac.'

	Sun's Declination.	Var. in 1 Hour.		Sun's Semi.
June 10	23° 2′ 13″ N.	12·37″	incr.	15′ 46″
Aug. 7	16 24 0 N.	41·40	decr.	15 48
May 3	15 43 50 N.	43·98	incr.	15 53
July 17	21 11 16 N.	24·69	decr.	15 46
Mar. 3	6 32 30 S.	57·65	decr.	16 10
May 7	17 0 10 N.	40·91	incr.	15 53
July 11	22 1 56 N.	20·51	decr.	15 46
Oct. 13	8 1 50 S.	55·97	incr.	16 4
Dec. 13	23 12 25 S.	9·25	incr.	16 17
Feb. 9	14 30 47 S.	48·46	decr.	16 14
May 17	19 25 38 N.	33·46	incr.	15 51
July 14	21 37 53 N.	23·09	decr.	15 46
Oct. 17	9 25 6 S.	54·80	incr.	16 5
Oct. 9	6 20 10 S.	57·08	incr.	16 4

Latitude by Meridian Altitude of the Moon.

Examples.

November 12, at $2^h\ 20^m$ P.M. mean time nearly, in longitude 60° 45′ W., observed the meridian altitude of the moon's lower limb to be 30° 30′ 40″ (Z. N.), the index correction +10′ 42″, and height of eye above the sea 16 feet. Required the latitude.

Ship, Nov. 12 .... $2^h\ 20^m$
long. in time............... 4 3W.

Greenwich, Nov. 12...... 6 23

NOTE.—The Greenwich date might also have been found from the time of meridian passage (*see* page 216) or more simply and accurately by taking time by chronometer.

Moon's Declination.	**Moon's Semi.**	**Hor. Par.**
Nov. 12, at 6^h...2° 44′ 20″ N.	Noon.........15′ 6″	55′ 20″
corr. 5 6	mid. 15 3	55 6
2 49 26	3	14
Var. in 10^m	corr. by insp. 2	7
133″	15 4	55 13
2·3	aug^n. 7	
399	semi. 15 11	
266		
305·9		
Var. in 23^m=5′ 6″		

Moon's alt. L. L.........	30°	30′	40″	
index corr.		10	42	+
	30	41	22	
dip.......................		3	56	−
	30	37	26	
semi.		15	11	+
	30	52	37	
corr. in alt...............		45	36	+
			11	+
true alt.	31	38	24	
zen. dist..................	58	21	36	N.
decl.	2	49	26	N.
Lat........................	61	11	2	N.

Note.—When the declination of the moon is changing rapidly, or the ship is steaming at high speed on courses near North or South true, the *meridian* altitude of the moon may differ considerably from the *maximum* altitude. In this case the time of meridian passage is calculated, and the altitude observed at that instant (*see* page 361).

16. January 10, at 7^h 40^m P.M. mean time nearly, in long. 5° 30′ E., the observed meridian altitude of the moon's lower limb was 10° 20′ 30″ (Z. N.), the index correction −2′ 20″, and height of eye 14 feet. Required the latitude.

Ans. Lat. 56° 37′ 46″ N.

17. February 4, at 5^h 40^m A.M. mean time nearly, in long. 72° 18′ W., the observed meridian altitude of the moon's lower limb was 40° 20′ 15″ (Z. N.), the index correction + 4′ 40″, and height of eye 15 feet. Required the latitude.

Ans. Lat. 25° 17′ 10″ N.

18. March 7, at 3^h 20^m P.M. mean time nearly, in long. 19° 20′ W., the observed meridian altitude of the moon's lower limb was 19° 17′ 18″ (Z. S.), index correction − 1′ 15″, and height of eye 16 feet. Required the latitude. *Ans.* Lat. 88° 0′ 44″ S.

19. July 5, at 1^h 7^m P.M. mean time nearly, in long. 33° 30′ E., the observed meridian altitude of the moon's upper limb was 25° 42′ 30″ (Z. N.), the index correction + 2′ 15″, and height of eye 20 feet. Required the latitude. *Ans.* Lat. 88° 22′ 37″ N.

20. August 12, at 5^h 4^m A.M. mean time nearly, in long. 94° 40′ E., the observed meridian altitude of the moon's upper limb was 72° 20′ 0″ (Z. S.), the index correction + 3′ 40″, and height of eye 22 feet. Required the latitude.

Time by chronometer 6^h 26^m 10^s; error on G. M. T. 4^h 19^m 10^s slow. *Ans.* Lat. 31° 53′ 5″ S.

21. December 27, at 9^h 12^m A.M. mean time nearly, in long. 95° 20′ W., the observed meridian altitude of the moon's upper limb was 19° 50′ 4″ (Z. S.), the index correction − 0′ 30″, and height of eye above the sea was 24 feet. Required the latitude.

Time by chronometer 11^h 12^m 20^s; error on G. M. T. 0^h 59^m 5^s fast. *Ans.* Lat. 87° 35′ 20″ S.

Elements from 'Nautical Almanac.'

	Moon's Declination.	Moon's Semi.		Hor. Par.
Jan. 10, at 7^h	22° 1′ 16″ S.	noon	16′ 0·1″	58 36·4″
Var. in 10^m	61·2″ decr.	mid.	15 54·2	58 14·9
Feb. 3, at 22^h	23° 20′ 51″ S.	mid.	16 8·2	59 6·1
Var. in 10^m	38·7″ incr.	noon	16 6·6	59 0·3
Mar. 7, at 4^h	18° 25′ 4″ S.	noon	15 33·4	56 58·5
Var. in 10^m	92·1″ decr.	mid.	15 29·3	56 43·7
July 4, at 22^h	24° 33′ 11″ N.	mid.	14 50·7	54 22·0
Var. in 10^m	22·7″ incr.	noon	14 52·9	54 30·1
Aug. 11, at 10^h	14° 4′ 13″ S.	noon	16 6·9	59 1·1
Var. in 10^m	125·5″ incr.	mid.	16 9·2	59 9·7
Dec. 26, at 22^h	17° 55′ 16″ S.	mid.	16 27·4	60 16·7
Var. in 10^m	118·3″ incr.	noon	16 33·2	60 37·6

NOTE.—The meridian altitude of the moon is often advantageously used in the daytime or in the twilight.

Latitude by Meridian Altitude of a Fixed Star.

Practical rule

The declination of a fixed star changes so slowly that we may, without any practical error, take it out of the *Nautical Almanac* (from the Catalogue of *apparent* places of stars) by *inspection*; a Greenwich date will not be necessary.

Examples

Feb. 13, the observed meridian altitude of *a* Hydræ was 35° 55′ 00″ (zenith north of star), the index correction was + 2′ 10″, and height of eye 20 feet. Required the latitude.

Observed altitude………	35°	55′	00″	
index correction ………		2	00	+
	35	57	00	
dip ……………………		4	24	−
	35	52	36	
refraction ………………		1	20	−
true altitude ……………	35	51	16	
	90			
true zenith distance……	54	8	44	N.
declination ……………	8	1	29	S.
latitude ………………	46	7	15	N.

22. May 21, the observed meridian altitude of *a* Böotis was 62° 42′ 10″ (Z. N.), the index correction − 4′ 4″, and height of eye 18 feet. Required the latitude. *Ans.* Lat. 47° 23′ 32″ N.

23. June 16, the observed meridian altitude of *a* Lyræ was 77° 1′ 50″ (Z. N.), index correction + 2′ 10″, and height of eye 16 feet. Required the latitude. *Ans.* Lat. 51° 39′ 4″ N.

24. May 6, the observed meridian altitude of *a* Virginis was 16° 52′ 5″ (Z. N.), index correction + 1′ 45″, and height of eye 20 feet. Required the latitude. *Ans.* Lat. 62° 50′ 4″ N.

25. Oct. 26, the observed meridian altitude of *a* Piscis Australis was 70° 10′ 0″ (Z. S.), the index correction − 4′ 5″, and height of eye 10 feet. Required the latitude.

Ans. Lat. 50° 21′ 26″ S.

26. May 4,[1] at $2^h\ 45^m$ A.M. mean time nearly, in long. 42° 10′ W., the observed meridian altitude of Jupiter's centre was 16° 42′ 10″ (Z. N.), index correction + 11′ 42″, and height of eye 20 feet. Required the latitude. *Ans.* Lat. 50° 30′ 38″ N.

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[1] The meridian altitude of a planet is worked like that of a star, the small corrections for semidiameter and parallax being neglected; the declination, however, given in the *Nautical Almanac* for each Greenwich mean noon requires correction (usually by inspection) for Greenwich date. The meridian altitudes of a planet invisible to the naked eye can often be observed in the daytime by setting the sextant to the approximate meridian altitude (*see* page 214), and at the time of meridian passage (*see* p. 209), sweeping the horizon through the *telescope* near the north or south point according to the position of the planet with regard to the zenith.

27. July 12, at $9^h\ 36^m$ P.M. mean time nearly, in long. 30° 30′ E., the observed meridian altitude of Venus's centre was 10° 10′ 50″ (Z. N.), the index correction − 4′ 4″, and height of eye 10 feet. Required the latitude. *Ans.* Lat. 57° 45′ 37″ N.

28. November 27, at $6^h\ 3^m$ A.M. mean time nearly, in long. 100° 0′ W., the observed meridian altitude of Mars' centre was 32° 40′ 10″ (Z. S.), index correction − 8′ 10″, and height of eye 16 feet. Required the latitude. *Ans.* Lat. 45° 44′ 58″ S.

29. Sept. 15, at $4^h\ 20^m$ A.M. mean time nearly, in long. 10° 6′ W., the observed meridian altitude of Saturn's centre was 19° 42′ 10″ (Z. N.), index correction − 6′ 45″, and height of eye 12 feet. Required the latitude. *Ans.* Lat. 88° 55′ 24″ N.

Elements from 'Nautical Almanac.'

May 21	... α Böotis	Decl.	19°	56′	57″	N.
June 16	... α Lyræ	„	38	38	55	N.
May 6	... α Virginis	„	10	23	40	S.
Oct. 26	... α Piscis Australis	„	30	23	53	S.

Jupiter, decl.	May 3	22°	43′	11″	S.	May 4	22°	43′	1″	S.
Venus, „	July 12	22	16	10	S.	July 13	22	15	49	S.
Mars, „	Nov. 28	11	48	44	N.	Nov. 29	11	40	44	N.
Saturn „	Sept. 14	18	24	50	N.	Sept. 15	18	24	38	N.

NOTE.—In selecting stars for observation on the meridian, preference should be given to those of high declination, their movement in altitude being very slow when near the meridian. Some observers prefer taking several observations of altitude from about five minutes before the meridian passage to five minutes after, unclamping the sextant each time, and taking the mean of the observations at the meridian altitude. (*See* also notes on pp. 360, 361).

Latitude by Meridian Altitude below Pole.

Given the meridian altitude of a heavenly body below the pole, and its declination; to find the latitude. **Theory**

Let NWSE represent the horizon, NZS the celestial meridian, P the pole of the heavens, and X a heavenly body on the meridian below the pole (fig. 123).

Then XN = meridian alt. of heavenly body,
PX = polar distance = 90° − decl.,
and PN = alt. of pole = latitude.

By the figure, PN = XN + XP,
∴ latitude = merid. alt. + polar distance.

Examples

Given the meridian altitude of *a* Centauri below the south pole =27° 25′ 50″, and its declination=60° 14′ 40″ S. Construct a figure, and find by calculation the latitude (fig. 124).

Let N W S E represent the horizon, and N Z S the celestial meridian. In S Z take S X=27° 25′ 50″; then X is the place of the star below the south pole. Take X P=29° 45′ 20″, the star's south polar distance; then P is the south pole, and P S=alt. of south pole=lat. required.

Fig. 123.

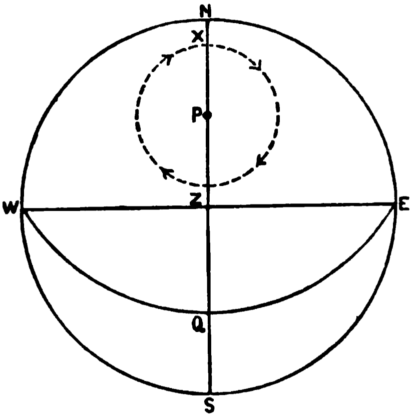
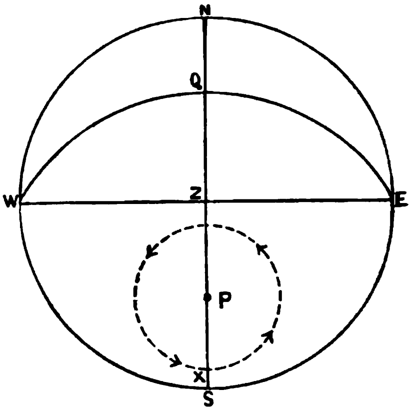


Fig. 124.



By formula, lat.=mer. alt.+polar distance,
∴ lat.=27° 25′ 50″+(90°−60° 14′ 40″)
=57° 11′ 10″ S.

Note.—When crossing the meridian *above* pole a heavenly body moves from east to west, and when crossing *below* pole, from west to east. Below pole the meridian altitude is of course the least observed.

30. Feb. 10, the meridian altitude of *a* Argûs below the pole was observed to be 6° 41′ 15″, index correction −2′ 10″, and height of eye above the sea 14 feet. Required the latitude.

Ans. Lat. 43° 50′ 18″ S.

31. Jan. 11, the observed meridian altitude of *a* Ursæ Majoris below the pole was 14° 14′ 30″, the index correction −4′ 5″, and height of eye 20 feet. Required the latitude.

Ans. Lat. 41° 29′ 47″ N.

32. April 20, the observed meridian altitude of η Argûs below the pole was 20° 14′ 15″, the index correction − 4′ 10″, and height of eye 10 feet. Required the latitude.

Ans. Lat. 51° 9′ 26″ S.

Elements from 'Nautical Almanac.'

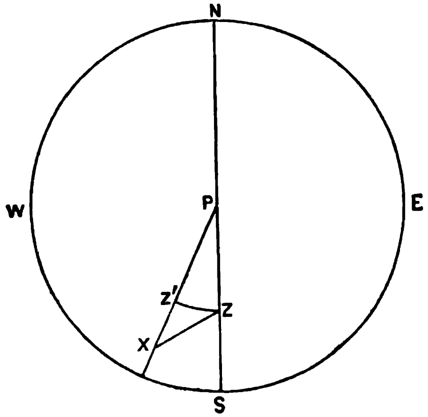
Feb. 10 ... α Argûs..............	Decl.	52°	37′	17″	S.
Jan. 11 ... α Ursæ Majoris ...	„	62	32	28	N.
Apr. 20 ... η Argûs..................	„	58	54	55	S.

Note.—On referring to fig. 116 on page 229 it is evident that the altitude of the pole or latitude of an observer is equal to half the sum of the meridian altitudes of a circumpolar star above and below pole if both altitudes are observed on the same side of the zenith, or to half the sum of (180° + inferior mer. alt. – superior mer. alt.) if on opposite sides of the zenith; this method of determining latitude is employed, among others, at astronomical observatories, but is of no use for purposes of navigation, since it practically involves the condition of the ship remaining stationary for a long interval.

LATITUDE BY ALTITUDE NEAR MERIDIAN ABOVE POLE.

The observation of the altitude of a heavenly body at the instant of its crossing the meridian of an observer being often prevented by clouds, the latitude in such a case may be obtained from an altitude *near* the meridian.

Fig. 125.



Every heavenly body being *always* on the meridian of *some* place, the problem resolves itself into finding from an observation what would be the altitude of the body at that instant to another observer, who, being in the same latitude as the first observer, has the sun on his meridian.

Thus in fig. 125 (drawn on plane of the equinoctial), if P be the pole, Z the zenith, and N P S the meridian of the first observer: at the time of observing the altitude of a heavenly body X, not on the meridian of Z, it must be on *some* meridian east or west of Z; in the case considered it will be on the meridian P X, and to an observer at Z′ *in the same latitude as* Z its meridian zenith distance will be Z′ X.

If, then, Z′ X can be calculated, the latitude of Z′, and therefore that of Z, will be known.

As in the problem of finding the latitude by meridian altitudes there will be a distinction of cases between heavenly bodies near the meridian above and below pole, but the latter (except for stars) is in practice of very rare occurrence, only happening with the sun, moon, or planets in very high latitudes.

Theory

Given the altitude of a heavenly body *near the meridian* above the pole, its hour angle and declination, and the estimated latitude of the observer: to find the correct latitude.[1]

Let X be a heavenly body, P the pole, and Z the zenith. Let the hour angle $ZPX = h$, latitude $ZQ = l$, zenith distance $ZX = z$, and declination of X or $XD = d$.

Then polar distance $PX = 90° - d$ when the lat. and decl. have the same name, and $90° + d$ when of different names.

Fig. 126.

(On plane of horizon.)

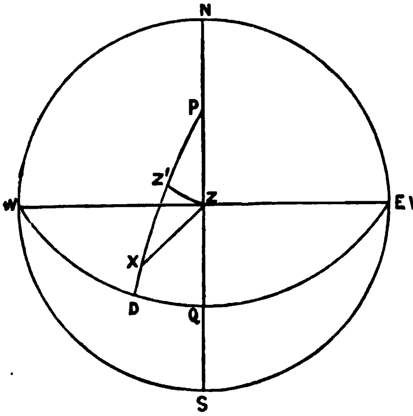
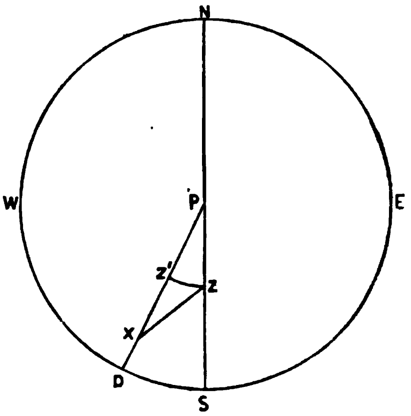


Fig. 127.

(On plane of equinoctial.)



$$\text{In triangle } ZPX, \cos ZPX = \frac{\cos ZX - \cos PX \cdot \cos PZ}{\sin PX \cdot \sin PZ}$$

$$\text{or } \cos h = \frac{\cos z - \sin l \cdot \sin d}{\cos l \cdot \cos d}, \text{ when } l \text{ and } d \text{ have same name,}$$

$$\text{or } \cos h = \frac{\cos z + \sin l \cdot \sin d}{\cos l \cdot \cos d}, \text{ when } l \text{ and } d \text{ have different names.}$$

In the first case,

$$\cos l \cdot \cos d \cdot \cos h = \cos z - \sin l \cdot \sin d,$$

$$\begin{aligned} \therefore \cos z - \sin l \cdot \sin d &= \cos l \cdot \cos d \cdot \cos h \\ &= \cos l \cdot \cos d \cdot (1 - \text{vers } h) \\ &= \cos l \cdot \cos d - \cos l \cdot \cos d \cdot \text{vers } h \end{aligned}$$

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[1] This method of investigation is apparently due to Cornelis Douwes, of Amsterdam, being published in 1754, and subsequently adapted to the tables of versines by Dr. Inman.

$$\therefore \cos z + \cos l \,.\, \cos d \,.\, \text{vers}\, h = \cos l \,.\, \cos d + \sin l \,.\, \sin d$$
$$= \cos (l \sim d)^1 = 1 - \text{vers}\, (l \sim d);$$
$$\therefore \text{vers}\, (l \sim d) = 1 - \cos z - \cos l \,.\, \cos d \,.\, \text{vers}\, h$$
$$= \text{vers}\, z - \cos l \,.\, \cos d \,.\, \text{vers}\, h$$

In the second case we shall find, in a similar manner, that

$$\text{vers}\, (l + d) = \text{vers}\, z - \cos l \,.\, \cos d \,.\, \text{vers}\, h.$$

But $l \sim d$, or $l + d$, is the meridian zenith distance of the heavenly body at some place z' in the same latitude l as the spectator at the instant when its declination was d. Denote this *meridian* zenith distance by z. Then

$$\text{vers}\, \mathrm{z} = \text{vers}\, z - \cos l \,.\, \cos d \,.\, \text{vers}\, h.$$

The meridian zenith distance being thus found, corresponding to the declination d for the time of observation, the problem is reduced to finding the latitude from the meridian altitude of a heavenly body and its declination; the rest of the steps will then be the same as given on page 233.

To simplify the formula

vers z = vers z − cos l . cos d . vers h, by adapting it to the table of versines,

Assume vers $\theta = \cos l \,.\, \cos d \,.\, \text{vers}\, h$.

$\therefore$ hav $\theta = \cos l \,.\, \cos d \,.\, \text{hav}\, h$

L hav θ = L cos l + L cos d + L hav $h - 20$

From which θ and therefore vers θ may be found.[2]

Then vers z = vers z − vers θ.

From which the meridian zenith distance z is found; and thence the latitude, for

$$\text{latitude} = \text{meridian zenith distance} \underset{\sim}{+} \text{decl.}$$

In the above expression for finding the latitude the latitude itself is involved. Now, as this quantity is only known approximately, it may be sometimes necessary to repeat one part of the operation, using the latitude last found, which will be nearer the truth than the given estimated latitude; and if the result thus determined differs much from the last one used, to make a further

---

[1] When d is greater than l the expression becomes cos $(d \sim l)$, the symbol $\sim$ meaning *difference between.*

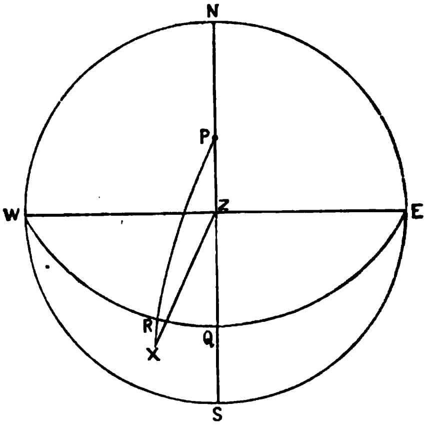
[2] The table of versines in Inman's Tables gives the values of the *natural* versines multiplied by one million; thus, the versine of 100°, as given in the table, is 1173648, but the real value of the natural versine is 1·173648. The haversine is (as implied by the name) half the versine. *See* explanation of Table 48.

calculation until the latitude deduced does not differ from the one last found: it is seldom, however, necessary to repeat the operation more than once. This will be seen in the following example, where the approximation has been carried beyond what was requisite in order to show the practical utility of the method, and that with an error of even one degree in the latitude, when the hour angle is well known, a sufficiently correct result will be obtained without a second approximation.

When the heavenly body considered is *east* of meridian, the angle at pole between the meridian and the circle of declination passing through the body is found by subtracting the hour angle from 24 hours. This angle will be termed the *angular distance from the meridian*.

Note.—The nearer to the meridian that the observation can be obtained, the more accurate will be the result, and if approximately equal altitudes can be observed on both sides of the meridian the mean result will be free from the effect of errors in the time.

Fig. 128.



Practically between the equator and lat. 30°, when the apparent time is uncertain to the extent of *one minute*, the observation should be made within 20 minutes of the time of meridian passage when lat. and decl. are of the same names, and within 30 minutes when of contrary names; between lats. 30° and 50° these limits may be extended to 40 minutes in the first case, and one hour in the second: these limits apply to heavenly bodies whose declinations do not exceed 24°.

Example

Given the altitude of a heavenly body $=27° \; 0'$, zenith north of the body, its declination $=11° \; 29' \; 8''$ S., and hour angle $=0^h \; 43^m \; 25^s$, the estimated latitude being 50° N. Construct a figure, and find by calculation the true latitude.

Let N W S E (fig. 128) represent the horizon, P the pole, Z the zenith, and X the heavenly body near the meridian. Then we have given declination X R $=11° \; 29' \; 8'' = d$, zenith dist. Z X $= 63° \; 0' = z$, hour angle Z P X $=0^h \; 43^m \; 25^s = h$, and estimated latitude $=90° -$ P Z $=50° = l$; to find the latitude correctly.

$$\text{hav } \theta = \text{hav } h \cdot \cos d \cdot \cos l,$$

$$\text{vers mer. zen. dist.} = \text{vers zen. dist.} - \text{vers } \theta,$$

$$\text{latitude} = \text{mer. zen. dist.} - \text{decl.}$$

		2nd approximation.	3rd approximation.
L cos d	9·991215		
„ hav h	7·951588		
	7·942803	7·942803	7·942803
„ cos l	9·808067	9·800853	9·800737
L hav θ	7·750870	7·743656	7·743540
∴ θ=8° 36′ 30″			
and vers θ=	11269	11084	11081
vers z=	546009	546009	546039
vers mer. zen. dist.=	534740	534925	534928
∴ mer. zen. dist.=	62° 16′ 23″ N.	62° 17′ 6″ N.	62° 17′ 6″ N.
declination=	11 29 8 S.	11 29 8 S.	11 29 8 S.
∴ latitude=	50 47 15 N.	50 47 58 N.	50 47 58 N.

As the last two results come out the same, we see that the approximation is carried far enough. When the latitude is known within a few minutes of the truth, a second approximation will seldom be necessary. Thus, in the above example, if we had supposed the estimated latitude to be 50° 28′ N., which we know from the above result is 20′ wrong, and had first worked with this, we should have found our *first* result to be 50° 47′ 40″ N., and this is only 18″ less than the truth; showing in this case that an error of 20′ in the latitude used in the calculation will not produce any important error in the latitude deduced from it.

This result arises from the fact that when near the meridian the *rate of change* of altitude of a heavenly body is not practically affected by even a considerable change of latitude (*see* page 369).

Before giving a complete example of the calculation of this problem we repeat what was said on page 225 with regard to finding the *sun's* hour angle:

To find the sun's hour angle. When observations have recently been obtained for longitude, the error of the chronometer on ship apparent time at that instant becomes known; on correcting this for the run of the ship in the interval, the error on apparent time (and therefore the sun's hour angle) becomes known at the time of taking the observation for latitude. When observations for longitude have *not* been recently obtained, to the Greenwich mean time (known by the chronometer) apply the longitude by dead reckoning in time, subtracting if west, and adding if east; the result will be ship mean time: to this apply the equation of time with its proper sign to reduce mean time into apparent time; the result will be the sun's hour angle.

Examples August 22, A.M., in latitude by account 50° 48′ N., and long. 1° 6′ W., a chronometer showed $11^h\ 50^m\ 22^s$, error on Greenwich mean time being $40{\cdot}2^s$ fast, when the observed altitude of the sun's lower limb near meridian was 50° 36′ 50″ (Z. N.), index correction + 30″, height of eye, 0. Required the latitude.

Greenwich Date and Hour Angle.

Chr. showed.........	$11^h\ 50^m\ 22^s$ A.M.	Obs. alt. L.L.	50° 36′ 50″
error, fast	40·2	index corr.....	30 +
	11 49 41·8		50 37 20
	12	corr. in alt....	42 −
Gr. date, Aug. 21...	23 49 41·8		50 36 38
long. in time.........	4 24·0 −	semid...........	15 51 +
ship mean time.....	23 45 17·8	sun's true alt.	50 52 29
equation of time....	2 39·1 −		90
ship app. time.......	23 42 38·7	sun's zen. dist.	39 7 31
	24		
∴ angular distance from meridian	= 0 17 21·3		

Sun's Decl.

Aug. 22......	11° 44′ 50″ N.	Var. in 1 hour......	30″
corr.........	8	Hours from noon...	·28
	11 44 58		8·4″

Equation of Time.

Aug. 22......	$2^m\ 39^s$ add	Var. in 1 hour.....	·628
Corr..........	·17	hours from noon...	·28
	2 39·17		5024
			1256
			·17584

		Vers...39° 7′...	0224137
L cos. decl.........	9·990803	31″	95
„ cos est. lat.	9·800737	vers zen. dist...	0224232
„ hav hour angle	7·155966	vers 3° 24′ 40″.	1770
„ hav θ	6·947506	vers mer. Z. D.	0222462
∴ θ=3° 24′ 40″		38° 57′	305
		51″	157

mer. Z. D... 38 57 51 N.
decl........... 11 44 59 N.
∴ Latitude 50 42 49 N.

33. Nov. 14, P.M., in lat. by account 57° 21′ S., and long. 1° 0′ W., a chronometer showed $0^h\ 25^m\ 27^s$, error on Greenwich mean time

being fast $5^m\ 56{\cdot}7^s$, when the observed altitude of the sun's lower limb was 50° 26′ 20″ (Z. S.), index correction − 2′ 20″, and height of eye 10 feet. Required the latitude.

Ans. Lat. 57° 16′ S.

34. June 30, A.M., in lat. by account 43° 00′ N., and long. 23° 30′ W., a chronometer showed $1^h\ 30^m\ 15^s$, error on Greenwich mean time being $7^m\ 32^s$ fast, when the observed altitude of the sun's lower limb was 69° 50′ 22″ (Z. N.), index correction + 2′ 20″, and height of eye 14 feet. Required the latitude.

Ans. Lat. 42° 56′ N.

35. July 10, A.M., in lat. by account 57° 24′ N., and long. 3° 40′ W., a chronometer showed $11^h\ 20^m\ 15^s$, error on Greenwich mean time being $30^m\ 30^s$ slow, when the observed altitude of the sun's lower limb was 54° 17′ 19″ (Z. N.), index correction − 2′ 40″, and height of eye 20 feet. Required the latitude.

Ans. Lat. 57° 25′ 25″ N.

36. May 20, A.M., in lat. by account 5° 48′ S., and long. 44° 30′ E., a chronometer showed $8^h\ 30^m\ 0^s$, error on ship apparent time at instant of observation $3^h\ 17^m\ 4^s$ slow, when the observed altitude of the sun's lower limb was 64° 3′ 50″ (Z. S.), index correction − 4′ 30″, and height of eye 20 feet. Required the latitude. *Ans.* Lat. 5° 38′ S.

37. June 16, P.M., in lat. by account 52° 25′ N., and long. 1° 6′ W., a chronometer showed $1^h\ 2^m\ 9^s$, when the observed altitude of the sun's lower limb was 60° 37′ 50″ (Z. N.), index correction − 2′ 10″, and height of eye 17 feet. At 9 A.M. observations showed the chronometer to be $0^h\ 48^m\ 17^s$ fast on ship apparent time, the run since then being S.E. (true) 40 miles. Required the latitude. *Ans.* Lat. 52° 24′ 15″ N.

Elements from 'Nautical Almanac.'

Sun's Declination.	Equation of Time.		Sun's Semi.
Nov. 14, 18° 18′ 5″ S........	$15^m\ 22{\cdot}9^s$	to be added.......	16′ 13″
Var. in 1 hour 38·77″ incr.	$\cdot 410^s$ decr.		
June 30, 23° 14′ 31″ N.	$3^m\ 3{\cdot}7^s$	„ subtracted...	15 46
Var. in 1 hour 9·45″ dec...	$\cdot 486^s$ incr.		
July 10, 22° 14′ 22″ N.	$4^m\ 59{\cdot}5^s$	„ subtracted...	15 46
Var. in 1 hour 19·34″ decr.	$\cdot 352^s$ incr.		
May 20, 20° 1′ 23″ N.	$3^m\ 44{\cdot}9^s$	„ added.........	15 50
Var. in 1 hour 30·94″ incr.	$\cdot 141^s$ decr.		
June 16, 23° 22′ 15″ N......	$0^m\ 18{\cdot}8^s$	„ subtracted...	15 46
Var. in 1 hour 4·95″ incr.	$\cdot 538^s$ incr.		

Note.—The latitude may be similarly determined from observations of stars, planets, or the moon, when near meridian, the hour angle being deduced from the formulæ on page 206, viz.:

Mean time = star's hour angle + star's R. A. − R. A. mean sun,

∴ hour angle = mean time + R. A. mean sun − R. A. of star, planet, or moon.

In the cases of the two latter the R. A. should be corrected for Greenwich date.

For the hour angle of a body near meridian *below* pole subtract 12^h from the hour angle found as if for *above* pole if this exceeds 12^h, or subtract it from 12^h if it is less. As these are less frequently observed, our examples have been taken from observations of the sun only. The effect of errors of latitude and time on observations near the meridian will be considered on p. 363.

Reduction to the Meridian.

The following method of obtaining the latitude by altitude near the meridian was given by Delambre in 1814.

Let X (fig. 129) represent a heavenly body observed at a time when near the meridian N S, the angular distance from the meridian being Z P X, or h. Let P Z represent the colatitude or $90° - l$, and P X the polar distance or $90° - d$, the zenith distance at time of observation being Z X.

From P as centre with radius P X describe an arc X m; then m will represent the position of the heavenly body when it comes on the meridian, if the change in declination is neglected.

From Z as centre with radius Z X describe an arc X O; then O will be a point on the meridian having the same altitude as X at time of observation, and m O (or x) will represent the star's change of zenith distance or altitude while moving from X to the meridian, and will therefore be the quantity or reduction to be added to the observed altitude in order to find the meridian altitude, and hence the latitude. From the spherical triangle P Z X we have

$$\begin{aligned}\cos \mathrm{ZX} &= \cos \mathrm{PZ} \cos \mathrm{PX} + \sin \mathrm{PZ} \,.\, \sin \mathrm{PX} \cos h\\ &= \sin l \sin d + \cos l \,.\, \cos d \,.\, \cos h.\end{aligned}$$

For $\cos h$ substitute its value $1 - 2\sin^2 \tfrac{1}{2}h$.

$$\begin{aligned}\cos \mathrm{ZX} &= \sin l \sin d + \cos l \,.\, \cos d - 2 \cos l \cos d \sin^2 \tfrac{1}{2}h\\ (1) \qquad &= \cos (l \underset{\sim}{+} d) - 2 \cos l \,.\, \cos d \sin^2 \tfrac{1}{2}h.\end{aligned}$$

Now $\mathrm{ZX} = \mathrm{ZO} = x + \mathrm{Z}m$

∴ $\cos \mathrm{ZX} = \cos x \cos \mathrm{Z}m - \sin x \,.\, \sin \mathrm{Z}m$

(2) $= (1 - \tfrac{1}{2}x^2)$[1] $\cos (l \underset{\sim}{+} d) - x \sin (l \underset{\sim}{+} d)$,

since x is only a small quantity and Z m (in this case) $= (l - d)$.

---

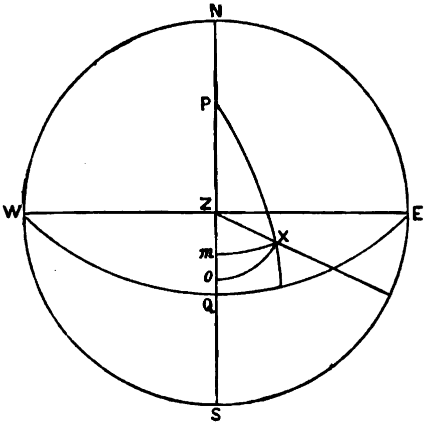
[1] More accurately, $\cos x = 1 - \frac{x^2}{1 \cdot 2} + \frac{x^4}{1 \cdot 2 \cdot 3 \cdot 4} - \text{\&c.}$

By equating the values of cos Z X as given in equations (1) and (2), we have by changing signs and reducing

$$(3)\qquad x \sin(l \underset{\sim}{+} d) + \tfrac{1}{2} x^2 \cos(l \underset{\sim}{+} d) = 2 \cos l \,.\, \cos d \sin^2 \tfrac{1}{2}h.$$

The second term of the left-hand side of this equation is small when the observation is made near the meridian, and under those circumstances may usually at sea be neglected; hence

Fig. 129.



$$x \sin(l \underset{\sim}{+} d) = 2 \cos l \,.\, \cos d \sin^2 \tfrac{1}{2}h.$$

$$x = \frac{2 \cos l \cos d}{\sin(l \underset{\sim}{+} d)} \sin^2 \tfrac{1}{2}h;$$

or, expressed in seconds,

$$x = \frac{\cos l \cos d}{\sin(l \underset{\sim}{+} d)} \cdot \frac{2 \sin^2 \frac{1}{2}h}{\sin 1''}$$

In Inman's Table No. 28 the value of $\frac{2 \sin^2 \frac{1}{2}h}{\sin 1''}$ can be found by inspection; $(l \underset{\sim}{+} d)$ is of course the meridian zenith distance, from which the latitude *at time of observation* is found.[1]

The reduction to the meridian is *additive* to the altitudes near meridian above pole and subtractive from those near meridian below pole.

The value of x evidently will *increase* as the meridian zenith distance $(l \underset{\sim}{+} d)$ *decreases*, and therefore the formula given should

---

[1] Since when h is small the arc may be substituted for the sine, the expression may be written thus:

$$x = \frac{\cos l \,.\, \cos d \sin^2 15'}{2 \sin(l \underset{\sim}{+} d) \sin 1''} h^2$$

where x is expressed in seconds of arc and h in *minutes* of time. A most comprehensive table from which the value of x deduced from this expression can be taken by inspection has been published by Staff-Commander Brent and Messrs. Walter and Williams, naval instructors. *See* also "Towson's" ex-meridian tables.

not be used when the heavenly body passes near the zenith, except when it is *very near* the meridian. Hence when the heavenly body has nearly the same declination as the observer, and cannot be observed when very nearly on the meridian, it is necessary to use the formula obtained from the more complete solution of equation (3), from which

$$x = \frac{\cos l \,.\, \cos d}{\sin (l \underset{\sim}{+} d)} \cdot \frac{2 \sin^2 \frac{1}{2}h}{\sin 1''} - \left(\frac{\cos l \,.\, \cos d}{\sin (l \underset{\sim}{+} d)}\right)^2 \cdot \frac{2 \cot (l \underset{\sim}{+} d) \sin^4 \frac{1}{2}h}{\sin 1''}$$

Practically, when the number of *minutes* in the first reduction, or left-hand term, is less than the number of minutes of time in the hour angle, Part 2, or the right-hand term, is unnecessary.

Example: In the example on page 246 worked by this method, the reduction of the observed altitude to the meridian altitude would be found as follows:

$$\log x = \text{L} \cos l + \text{L} \cos d - \text{L} \sin (l-d) + \log[1]\; 590'' - 10$$

$$\begin{array}{r} =9{\cdot}800737 \\ 9{\cdot}990803 \\ \underline{2{\cdot}770852} \\ 12{\cdot}562392 \\ \underline{9{\cdot}799339} \\ {\cdot}2{\cdot}763053 \end{array}$$

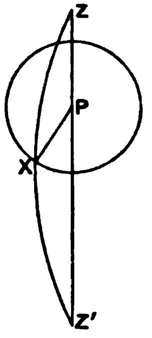
$\therefore x = 579{\cdot}5''$ or 9′ 39″.

This added to the observed altitude gives the value of the meridian altitude 51° 2′ 8″ and that of the meridian zen. dist. 38° 57′ 52″, agreeing with the result obtained before.

LATITUDE BY ALTITUDE NEAR MERIDIAN BELOW POLE.

In this case the heavenly body is referred to the *nadir*, or point opposite to the zenith, instead of the *zenith*, and the result is the meridian nadir distance of the body, which is equal to its distance from the horizon + distance of nadir from the horizon, that is, to *mer. alt.* + 90°; and *mer. alt.* + 90° − *decl.* = *lat.*

FIG. 130.



Let Z′ be the nadir, Z the zenith; then Z Z′ = 180°, and Z X Z′ = 180°, X being the body near the meridian under the pole. Let h = hour angle which the body is W. from the upper part of the meridian, that is, the angle Z P X, and h_1 = the hour angle which X is from the lower part of the meridian or angle Z′ P X; then $h_1 = 12^h - h$ or $h - 12^h$. Let $z' =$ nadir dist. or Z′ X; $z =$ zen. dist. or Z X; then $z' = 180° - z = 90° + (90° - z) = 90° +$ *alt.* Let M′ Z′ denote mer. nadir dist.; then M′ Z′ $= 90° +$ *mer. alt.*; also P Z′ $= 180° -$ P Z. In this case by trigonometry.

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[1] **Taken from Inman's Tables, No. 28; the reader will observe that Part 2 or the 'second reduction' is of the opposite sign to Part 1.**

L *hav* $\theta =$ L cos $l +$ L cos $d +$ L *hav* $h_1 - 20 =$ L cos $l +$ L cos $d +$ L *hav* $(12^h - h) - 20$. And in nat. versines . . . *vers* M′ Z′ $=$ *vers* $z' -$ *vers* θ; or *vers* $(90° +$ *mer. alt.*$) =$ *vers* $(90° +$ *alt.*$) -$ *vers* θ.

For an illustration of a computed example, *see* ‘Wrinkles on Navigation.’

Latitude by Pole-star.

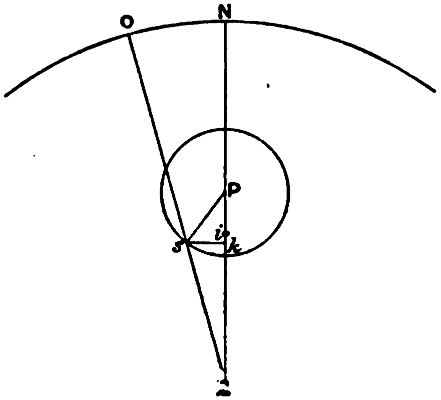
The latitude of an observer in the northern hemisphere can be found at any time of the night by applying a correction to the true altitude of the Pole-star (*Polaris*).

This correction, which may be otherwise obtained as in the case of *any* star near the meridian above or below pole, is simplified in the case of *Polaris* by its proximity to the pole.

In fig. 131 if P represent the north pole of the heavens and Z the zenith of the observer, the apparent path of *Polaris*, owing to the rotation of the earth, will be that shown as a circle described round P, with a radius equal to the polar distance of the star (p), at present (1898) 1° 14′.

Let s be the star, $s\,k$ a perpendicular drawn from the star to the meridian Z N, P s the star's polar distance $= p$, s P k its hour-angle $= h$.

Fig. 131.



In Z N conceive a point i to be taken, so that N i $=$ O $s = a$, the star's altitude.

Then—

the lat. P N $=$ N $i + i\,k -$ P k

But—

P $k'' = p''$ cos h and $s\,k'' = p''$ sin h very nearly.

$\therefore\ l = a - p''$ cos $h + i\,k$

To find an expression for $i\,k$:

In the right-angled triangle Z $s\,k$—

$$\cos z s = \cos z k \cos s k$$

And since $z i = z s$ by construction

$$\left.\begin{array}{r}\cos z i, \text{ or}\\ \cos (z k + i k)\end{array}\right\} = \cos z k \cos s k$$

Hence $\cos z k \cos i k - \sin z k \sin i k = \cos z k \cos s k.$

But $i k$ being very small,

$\cos i k = 1$ and $\sin i k = (i k)'' \sin 1''$ nearly. Also $z k = z i$ nearly.

Therefore, by subtitution—

$$\cos z i - \sin z i \, (i k)'' \sin 1'' = \cos z i \cos s k \text{ very nearly.}$$

And $\sin z i \, (i k)'' \sin 1'' = \cos z i - \cos z i \cos s k$.

Hence, dividing by $\sin z i$—

$$\begin{aligned}(i k)'' \sin 1'' &= \tan a \, (1 - \cos s k)\\ &= \tan a \sin^2 \frac{s k}{2}\\ &= \tan a \, 2 \sin^2 1'' \left(\frac{s k''}{2}\right) \text{ nearly.}\\ &= \tan a \frac{\sin^2 1''}{2} \, (p'' \sin h)^2\\ &= \tfrac{1}{2} \tan a \sin^2 1'' \, (p'' \sin h)^2\\ \text{and } i k'' &= \tfrac{1}{2} \tan a \sin 1'' \, (p'' \sin h)^2\end{aligned}$$

Hence, finally,

$$l = a - p'' \cos h + \tfrac{1}{2} \sin 1'' \, (p'' \sin h)^2 \tan a.$$

The values of $p'' \cos h$ and $\frac{1}{2} \sin 1'' \, (p'' \sin h)^2$ tan alt. are calculated for different values of h, and form Tables I. and II. of the tables given in the *Nautical Almanac* for determining latitude by Pole-star; but as these tables are calculated for a *constant* value of $p = 74'$ and the right ascension of the star $1^h \, 21^m$, an additional table is given (Table III.), depending on the difference between the true and assumed values[1] of the polar distance and right ascension; the values in this table are increased by 1′, so as to be always additive, and hence the necessity for subtracting 1′ from the true altitude before applying the corrections from the *Nautical Almanac*.

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[1] The annual *variation* of declination of *Polaris* is 19″, and that of right ascension 22^s; the tables in the *Nautical Almanac* were first inserted in 1834, and are calculated from the formulæ given by Littrow in 1822. Originally, only what is now known as the first correction was applied (given in the *Seaman's Kalendar*, 1602), but in 1810 good tables were published by Mr. J. Stevens, of the East India Company's service.

NOTE.—The value of $p \cos h$ or Table I. in the *Nautical Almanac*, applied alone to the *true* altitude of *Polaris*, will (except in latitudes greater than 60°) generally give the latitude within an error of 2′ (*see* also Inman's Tables, No. 31).

The Pole-star can be observed for latitude at any time of night in the northern hemisphere, but is most favourably situated when on or near the meridian, at which times the middle star in the tail of the Great Bear is vertically above or below *Polaris*.

On March 6, in longitude 37° W., at $7^h\ 43^m\ 35^s$ P.M. ship mean time, suppose the altitude of the Pole-star, when corrected for the error of the instrument, refraction, and dip of the horizon, to be 46° 17′ 28″. Required the latitude. **Examples**

Ship mean time	7^h	43^m	35^s
diff. long. (37° W.) in time	2	28	0
Greenwich mean time	10	11	35

Sidereal time at Greenwich mean noon	22^h	56^m	28^s
ship mean time	7	43	35
acceleration[1] for $10^h\ 12^m$		1	41
sidereal time of observation	6	41	44
True altitude	46°	17′	28″
subtract		1	0
reduced altitude	46	16	28
with *argument*[2] $6^h\ 41^m\ 44^s$, *first correction*[3]	0	12	40
approximate latitude	46	3	48
arguments, 49° 17′ $6^h\ 42^m$ } *second correction*[4]	+	0	55
arguments, March 6, $6^h\ 42^m$, *third correction*[5]	+	1	3
Latitude of the place	46	5	46 N.

38. April 10, at $1^h\ 30^m$ A.M. mean time nearly, in longitude 64° 30′ E., the observed altitude of the Pole-star was 52° 30′ 40″, index correction − 1′ 20″, and height of eye above the sea 15 feet. Required the latitude. *Ans.* 53° 36′ N.

39. June 15, at $2^h\ 20^m$ A.M. mean time nearly, in longitude 10° 20′ W., the observed altitude of Polaris was 46° 10′ 30″, index correction + 3′ 10″, and height of the eye 19 feet. Required the latitude. *Ans.* Lat. 45° 55′ N.

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[1] Taken from the *Nautical Almanac* or from Inman's Tables; the elements in this example are arranged as in that given in the *Nautical Almanac*.

[2] The *argument* of a table is the value of an element on which another element depends.

[3] By interpolation from Table I., *Naut. Alm.*

[4] By interpolation from Table II., *Naut. Alm.*

[5] By interpolation from Table III., *Naut. Alm.*

40. July 20, at $11^h\ 40^m$ P.M. mean time nearly, in longitude 42° E., the observed altitude of Polaris was 35° 30′ 40″, index correction − 4′ 10″, and height of eye above the sea 15 feet. Required the latitude. *Ans.* Lat. 35° 18′ N.

41. March 1, at $3^h\ 10^m$ A.M. mean time nearly, in longitude 16° 20′ E., the observed altitude of Polaris was 33° 27′ 30″, the index correction was − 2′ 10″, and the height of the eye above the sea was 24 feet. Required the latitude. *Ans.* Lat. 34° 36′ N.

42. August 27, at $2^h\ 40^m$ A.M. mean time nearly, in longitude 165° 45′ E., the observed altitude of Polaris was 39° 47′ 50″, the index correction was + 1′ 20″, and the height of the eye above the sea 20 feet. Required the latitude. *Ans.* Lat. 39° 1′ N.

43. September 9, at $10^h\ 47^m$ P.M. mean time nearly, in longitude 12° 30′ W., the observed altitude of Polaris was 37° 4′ 20″, the index correction was − 2′ 10″, and the height of the eye above the sea 20 feet. Required the latitude. *Ans.* Lat. 36° 6′ N.

Elements from 'Nautical Almanac.'

April 9,	Sidereal Time at Green. mean noon.........	1^h	11^m	47^s
June 14,	„ „ „	5	32	0
July 20,	„ „ „	7	54	0
Feb. 29,	„ „ „	22	34	50
Aug. 26,	„ „ „	10	20	33
Sept. 9,	„ „ „	11	15	45

LATITUDE BY DOUBLE ALTITUDE.

The methods of finding the latitude by two observations of the same heavenly body separated by a known interval of time, or by simultaneous observations of *two* heavenly bodies, is commonly, though erroneously, termed the *double* altitude. There have been many solutions given of this celebrated problem, a selection from them being found in the Appendix.

The cases now considered will be solved by what is termed the Direct Method.

First. Latitude by two altitudes of a heavenly body (usually the sun), knowing the time elapsed between the observations.

Second. Latitude by altitudes of two heavenly bodies (usually stars) observed at the same instant of time.[1]

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[1] In 1537 Pedro Nunez gave a solution of this problem (Case 1) on a *globe*, using the differences of bearing instead of the polar angle; in 1594 Robert Hues solved the problem (Case 1) by using the same elements as practised now, the means of measuring

Theory

Case 1.—Given the altitudes of the sun, or any other heavenly body, observed at two different times, and the elapsed time between the two observations: to find the latitude.

Let N W S E represent the horizon, N Z S the celestial meridian, P the pole, and Z the zenith of the spectator. Let X and Y be the places of the heavenly body when its altitudes were observed. Draw the circles of altitude Z X and Z Y, and circles of declination P X and P Y, through X and Y. Draw also X Y, the arc of a great circle.

In fig. 132 the arc X Y, drawn through the two places of the heavenly body, is supposed not to pass, when produced, between the zenith and pole, and in fig. 133 it does thus pass.

Fig. 132.

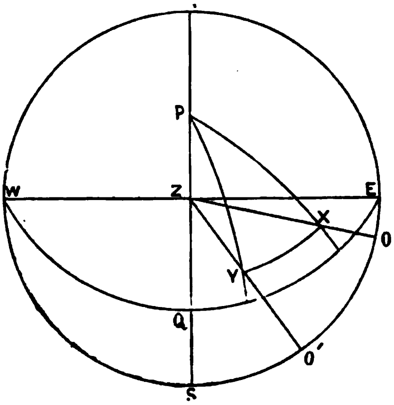
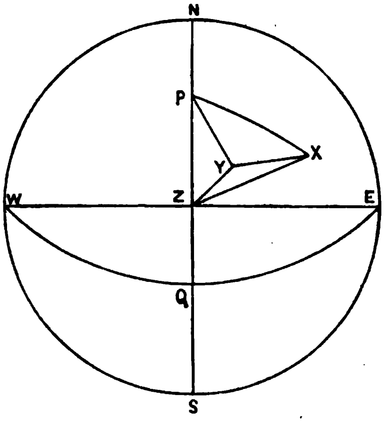


Fig. 133.



In the two spherical triangles P X Y and Z X Y, there are given the two polar distances P X and P Y (known from the *Nautical Almanac*), the angle X P Y (the interval between the observations, which will be given sufficiently accurately if noted by a chronometer whose rate is known), and the two zenith distances Z X and Z Y (from the observed altitudes); to find the colatitude P Z, and thence the latitude. The successive steps in the calculation are as follows:

(1) In the spherical triangle P X Y, are given the polar distances P X and P Y, and included angle X P Y, to calculate X Y. Call this arc 1.

---

the time between the observations having become available through the invention of watches.

Brinkley, in the Appendix to *Nautical Almanac* for 1825, gave one of the earliest applications of the problem to star observations (Case 2).

The method of solving Case 1 given in the text is due to N. F. Duillier, and was published by him in 1728.

(2) In the spherical triangle P X Y, are given the polar distances P X and P Y, and arc 1, just found, to calculate the angle P X Y. Call this angle P X Y arc 2.

(3) In the spherical triangle Z X Y, are given the zenith distances Z X and Z Y, and arc 1, to find the angle Z X Y. Call this angle Z X Y arc 3.

In the case of fig. 133 add arc 3 to arc 2, and in the case of fig. 131 subtract arc 3 from arc 2; the sum or remainder is angle P X Z, or arc 4.

(4) *Lastly.* In the triangle P X Z, are given P X and Z X, the polar dist. and zenith dist. at the position of X, and the included angle P X Z, or arc 4, to calculate P Z, the colatitude and thence Z Q, the latitude.

This rule for finding the latitude, although long, is direct, and free from any ambiguity of cases, and has also the great advantage of being a simple application of two of the common rules of Spherical Trigonometry.

Limits The following limitations of the application of the double altitude problem were given by Dr. Inman (*see* also page 374):

(1) *Both observations E. or both W. of meridian.* The difference of the two bearings measured from the elevated pole (or change of bearing) should exceed the lesser bearing, and the nearer the difference of bearings to 90° the better.

By the *greater* bearing is to be understood that nearer to the prime vertical, and by the *lesser bearing* is to be understood that which is nearer to the meridian.

(2) *One observation E. and the other W. of meridian.* The limit just given in (1) will hold good if for the greater bearing be substituted the bearing opposite thereto, as N.E. for S.W.

(3) When the bearings on different sides of the meridian are equal, *either* may be considered as the *lesser*.

(4) The nearest arc on the celestial concave drawn between the places of the body or bodies should not pass (if necessary when produced) near the zenith or celestial pole, but *distinctly* on the same side of both, or distinctly between them. This arc may be traced by the eye.

(5) The distance run by the ship in the interval of the observations should not be more than 12 leagues or 36 miles; the interval itself should not be more than 8 or 9 hours. In determining the true course and distance in the interval, care should be taken to make a proper allowance for any currents or tides by which the place of the ship may have been affected.

Example

October 11, in latitude by account 54° N. and long. 83° 15′ W. the following double altitude of the sun was observed:

Mean Time nearly.	Chro.	Alt. Sun's L. L.	Compass Bearing.
$7^h\ 45^m$ A.M.	$11^h\ 40^m\ 15^s$	9° 0′ 20″	S. 70° E.
10 35 A.M.	2 13 20	25 3 30	S. 22 E.

The run of the ship in the interval was S. by W. by compass 15 miles, index correction + 5′ 10″, and the height of eye above the sea was 18 feet. Required the true latitude at the second observation.

At Greater Bearing.		At Lesser Bearing.	
Ship, Oct. 10.........	$19^h\ 45^m$	Ship, Oct. 10.........	$22^h\ 35^m$
long. in time	5 33 W.		5 33 W.
Oct. 10	25 18	Oct. 10	28 8
Gr. Oct. 11[1]	1 18	Gr. Oct. 11	4 8
	or 1·3 hrs.		or 4·13 hrs.

Decl. at Greater Bearing.		Decl. at Lesser Bearing.	
Oct. 11............	7° 4′ 38″ S.	Oct. 11............	7° 4′ 37″ S.
corr.	1 14	corr.	3 55
	7 5 52		7 8 32
Var. in 1 hour......	56·87″ incr.		56·87″ incr.
Hours from noon...	1·3		4·13
	17061		17061
	5687		5687
	73·931 = 1′ 14″		22748
			234·8731 = 3′ 55″
N. pol. dist. ...	97° 5′ 52″ at greater bearing.	N. pol. dist. ...	97° 8′ 32″ at lesser bearing.

---

[1] The Greenwich dates are in ordinary cases more usually and accurately found from the known errors of chronometer on G.M.T. at each time of observation; but the rule in the text is retained, since it is important to observe in double altitudes that they are applicable where the Greenwich date is only *roughly* known, provided the *interval* between the observations is accurately measured by a good watch.

Sun's Altitude at Greater Bearing.

Obs. alt.	9°	0′	20″
index corr.		5	10+
	9	5	30
dip		4	11−
	9	1	19
corr. in alt.		5	33−
	8	55	46
semi.		16	3+
	9	11	49
corr. for run (*see* p. 222)		2	12+
true alt.	9	14	1
	90		
Z. D.	80	45	59

at greater bearing.

Sun's Altitude at Lesser Bearing.

Obs. alt.	25°	3′	30″
index corr.		5	10+
	25	8	40
dip................		4	11−
	25	4	29
corr. in alt.		1	54−
	25	2	35
semi...............		16	3+
true alt.	25	18	38
	90		
Z. D.	64	41	22

at lesser bearing.

To find Arc 1.

	Chro. Times.		
	11ʰ	40ᵐ	15ˢ
	14	13	20
Polar angle[1]	2	33	5

L sin pol. dist. at G. B.	9·996661
„ „ „ „ at L. B.	9·996617
„ hav pol. angle.........	9·031223
„ hav arc 1	9·024501
Arc 1[2]	37° 58′ 0″

[1] When great accuracy is required, this elapsed time should be corrected for rate of chronometer, and also for the change in the equation of time in the interval, since the interval in *apparent* time is what is required; but these corrections are seldom made. Similarly, if the same *star* is observed twice, the interval should be converted into sidereal time.

An interval of *mean* time is converted into the corresponding interval of *apparent* time by applying to it the *change* of equation of time during that interval.

Thus, an interval 4 hours mean time is corrected by applying to it 4 times the var. in 1 hour of the equation of time. If in the *Nautical Almanac* we see that the equation is *decreasing* and to be *subtracted* from mean time, or *increasing* and to be *added*, the correction thus found will be *added* to 4 hours, otherwise it will be *subtracted*; the result being the required interval expressed in apparent time.

In a star double altitude the interval shown by chronometer (mean time) would be converted into sidereal time, as explained on page 192.

[2] To find arc 1 *accurately*. To the versine of the arc found as above add the versine of the difference of polar distances; the sum will be the versine of arc 1. This is only necessary to be done in Case 2 on page 263.

To find Arc 2.

Arc 1	37° 58′ 0″	log. cosec	0·210982
pol. dist. at G. B.	97 5 52	„ cosec	0·003339
diff.	59 7 52		
pol. dist. at L. B.	97 8 32		
sum	156 16 24	½ L hav	4·990618
diff.	38 0 40	½ „ hav	4·512779
		L hav arc 2	9·717718
		Arc 2	92° 31′ 45″

To find Arc 3.

Arc 1	37° 58′ 0″	log. cosec	0·210982
Z. D. at G. B.	80 45 59	„ cosec	0·005664
diff.	42 47 59		
Z. D. at L. B.	64 41 22		
sum	107 29 21	½ L hav	4·906540
diff.	21 53 23	½ „ hav	4·278481
		L hav arc 3	9·401667
		Arc 3	60° 17′ 0″

To find Arc 4.

Arc 2	92° 31′ 45″
arc 3	60 17 0
Arc 4	32 14 45
Pol. dist. at G. B.	97° 5′ 52″
zen. dist. at G. B.	80 45 59
diff.	16 19 53

To find Arc 5.

L sin pol. dist. at G. B.	9·996661
„ sin zen. dist. at G. B.	9·994336
„ hav arc 4	8·887148
„ hav arc 5	8·878145
Arc 5	31° 54′ 15″

Vers	31° 54′ 15″	0151066
„	16 19 53	0040347
„ colat.		0191413
	∴ colat.	36° 2′ 31″
		90
Latitude		53 57 29 N.

44. June 3, in latitude by account 52° N., and long. 72° E., the following double altitude of the sun was observed:

Mean Time nearly.	Chro.	Alt. Sun's L. L.	Compass Bearing.
9h 50m A.M.	9h 52m 28s	51° 17′ 45″	SE. by S.
11 15 A.M.	11 14 29	59 32 15	S. by E.

The run of the ship in the interval was W. by S. by compass 10 miles, index correction −0′ 40″, and height of eye 12 feet. Required the true latitude at the second observation.

Ans. Arc 1, 18° 57′ 45″; arc 2, 86° 3′ 30″;
arc 3, 52° 9′ 15″; lat. 50° 48′ N.

45. April 11, in latitude by account 56° 20′ N., long. 10° 30′ E., the following double altitude of the sun was observed:

Mean Time nearly.	Chro.	Alt. Sun's L. L.	True Bearing.
11h 0m A.M.	7h 5m 10s	40° 10′ 15″	S. 23° E.
2 0 P.M.	10 6 10	35 15 40	S. 57° W.

The run of the ship in the interval was NNE. true 29 miles, index correction + 2′ 10″, and height of eye 18 feet. Required the true latitude at the second observation.

Ans. Arc 1, 44° 45′ 45″; arc 2, 86° 39′ 30″;
arc 3, 66° 12′ 15″; lat. 56° 56′ N.

46. April 13, in latitude by account 41° 20′ N., long. 156° 15′ E., the following double altitude of the sun was observed:

Mean Time nearly.	Chro.	Alt. Sun's L. L.	True Bearing.
10h 45m A.M.	7h 30m 20s	53° 0′ 20″	SE. by S.
2 45 P.M.	11 29 40	40 59 10	SW. by W.

The run of the ship in the interval was SSE. true 25 miles, index correction −5′ 20″, and height of eye 14 feet. Required the true latitude at second observation.

Ans. Arc 1, 59° 3′ 45″; arc 2, 85° 2′ 30″;
arc. 3, 43° 51′ 30″; lat. 41° 23′ 15″ N.

47. April 22, in latitude by account 50° 48′ N., and long. 148° 30′ E., the following double altitude of the sun was observed:

Mean Time nearly.	Chro.	Alt. Sun's L. L.	Compass Bearing.
10h 0m A.M.	10h 2m 25s	44° 20′ 0″	SE. by S.
11 24 A.M.	11 24 34	50 20 0	S. by E.

The run of the ship in the interval was 0, index correction +40′, and height of eye 0. Required the true latitude at second observation.

Ans. Arc 1, 20° 5′ 30″; arc 2, 87° 48′ 0″;
arc 3, 62° 23′ 30″; lat. 50° 44′ 30″ N.

48. October 15, in latitude by account 53° N., and long. 54° E., the following double altitude of the sun was observed:

Mean Time nearly.	Chro.	Alt. Sun's L. L.	True Bearing.
$11^h\ 20^m$ A.M.	$11^h\ 15^m\ 50^s$	27° 31′ 50″	S. 12° E.
1 20 P.M.	0 50 32	25 45 5	S. 22° W.

The run of the ship in the interval was S. by W. true 14 miles, index correction +2′ 55″, and height of eye above the sea 15 feet. Required the true latitude at second observation.

Ans. Arc 1, 23° 24′ 15″; arc 2, 91° 43′ 15″;
arc 3, 79° 10′ 30″; lat. 53° 17′ 45″ N.

49. October 24, in latitude by account 50° 40′ S., and long. 142° W., the following double altitude of the sun was observed:

Mean Time nearly.	Chro.	Alt. Sun's L. L.	Compass Bearing.
$10^h\ 0^m$ A.M.	$10^h\ 12^m\ 34^s$	44° 20′ 0″	SE. by S.
11 24 A.M.	11 34 34	50 20 0	S. by E.

The run of the ship in the interval was 0, index correction 0, and height of eye above the sea 0. Required the true latitude at second observation.

Ans. Arc 1, 20° 3′ 0″; arc 2, 87° 48′ 0″;
arc 3, 62° 23′ 0″; lat. 50° 45′ 30″ S.

Elements from 'Nautical Almanac.'

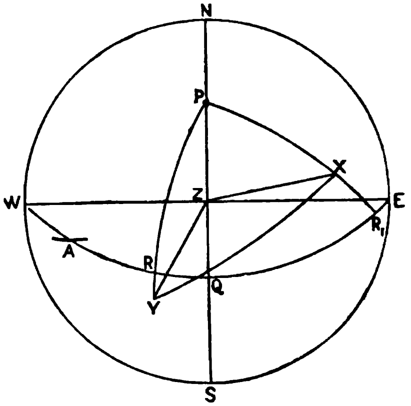
	Sun's Declination.			Sun's Semi.
June 3,	22° 16′ 34″ N. ...	Var. in 1 hour	18·05″ incr....	15′ 47″
April 11	8 11 21 N. ...	„ „	54·93 „ ...	15 58
„ 12,	8 33 22 N. ...	„ „	54·19 „ ...	15 57
„ 21,	11 44 26 N. ...	„ „	50·19 „ ...	15 56
Oct. 15,	8 40 28 S. ...	„ „	55·45 „ ...	16 4
„ 24,	11 49 32 S. ...	„ „	52·01 „ ...	16 7

CASE 2.—Given the altitudes of two heavenly bodies observed at the same time: to find the latitude.

Let N W S E represent the horizon, N Z S the celestial meridian, P the pole, Z the zenith, W Q E the equinoctial, and A the first point of Aries. Let X and Y be the places of the two heavenly bodies whose altitudes are observed. Draw the circles of altitude Z X and Z Y, and circles of declination P X and P Y, through X and Y; draw also X Y, the arc of a great circle joining the two bodies.

In the two spherical triangles P X Y and Z X Y, there are given the two polar distances P X and P Y (known from the *Nautical Almanac*), the angle X P Y, the difference of the right ascensions of X and Y (for R R′ measures the angle X P Y, and R R′ = A R′ − A R = R A of X − R A of Y), and the two zenith distances Z X and Z Y (known from the observed altitudes), to find the colatitude P Z, and thence the latitude. The successive steps of the calculation are as follows:—

FIG. 134.



(1.) In the spherical triangle P X Y, are given the polar distances P X and P Y, and the included angle X P Y, to calculate X Y, or arc 1.

2.) In the spherical triangle P X Y, are given the polar distances P X, P Y, and arc X Y, or arc 1, to calculate the angle P X Y, the angle at the heavenly body whose bearing is the greater. Call this angle arc 2.[1]

(3.) In the spherical triangle Z X Y, are given the zenith distances Z X and Z Y, and arc 1, or X Y, to find the angle Z X Q, the angle at the greater bearing. Call this angle arc 3.

(4.) When the arc X Y does not pass between the zenith and the pole, subtract arc 3 from arc 2, the remainder is angle P X Z. Call this arc 4. When the arc X Y does pass between the zenith and the pole, add arc 3 to arc 2; the sum will be angle P X Z, or arc 4.

(5.) *Lastly.* In the spherical triangle P X Z, are given P X and X Z, the polar distance and zenith distance at the greater bearing, and the included angle P X Z, or arc 4, to calculate P Z the colatitude; and thence Z Q, the latitude, is known.

Examples

January 1, in latitude by account 38° 10′ N., the following altitudes of the stars *a* Pegasi and *a* Aquilæ were taken at the same instant:

Obs. Alt. *a* Pegasi.	True Bearing.	Obs. Alt. *a* Aquilæ.	True Bearing.
29° 49′ 27″	S. 78° E.	57° 29′ 50″	S. 22° E.
In. corr. −15″		In. corr. −15″	

---

[1] Arcs 1 and 2 for 27 pairs of suitable stars can be taken by inspection from Shadwell's *Star Tables*, which also gives the rule for application of arcs 2 and 3 to find arc 4.

The height of the eye was 41 feet. Required the latitude.

At Greater Bearing. α Pegasi.		At Lesser Bearing. α Aquilæ.	
Observed alt.	29° 49′ 27″	Observed alt.	57° 29′ 50″
index correction...	15 —	index correction ...	15 —
	29 49 12		57 29 35
dip	6 18 —	dip	6 18 —
	29 42 54		57 23 17
refraction	1 42 —	refraction	0 37 —
true alt.	29 41 12	true alt.	57 22 40
	90		90
zenith distance ...	60 18 48	zenith distance ...	32 37 20
Star's declination .	14° 22′ 50″ N.	Star's declination .	8° 28′ 2″ N.
	90		90
pol. dist. at G. B.	75 37 10	pol. dist. at L. B.	81 31 58
R. A. α Pegasi ...	22^h 57^m 6^s	Pol. dist. at G. B.	75° 37′ 13″
R. A. α Aquilæ ...	19 43 15	pol. dist. at L. B.	81 31 58
polar angle	3 13 51	diff. pol. dists. ...	5 54 48

To find Arc 1.

L sin polar distance at greater bearing	9·986177
„ sin polar distance at lesser bearing	9·995241
„ haversine polar angle	9·226458
„ haversine arc A	9·207876
arc A	47° 22′ 30″
vers arc A	0322696
	107
vers difference polar distances	5319
vers arc 1	0328122
arc 1	47° 47′ 16″

To find Arc 2.

Arc 1............	47° 47′ 16″	............log. cosec	0·130382
P. D. at G. B.	75 37 10	.... „ cosec	0·013823
difference	27 49 54	.... ½ L hav S. ...	4·911662
P. D. at L. B.	81 31 58	.... ½ „ hav D. ...	4·654808
sum (S)	109 21 52	L haversine, arc 2 ...	9·710675
difference (D)	53 42 4	∴ arc 2	91° 34′ 0″

To find Arc 3.

Arc 1...............	47° 47′ 16″	............log. cosec	0·130382
zen. dist. at G. B.	60 18 48	.... „ cosec	0·061110
difference	12 31 32	.... ½ L hav S....	4·584171
zen. dist. at L. B.	32 37 20	.... ½ „ hav D...	4·241725
sum (S)............	45 8 52	L haversine, arc 3 ...	9·017388
difference (D) ...	20 5 48	∴ arc 3	37° 38′ 30″

To find Arc 4. *To find Arc* 5.

Arc 2	91° 34′ 0″	L sin pol. dis. at G. B.	9·986177
arc 3................	37 38 30	„ sin Z. D. at G. B. ...	9·938890
∴ arc 4............	53 55 30	„ haversine, arc 4	9·312977
		„ haversine, arc	9·238044
		arc	49° 9′ 15″
		vers arc	0345919
P. D. at G. B. ...	75° 37′ 10″		55
Z. D. at G. B. ...	60 18 48	...vers difference	0035443
difference	15 18 22		28
		vers colat..................	0381445
			363
		colat............51° 47′ 22″	82
		90	
		∴ latitude...38 12 38 N.	

50. September 17, in latitude by account 36° 45′ N., the following altitudes were observed at the same time:

Obs. Alt. α Orionis.	True Bearing.	Obs. Alt. α Leonis.	True Bearing.
55° 1′ 30″	SE. by E.	45° 13′ 30″	SSW.

The index correction was + 55″, and height of eye 8 feet. Required the true latitude.

Ans. Arc 1, 62° 27′ 10″; arc 2, 88° 14′ 45″;
arc 3, 38° 14′ 0″; lat. 36° 44′ N.

51. February 20, in latitude by account 36° 40′ N., the following altitudes were observed at the same time:

Obs. Alt. Sirius.	True Bearing.	Obs. Alt. Spica.	True Bearing.
27° 50′	S. 45° W.	12° 56′	S. 66° E.

The index correction was + 1′, and height of eye above sea 10 feet. Required the true latitude.

Ans. Arc 1, 96° 10′ 30″; arc 2, 108° 5′ 15″;
arc 3, 59° 39′ 15″; lat. 36° 36′ 45″ N.

52. May 1, in latitude by account 41° 20′ N., the following altitudes of stars were taken at the same instant. Required the true latitude.

True Alt. α Pegasi.	True Bearing.	True Alt. α Tauri.	True Bearing.
62° 44′	S. by E.	19° 26′ 20″	E.

Ans. Arc. 1, 79° 0′ 15″; arc 2, 78° 3′ 30″;
arc 3, 26° 55′ 0″; lat. 41° 22′ 30″ N.

53. March 2, in latitude by account 41° 20′ N., long. 60° E., the altitudes of the two following stars were observed at the same time. Required the true latitude.

True Alt. α Andromedæ.	True Bearing.	True Alt. α Tauri.	True Bearing.
73° 14′	S. 12° E.	18° 27′ 30″	E.

Ans. Arc 1, 62° 9′ 30″; arc 2, 66° 11′ 0″;
arc 3, 15° 8′ 45″; lat. 41° 23′ N.

54. January 2, in latitude by account 32° 10′ N., the following altitudes of the stars α Pegasi and α Aquilæ were observed at the same instant:

Obs. Alt. α Pegasi.	True Bearing.	Obs. Alt. α Aquilæ.	True Bearing.
22° 49′ 27″	E. by S.	57° 29′ 50″	SSE.

The index correction − 15″, and height of eye above the sea 41 feet. Required the true latitude.

Ans. Arc 1, 47° 46′ 45″; arc 2, 91° 34′ 0″;
arc 3, 31° 24′ 30″; lat. 32° 43′ N.

55. December 27, the following altitudes were observed at the same instant, in latitude by account 37° 10′ N. Required the true latitude.

True Alt. β Orionis.	True Bearing.	True Alt. α Hydræ.	True Bearing.
31° 5′ 11″	S. 56° W.	39° 47′ 33″	S. 40° E.

Ans. Arc 1, 62° 30′ 0″; arc 2, 94° 42′ 0″;
arc 3, 58° 5′ 0″; lat. 37° 13′ N.

Elements from 'Nautical Almanac.'

	Star's Right Ascension.			Star's Declination.			
α Leonis	10^h	0^m	$5{\cdot}8^s$	12°	43′	24″	N.
α Orionis	5	46	47·9	7	22	23	N.
Spica	13	17	7·4	10	21	30	S.
Sirius	6	38	23·8	16	30	52	S.
α Tauri	4	27	3·0	16	11	30	N.
α Pegasi	22	57	4·0	14	22	24	N.
α Andromedæ	0	0	23·4	28	14	11	N.
α Pegasi	22	57	8·5	14	23	8	N.
α Aquilæ	19	43	18·2	8	28	12	N.
β Orionis	5	7	15·5	8	23	5	S.
α Hydræ	9	20	8·2	8	0	15	S.

The limits for favourable observation of a pair of stars are similar to those given for the sun on page 256, and a list of suitable couplets is given in Shadwell's 'Star Tables.'

In selecting a couplet of stars or a star and planet, for simultaneous observation, those should be avoided which at their meridian passage pass very near the zenith, or which pass the meridian the one north and the other south of the zenith.

The heavenly bodies selected ought to pass the meridian either *both* between the zenith and the elevated pole, or *both* on the equatorial side of the zenith.

When there is only *one* observer, the altitude of the body farthest from the meridian should be taken first, then that of the other body, and then again that of the first observed, the altitude of this at the times of observing that nearest the meridian being obtained by proportion, the times of observation being carefully noted.

If it is proposed to find the *time* from either observation, one of the stars should be on or near the prime vertical, or in the case of the sun the observation employed which is farthest from the meridian; the hour angle or angular distance from the meridian is readily found from the triangle P Z X (fig. 134), in which $\sin \text{Z P X} = \frac{\sin \text{Z X}}{\cos \text{lat}} \sin \text{arc } 4$. It will also be remarked that a double altitude can be calculated from an observation of a star or planet at twilight combined with that of the sun at some preceding or following date, the polar angle being the difference of their right ascensions at the instant of taking the second observation.

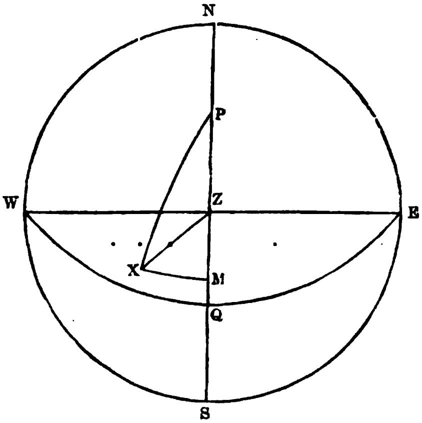
Note.—It appears unnecessary to investigate any further problems depending upon the direct solution of double altitudes, such as those given by Ivory in 1821 and Riddle in 1822, since in practice the methods given in Chapter XIV. are so much more commonly employed.

APPENDIX.

As an appendix to the chapter on latitude, we give some methods of less frequent use than those already described, but which it appears desirable to retain as being of occasional service.

Latitude by Altitude near Meridian, above Pole, without using estimated Latitude.

Fig. 135.



A *direct* method of finding the latitude from an observation near the meridian, in which the latitude by account is not required, may be obtained from the following investigation :[1]

Given the altitude of a heavenly body off the meridian above the pole, its hour angle and declination ; to find the latitude.

Let N W S E represent the horizon, N Z S the celestial meridian, and X a heavenly body near the meridian, P the pole, and Z the zenith.

Let hour angle Z P X $= h$,
polar distance P X $= p$,
zenith distance Z X $= 90° - a$,
a being the altitude.

From X drop a perpendicular X M upon the meridian.
Let P M $= x$, Z M $= y$, and X M $= s$.

Then the colatitude P Z $= x - y$, when the perpendicular X M does not fall between P and Z, as in the figure; and colatitude P Z $= x + y$, when the perpendicular falls between P and Z : this will be seen by constructing a figure. (The position of the perpendicular X M with respect to the points P and Z may in most cases be easily determined by the observer when the altitude is taken.)

1. To find P M or x. In the right-angled triangle P M X,

$$\cos h = \cot p \,.\, \tan x, \ \therefore \ \tan x = \tan \mathrm{p} \,.\, \cos h \ \ldots\ldots \ (1)$$

2. To find Z M or y. In the right-angled triangle X P M, $\cos p = \cos s \,.\, \cos x$.

In the right-angled triangle Z X M, $\cos$ Z X, or $\sin a = \cos y \,.\, \cos s$.

Dividing one equation by the other, to eliminate $\cos s$, we have $\frac{\cos s}{\sin a} = \frac{\cos x}{\cos y}$,

$$\therefore \ \cos y \,.\, \cos p = \cos x \,.\, \sin a,$$
$$\text{or } \cos y = \sec p \,.\, \cos x \,.\, \sin a \ \ldots\ldots \ (2)$$

Formulæ (1) and (2) will determine x and y, and thence the colatitude $= x \pm y$ is known.

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[1] This method was given in Robertson's *Navigation*, 1796.

In finding the latitude by formulæ (1) and (2) it will be necessary to attend to the magnitude of the polar distance; for tan x will be positive or negative according as the polar distance is less or greater than 90°; but the value of tan x, and also that of cos y, are readily found in the usual manner by putting the proper signs, + or −, over the given quantities.

When the declination is small, or p nearly equal 90°, the tan and sec must be taken out *accurately* from the logarithm tables.

This method is of course equally applicable to a heavenly body near meridian *below* pole, and when the *time* is well known may be used in either case when the body is even at a considerable distance from the meridian.

When the declination (N. or S.) is *less* than 1°, and the hour angle or the angular distance from the meridian *less* than 30 minutes, the following simple rule is applicable:

In fig. 135, under the supposed circumstances, P M = P X (very approximately), and X M = h (very approximately);

$$\therefore \cos \text{Z X} = \cos \text{X M} \cos \text{Z M},$$
$$\text{or } \sin a = \cos h \cos \text{Z M};$$
$$\therefore \cos \text{Z M} = \sin a \,.\, \sec h.$$

From this the value of Z M is found, and on applying to that the declination the latitude is determined.

EXAMPLE.—On Dec. 8, at sea, the following observation was taken near the meridian. Time by watch, $11^h\ 20^m\ 23^s$; corresponding true altitude of the sun, 29° 26′ 45″; comparison with chronometer (Z − W), $0^h\ 8^m\ 7^s$; chron. slow on G. M. T., $0^h\ 10^m\ 32^s$; approximate latitude, 37° N.; longitude, 7° 30′ E.

Time by watch, W	$23^h\ 20^m\ 23^s$	Eq. time add to M. T.	$7^m\ 46^s$	d 22° 46′ 3″ S.
				p 112 46 3
Z − W	8 7	tan x = tan p cos h and cos y = sin a cos x sec p		
Chron. Z	23 28 30			
Error	10 32		10·377067	9·691612
G. M. T.	23 39 2		9·998832	9·588665
Longitude	30 0 E.		10·375899	10·412297
L. M. T.	24 9 2	$x=$	112° 49′ 15″	9·692574
Eq. time	7 46	$y=$	60 29 0	60° 29′ 0″
Hour-angle, h	0 16 48 W.	$x-y=$	52 20 15	
		True latitude	37 39 45 N.	

A method by which in the process of calculation the azimuth is obtained as well as the latitude is as follows (*see* fig. 135, page 267).

In spherical triangle P Z X.

$$\text{Sin az} = \frac{\text{Sin } h \,.\, \text{Sin } p}{\text{Sin z}} \quad \ldots . \ (1)$$

In right-angled spherical triangle P X M.

$$\tan \text{P M} = \cos h \,.\, \tan p \quad \ldots . \ (2)$$

In right-angled spherical triangle Z X M.

$$\tan \text{Z M} = \cos \text{az.}\ \tan \text{z} \quad \ldots . \ (3)$$

Thence $\text{latitude} = 90 - (\text{P M} - \text{Z M})$

DOUWES'S METHOD OF DOUBLE ALTITUDES.

The following method of obtaining the latitude from a double altitude of the sun was proposed in 1754 by Cornelis Douwes, examiner of sea officers and pilots under the College of Admiralty at Amsterdam, and was formerly much used at sea.[1]

In fig. 136, drawn on the plane of the horizon, let P and Z represent the pole and zenith, and S_1 and S_2 two positions of the sun, whose zenith distances z_1 and z_2 have been obtained at an interval apart represented by the hour angle S_1PS_2, which has been measured by a watch.

Let the unknown angular distances from the meridian ZPS_1, ZPS_2, be represented by h_1, h_2, and let it be assumed that the polar distance PS_1 does not change during the time of observation.

$$\text{In triangle } PZS_1 \quad \cos z_1 = \cos PZ \cos PS_1 + \sin PZ \sin PS_1 \cos h_1 \quad (1)$$

$$PZS_2 \quad \cos z_2 = \cos PZ \cos PS_1 + \sin PZ \sin PS_1 \cos h_2 \quad (2)$$

$$\therefore \cos z_1 - \cos z_2 = \sin PZ \sin PS_1 (\cos h_1 - \cos h_2)$$

$$2 \sin \tfrac{1}{2}(z + z_1) \sin \tfrac{1}{2}(z - z_1) = 2 \sin PZ \sin PS_1 \sin \tfrac{1}{2}(h + h_2) \sin \tfrac{1}{2}(h - h_2)$$

$$\therefore \sin \frac{h_1 + h_2}{2} = \frac{\sin \tfrac{1}{2}(z + z_1) \sin \tfrac{1}{2}(z - z_1)}{\sin PZ \sin PS_1 \sin \tfrac{1}{2}(h_1 - h_2)}.$$

From the original equation (2) we have—

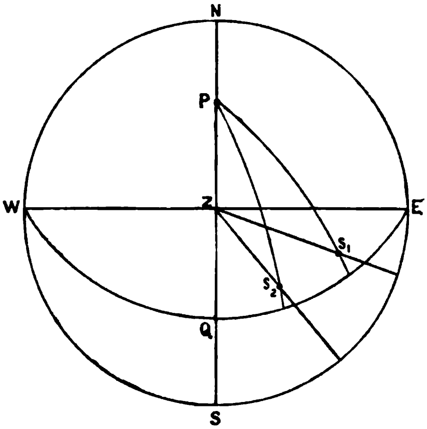
$$\cos z_2 = \cos PZ \cos PS_2 + \sin PZ \sin PS_2 \cos h_2$$

$$= \cos (PS_2 - PZ) - 2 \sin PS_2 \sin PZ \sin^2 \frac{h_2}{2}$$

$$\therefore \cos (PS_2 - PZ) = \cos z_2 + 2 \sin PS_2 \sin PZ \sin^2 \frac{h_2}{2}$$

From this, with an assumed value of P Z, the value of (p S_2 − P Z), and hence of P Z, can be determined.

FIG. 136.



Should the resulting latitude differ much from that assumed, the problem must be reworked with that new latitude.

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[1] Mr. Douwes received *50l.* from the Commissioners of Longitude for his solution of this problem, which long continued in use in the Royal Navy, although the direct solution had been published much earlier (1728).

As this was the first, and for many years the only, method of solving the double altitude problem used at sea, it is retained in this book from its historical interest, though long since superseded in practice by better methods.

Douwes's method is only applicable where the *same* heavenly body has been observed twice, and is much simplified by the use of special tables given in the Requisite Tables of 1771.

The above method with auxiliary tables was published in the *Nautical Almanac* of 1771, and the following were given as the limitations of application:—

'1. The operation is the same whether the sun has north or south declination and also whether the ship be in north or south latitude.

'2. The observations must always be taken between nine o'clock in the morning and three in the afternoon; and the nearer the greater altitude is to noon, the better.

'3. If both observations are in the forenoon, the interval must not be much less than half the distance of the first observation from noon.

'4. If both observations are in the afternoon, the interval between them must not be much less than the distance of the first observation from noon.

'5. If one observation be in the forenoon, and the other in the afternoon, the interval must not exceed 4½ hours.

'6. The above limitations are founded on a supposition that the sun's meridional zenith distance is not less than the latitude of the place; but if the latitude of the place should be double the sun's meridional zenith distance, the first of two altitudes taken in the forenoon must not be before half-past nine, nor the second before three-quarters past ten. The first of two taken in the afternoon must not be later than a quarter past one, nor the second after half-past two. If one be taken in the morning and the other in the afternoon, that in the morning must not be taken before half-past nine o'clock, and the interval between them must not exceed 3½ hours.

'7. If the latitude of the place be three times the sun's meridional zenith distance, the first of two observations taken in the forenoon must not be before ten o'clock, nor the second before eleven. The first of two taken in the afternoon must not be later than one o'clock, nor the second after two. If one observation be taken in the forenoon, and the other in the afternoon, that in the morning must not be before ten, and the interval between them must not exceed 3 hours.

'8. If the latitude be five times the sun's meridional zenith distance, the first of two observations taken in the forenoon must not be before half-past ten o'clock, nor the second before a quarter after eleven. The first of two taken in the afternoon must not be later than three-quarters past twelve, nor the second later than half-past one o'clock. If one be taken in the forenoon, and the other in the afternoon, the morning one must not be before half-past ten, and the interval between them must not exceed 2¼ hours.

'9. If the latitude be twelve times the sun's meridional zenith distance, the first of two observations taken in the forenoon must not be before eleven o'clock, nor the second before half-past eleven. The first of two taken in the afternoon must not be after half-past twelve, nor the second after one o'clock. If one be in the forenoon, and the other in the afternoon, the morning one must not be before eleven o'clock, and the interval between them not more than an hour and a half.

'10. If the preceding remarks be attended to, the latitude found by the calculation will be at least five times nearer the truth than the latitude by account; that is, the error in the computed latitude will not be above a fourth part of the difference between them: and hence a judgment may be formed whether it will be necessary to repeat the computation with the latitude last found or not.'

Latitude by Three Altitudes at Equal Intervals apart.

The problem of finding the latitude from three observations of the sun near the meridian when the apparent time is unknown received much attention in the eighteenth century; the only case of any practical use is that where the three altitudes are separated by *equal* intervals of time.

The following simple solution is that given by Chauvenet,[1] and depends on the fact that, since near the meridian altitudes *vary* as the squares of the hour angles or angular distances from the meridian (*see* page 371), the meridian altitude will *equal* either of the observed altitudes plus the square of the hour angle multiplied by some constant quantity k.[2]

Thus, if h, h', and h'' are three altitudes of the sun observed near the meridian at equal intervals of time x apart, then if τ represents the unknown hour angle of the middle observation the hour angles of the first and last will be $\tau - x$ and $\tau + x$, and if a is the meridian altitude

$$a = h + k(\tau - x)^2$$
$$a = h' + k\,\tau^2$$
$$a = h'' + k\,(\tau + x)^2.$$

Subtracting half the sum of the first and third equations from the second, we find

$$k\,x^2 = h' - \tfrac{1}{2}\,(h + h'').$$

The difference of the first and third equations gives

$$k\,\tau = \frac{\frac{1}{4}(h \sim h'')}{x}$$

$$\therefore\ k\,\tau^2 = \frac{[\frac{1}{4}\,(h \sim h'')]^2}{k\,x^2} = \frac{[\frac{1}{4}\,(h \sim h'')]^2}{h' - \frac{1}{2}\,(h + h'')}$$

This value of $k\,\tau^2$, which may be substituted in the second equation, gives a, or the required meridian altitude.

This method can also be conveniently applied to observations of the stars and planets under favourable circumstances.

Example.—On Nov. 25, at $8^h\ 48^m$ A.M. mean time nearly, the following true alts. of the planet Venus, near the meridian, were observed at equal intervals respectively of $7^m\ 42^s$, to determine the latitude.

True Alt. Venus.
h 77° 46′ 20″
h' 77 6 20
h'' 76 11 40

Planet's dec. 4° 52′ 34″ S.

Here $h \sim h'' = 1°\ 34'\ 40''$

$= 94'{\cdot}67$

$\frac{1}{4}\,(h \sim h'') = 23{\cdot}67$

$$\begin{array}{rrrr} \frac{h+h''}{2} & 76° & 59' & 0'' \\ h' & 77 & 6 & 20 \\ \hline h' - \frac{h+h''}{2} = & & 7 & 20 \\ \text{Or} & & 7'{\cdot}\,33 & \end{array}$$

$$\text{Mer. alt.} = h' + \frac{\{\frac{1}{4}\,(h \sim h'')\}^2}{h' - \frac{h+h''}{2}}$$
$$= h' + \frac{(23{\cdot}67)^2}{7{\cdot}33}$$
$$= h' + \frac{560{\cdot}27}{7{\cdot}33}$$
$$= h' + 76'{\cdot}43$$
$$= 77°\ 6'\ 20'' + 1°\ 16'\ 26''$$
$$= 78°\ 22'\ 46''$$

Whence in the usual manner we obtain,—

Mer. Z D	= 11°	37′	14″	N.
Decl.	= 4	52	34	S.
latitude	6	44	40	N.

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[1] *Spherical and Practical Astronomy*, vol. i.; it was also explained by La Caille in 1760.

[2] k represents $\dfrac{\cos l \cos d \sin^2 15'}{2 \sin (l \pm d) \sin 1''}$ (*see* page 249).

NOTE.—It is only necessary to remark further, that the observations in this method are not restricted to the same side of the meridian—the only requisite condition being the equality of the intervals between them.

Latitude by Combined Altitudes.

The following method of obtaining the latitude on the principle of the star double altitude, in which the elapsed (sidereal) time is equal to the difference of right ascensions of the two stars, was originally given in Lynn's *Navigation*, 1825, and with the aid of Shadwell's 'Star Tables' affords a short and simple mode of obtaining the latitude.

In figs. 137 and 138 let P and Z represent the pole and zenith, W and E being two stars having a small difference of right ascension. Let P W, P E represent portions of two circles of declination, and Z W, Z E, Z E_1 portions of circles of altitude.

If the altitude of W, the western star, is observed, and after a sidereal interval equal to their difference of right ascension the altitude of the eastern star E is observed, E will evidently at that instant, owing to the rotation of the earth, occupy a position E_1 on the *same* circle of declination as that which passed through the original position of W.

FIG. 137.

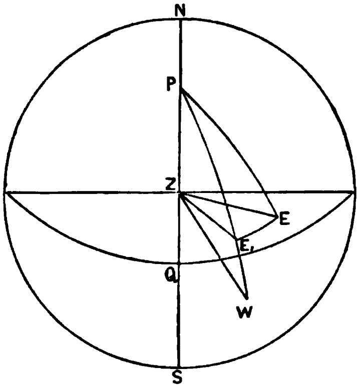
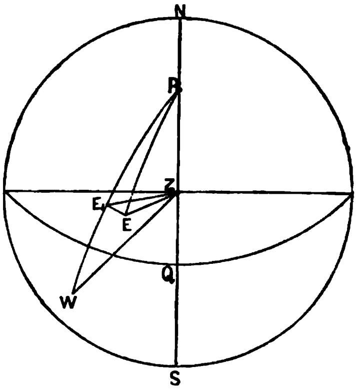


FIG. 138.



Then in the triangle Z E_1 W are known the three sides, viz. the two zenith distances Z E_1, Z W, and E_1 W (the difference of declination of the stars), from which the angle E_1, W Z can be found.

Again, in the triangle P Z W the two sides P W, Z W are known, being the polar distance and zenith distance of the star W, and the included angle P W Z having been found as above, the colatitude P Z can be determined.

The accuracy of this method of observing star double altitudes is greatest when the difference of bearing of the two stars is as nearly 90° as possible, a condition which may be approximated to by selecting stars which have a small difference of right ascension and a considerable difference of declination.

In order that errors of altitude may affect the resulting latitude least it is desirable that, when practicable, one of the stars should have a low altitude near the meridian, and the other a high altitude near the prime vertical; the altitude of the star first observed requires, of course, the usual correction for run in the interval.

The calculation is simplified by a table given in Shadwell's 'Star Tables.'

Latitude by Change of Altitude near the Prime Vertical.

On page 370 it will be shown that on or near the prime vertical the change of altitude of a heavenly body varies as the change in the hour angle or change in alt. = 15 change in hour angle × sin azimuth × cos lat.

$$\therefore \cos \text{lat.} = \frac{\text{change in alt.}}{15 \,.\, \text{change in time}} \text{ cosec azimuth.}$$

When very near the prime vertical, or within a few minutes of it, cosec azimuth may be considered to equal unity, and this formula gives a method of obtaining the latitude without any reference to the *Nautical Almanac.*

Thus, if during an interval of $3^{m}\ 31^{s}$ the altitude when very near the prime vertical changes 31′ 30″,

$$\cos \text{lat.} = \frac{1890''}{15 \times 211} \therefore \text{lat.} = 53^\circ\ 20'.$$

The best results would be obtained with only one setting of the sextant by observing on such occasions how long it takes for the sun to rise or fall its own diameter. This method can only at sea be considered a very rough one.

CHAPTER X.

Chronometers—Principle of balance—Longitude by chronometer—Sun chronometer—Star chronometer—Lunar observations—Sun lunars—Star lunars—Lunars by calculated altitudes—Longitude by equal altitudes.

Chronometer

A CHRONOMETER may be considered as a watch of superior construction, having a special mechanism fitted to it, by the action of which the rate of losing or gaining is made approximately uniform at all temperatures. Could this be *accurately* constructed the indications of such a chronometer when once set to Greenwich time and afterwards corrected for accumulated rate would always show the exact time at Greenwich independent of the position of the ship.[1]

As in ordinary watches, the moving power of a chronometer is a powerful coiled steel spring, which in seeking to uncoil imparts a circular motion to the drum in which it is placed. This circular movement is transferred to the hour, minute, and second hands by means of toothed wheels, the motions of which are regulated by the escapement, whose movement only allows one tooth of the wheel against which it presses to escape at a time.

Balance

The movement of the escapement is regulated by the balance and spiral spring (hair spring), which latter acts the part of a pendulum to a clock; one end of the spiral spring being fixed, its elasticity causes it, when drawn to one side of its position of equilibrium, to spring back, causing each time a movement of the escapement which enables the wheel against which it presses to move over the space of one tooth; the spiral spring is attached to the balance wheel and imparts to it its own motion; the balance wheel moves the escapement.

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[1] Watches were known in 1530, when it was proposed by Gemma Frisius to employ them for measuring differences of longitude; their great variations with changes of temperature prevented this being carried out until the invention of the compensation balance by Harrison in 1735.

An idea of the uncertainty at that time (1735) with regard to a ship's position at sea may be formed from the terms of the reward originally offered by the Government in 1714 for the discovery of the best way of ascertaining longitude. A sum of 20,000*l.* was offered for any method which would give the longitude at sea within 30 miles, 15,000*l.* if within 40 miles, and 10,000*l.* if true within 60 miles.

Harrison's watch (that for which he ultimately received the full award) lost 1 min. 54·5 secs. between November 6, 1761, and April 2, 1762, during an experimental voyage from England to the West Indies and back; this watch is preserved in good working order at the Royal Observatory, Greenwich.

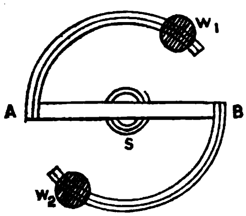
The reader should refer to a very interesting paper on the 'Ship's Chronometer' in the Journal of the U.S. Institution, Feb. 1890.

Compensation

In an ordinary watch no provision is made for the changes in the spiral spring caused by temperature; thus in a warm place the spring is not so elastic as it is at a mean temperature, and therefore it rebounds more slowly, and hence the wheelwork moves more slowly and the watch *loses*, while the reverse of this occurs in a cold place; to obviate this change of rate the compensated balance was invented by Harrison in 1735 and modified to its present form by Le Roy, the principle involved being that when the spring was less elastic it should be given less work to do, and *vice versâ*.

In fig. 139, S represents the spiral spring attached to the balance A B; this instead of forming a complete wheel made of one metal, as in ordinary watches, is made of two different metallic strips[1] joined together, each forming a nearly semicircular arc and carrying upon it a small weight W_1 W_2.

Fig. 139.



The metals are selected from those which expand and contract at different known rates, and are so proportioned that when, owing to an increase of temperature, the spiral spring becomes *less* elastic, the arms of the balance bend inwards (owing to the unequal expansions of the metals), the weights W_1, W_2 are brought nearer the centre, and thus the weakened spiral spring has less work to do in moving backwards and forwards the attached balance, and therefore the rate at which it moves the escapement is the same as at first: in a similar way when, owing to a decrease of temperature, the spiral spring becomes *more* elastic, the arms of the balance by bending outwards carry the attached weights farther from the centre, thereby increasing the work performed by the spring in moving them to and fro, thus again equalising the rate of movement of the escapement.

The adjustment of the position of these weights is a matter of great delicacy, but as the chronometers supplied to H. M. ships are very carefully selected and tested at the Royal Observatory, Greenwich, they are for the practical purposes of navigation considered to be perfectly adjusted for *all* temperatures, and usually no corrections are applied for changes of climate.[1]

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[1] Usually brass and steel, the steel strip being *inside*.

Box chronometers

Every large vessel is supplied from the Royal Observatory with three equally good large (or box) chronometers,[2] some being constructed to run two days others eight days. The former are wound *daily* at the same hour and the latter usually *weekly*, the winding being gently continued until the key cannot be turned any more.

These chronometers are never disturbed from their selected positions in the chronometer room.

The object of supplying ships with *three* chronometers is that by comparing day by day the *differences* between the comparisons of each of the three pairs a good idea can be formed of their respective performances. Thus, with three chronometers, Z, A, and B, the quantities Z—A, Z—B and A—B (representing the indications of these chronometers at the same instant) are recorded every day in the Chronometer Journal. The differences between these quantities on successive days (second differences) should always be equal to one another if each chronometer of the pair is steadily maintaining its rate, and from the regularity or otherwise of these second differences it can be seen which of the three is working best. It is unnecessary to *directly* observe A—B, since (Z—B) – (Z—A) = A—B.

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[1] Tests for chronometers supplied to the Royal Navy were first established at the Royal Observatory by an Admiralty order of June 26, 1821, when the Government offered to purchase the best of each year for 300*l.* and the next best for 200*l.*, provided at least ten were sent for competition; these rewards have been discontinued since 1835. For the first few years the chronometers were subjected to no change of temperature beyond that experienced in an ordinary room; but they are now, during this period of trial (six months), subjected to a heat test by being placed in a chamber heated to 95° F., those being rejected whose weekly rate *varies* more than ten seconds.

A chronometer under trial in 1886 at the mean open air temperature of 50° F. had a mean *weekly* rate of 1·6 seconds losing; and the same chronometer, when tested at a mean temperature of 92° F., had a mean *weekly* rate of 2·9 seconds losing.

In practice, therefore, in the Royal Navy no correction to rate for change of temperature is applied unless experience shows that the compensation is not so perfect as intended. In the mercantile marine, whose chronometers are generally tested at the Observatory at Bidston, Liverpool, the plan is adopted as originally proposed by Pierre Le Roy in 1754 of finding the rate for each 5° change of temperature; for a description of the method employed the reader is referred to *Wrinkles in Navigation*, and as an illustration of the success attending this mode of treatment we append the following extract from the *Monthly Notices of R.A.S.* for February 1886:

'During 1885 264 chronometers were tested at the Liverpool Observatory, and from a discussion of the records of performance of the chronometers of the Pacific Steam Navigation Company during twenty-six voyages from Liverpool to Valparaiso and back, by giving equal weight to each of the three chronometers carried by each ship, the mean error of longitude for an average voyage of 101 days was under three miles.'

[2] Flagships have five.

The practical benefit of a system of daily comparisons is that a guard may be kept on the steadiness of the rates. For instance, take the case of the chronometer marked A; the record gives the daily values of Z—A. If the daily rates happened to be the same, Z—A would remain a constant quantity; if, as is the general rule, the rates were different, Z—A would vary by a quantity equal to the algebraic difference of their rates; if the rates remained uniform, the second difference would remain constant; therefore, if the second difference should not remain uniform, it would show the rate of Z, of A, or of both was changing, and in case there is a third chronometer it may be useful in pointing out which one of the other two was going astray.

If, then, on examination of the records of the daily differences Z—A and A—B, it be observed that the second difference of the daily value of A—B remained constant, while that of Z—A was irregular, we may safely infer that chronometers A and B are going steadily, and that Z is altering; if, on the other hand, we perceived that the second differences of Z—A and A—B were both irregular, while that of Z—B remained steady, it would indicate that A was the culprit, and that Z and B retained their original rates. The performances of any other chronometers could be similarly investigated.

Of course, with but three, a direct comparison can be readily made between A and B; but in case of a larger number the system of indirect comparison is absolutely essential.

The following will give the reader an idea of suitable forms for Chronometer Journals:

Form No. I.

Chronometer Journal for Record of Daily Comparison of Chronometers.

Date.	Bar.	Therm.	Chron. A.	2nd Diff.	Chron. B.	2nd Diff.	Chron. C.	2nd Diff.	Initials of Comp.	Remarks.
Sun. Jan. 1st	in. 29·91	79°	h. m. s. z 2 15 0 10 50 7 3 24 53		h. m. s. 2 15 30 7 20 1 6 55 29		h. m. s. 2 16 0 1 11 54 1 4 6		C. S. W. B.	Heavy gale, much motion.
Mon. 2nd	30·32	75	z 2 39 0 11 14 5·3 3 24 54·7	s 1·7	2 39 30 7 43 48·8 6 55 41·2	s 12·2	2 40 0 1 35 55·5 1 4 4·5	s 1·5	C. S. G. C.	Confused cross sea.

FORM NO. II.

Chronometer Journal for fair copy of comparisons, if required.

Date	Z–A	2nd Diff.	Z–B	2nd Diff.	Z–C	2nd Diff.	Bar. and Therm.	Remarks
Jan.	h. m. s.		h. m. s.		h. m. s.		in.	
Sun. 1	3 24 53		6 55 29		1 4 6		29·91 79°	Heavy gale, much motion.
		s 1·7		s 12·2		s 1·5		
Mon. 2	3 24 54·7		6 55 41·2		1 4 4·5		30·32 74	Confused cross sea.
		0·3		10·8		0·8		
Tues. 3	3 24 55		6 55 52		1 4 3·7		30·41 70	Less motion.
		0·8		10·0		1·0		
Wed. 4	3 24 55·8		6 56 2		1 4 2·7		30·34 73	Fine weather.

It must be noted that, for the purposes of comparison, it is best to consider the standard z *fast* of all other chronometers, no matter what may be the actual indications; and also that all the chronometers may be treated in a uniform manner, considered either all *fast* or all *slow* of local or Greenwich mean time, regard then being had to the algebraic signs of the quantities.

Since, as has been already observed, change of temperature is supposed generally to be the principal cause of marked changes of rate, it is desirable that the temperature of the chronometer-box be duly recorded, so that if decided fluctuations of rate are observable in any of the chronometers, their cause may be sought among the records of the changes of temperature. A thermometer should therefore be placed within the chronometer-box, and its reading be recorded in the Chronometer Journal at the time of winding. The height of the barometer may also with propriety be noted at the same time.

Hack watch

The times of making all observations (except meridian altitudes of the heavenly bodies other than the moon) are noted by a smaller portable chronometer termed a hack watch, which is usually constructed to beat 5 times in 2 seconds (box chronometers generally beat half-seconds), and which, being more liable to change from being carried about than the box chronometers, is compared with their indications at as near the time of observation as possible.

A list of the rates of the chronometers as obtained while at the Royal Observatory is supplied with the instruments, and after placing them in the chronometer room on board and setting them going, every opportunity is taken of determining their daily errors and rates in harbour and at sea.

The methods of doing this will be described in Chapter XII.

Longitude and time

On page 180, when explaining the connection between Greenwich and local *time*, it was shown that this depended entirely on the relative positions of the meridians passing through Greenwich and through the place of an observer.

The longitude of a place may, therefore, conversely be defined as the angle at the pole, expressed in degrees or hours (mean solar or sidereal), included between the meridian of the place and that of Greenwich; this angle is measured in both directions from that latter meridian.

The *difference* of longitude between two places is the angle at the pole included between their meridians. This is equivalent to defining difference of longitude as difference of the right ascensions of the celestial meridians of two places or the difference of corresponding local times.

If, then, by astronomical or other means we can in any position on the globe determine at the same instant both the ship time and the Greenwich time, the difference between them, expressed in hours and fractions of an hour, usually of mean solar time, will be the longitude in time, or the same quantity multiplied by 15 will give the longitude in arc.

Greenwich mean time

The Greenwich mean time under all ordinary circumstances is known at any instant at sea by observing the time shown by the chronometer, and applying to that the original error on Greenwich mean time, together with any error which may have accumulated since that original error was ascertained.[1]

The Greenwich mean time may also be known from *astronomical* observations if the observer at any part of the world knows the Greenwich mean time at which some phenomenon occurs which is also visible to him.

Such phenomena are eclipses of the sun, eclipses of Jupiter's satellites, occultations or eclipses of stars by the moon, and lunar distances, or angular distances of the moon from the sun and certain stars and planets.

The first three of these astronomical methods have not hitherto been of any practical utility at sea, owing to the difficulty arising from the motion of the ship preventing such observations being accurately made with a telescope; they are, however, of great use in determining positions on shore, and are fully described in works on Practical Astronomy.[2]

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[1] The mean of *all* the box chronometers is at first employed, but when time has shown any one superior to the others that is selected as the standard.

[2] As an illustration of the results attainable by shore observations of eclipses of

The last method (lunar distances), affording the only mode at sea of obtaining Greenwich mean time independently of a chronometer, will be described after the more ordinary method of finding the longitude has been illustrated.

Ship mean time

The ship mean time (which is compared with the results of either of the two methods of obtaining Greenwich mean time) is found by observations of some suitably situated heavenly body, from which its hour angle can be calculated.

The theory of the methods by which such hour angles are obtained and by which they are converted into mean time has been fully explained in the chapter on Hour Angles, but for more convenient reference we give the following detailed and consecutive proof of the various parts of the important problem known as the 'Sun Chronometer.'

NOTE.—Observations of hour angles for the determination of longitude are most accurately made when the heavenly body employed is on the prime vertical, or, in the event of its not crossing that circle, either (1) when the angle at the body between the pole and zenith is 90°, and it is therefore not moving in azimuth, or (2) when it attains an altitude of not less than 8° or 10° (*see* pp. 365, 366) and Inman's Tables, Nos. 37, 38.

When the declination of the heavenly body observed does not fall within the limits of these tables, the *time* before or after that of meridian passage when it is on the prime vertical is found by adding L cot lat. to L tan decl., the sum (rejecting ten) being L cos hour angle; the true altitude when on that great circle being found by adding to the L sin decl. the L cosec lat., the sum (rejecting ten) being the L sin true alt. required.

When circumstances do not admit of making observations on the prime vertical, fairly good results may be obtained between the equator and lat. 30° when the bearing of the body observed is not within 30° of the meridian, and between lats. 30° and 50° when it is not within 40°.

LONGITUDE BY SUN CHRONOMETER.

The rule for finding the longitude by means of an altitude of the sun and the time at Greenwich determined by means of a chronometer involves proofs of the following:

1. The investigation of a rule for finding the apparent time.
2. Proof that mean time = apparent time ± equation of time.
3. Proof that longitude in time equals the difference between Greenwich mean time and mean time at place—west if Greenwich mean time is the greater, east if Greenwich time is the less.

the sun the following may be of interest: In 1766 Captain Cook determined the longitude of Eclipse Island (one of a small group off the south coast of Newfoundland) to be 57° 36′ 30″ W., and in 1874 the same spot by telegraphic determinations was found to be in 57° 36′ 52″ W.

1. Let N W S E represent the horizon (figs. 140, 141); N Z S the celestial meridian, and W Z E the prime vertical.

Let X be the place of the heavenly body. Draw the circle of declination P X R, and the circle of altitude Z X.

Let declination$=$R X$=d$; zenith distance$=$Z X$=z$; latitude $=$Z Q$=l$; and let Z P X$=h$.

From spherical trigonometry we have

$$\text{Hav Z P X}=\sqrt{\frac{\text{hav (Z X}+\text{P X}\sim\text{P Z)}\ .\ \text{hav (Z X}-\text{P X}\sim\text{P Z).}}{\sin\ .\ \text{P X}\ .\ \sin\ .\ \text{P Z.}}}$$

Now P X$\sim$P Z$=l\pm d$.

$+$ when latitude and declination are of opposite names.

$\sim$ when latitude and declination are of same name.

Fig. 140.

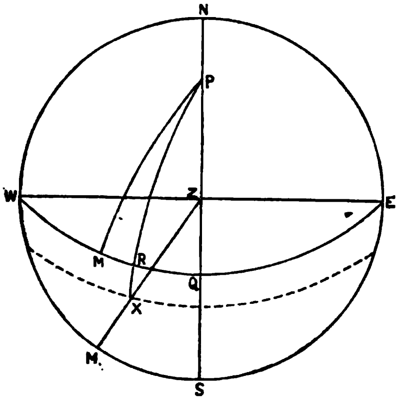
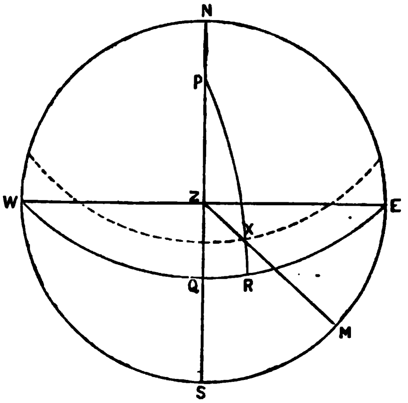


Fig. 141.



Therefore

$$\text{Hav } h=\sqrt{\frac{\text{hav }(z+l\pm d)\ .\ \text{hav }(z-l\pm d).}{\cos d\ .\ \cos l.}}$$

Or

$$\text{Hav h}=\sec d\ .\ \sec l\ .\ \sqrt{\text{hav }(z+l\pm d)\ .\ \text{hav }(z-l\pm d).}$$

But when the sun is west of the meridian, apparent time$=h$.
And when the sun is east of the meridian, apparent time$=24^{\text{h}}-h$.
Also hav $h=$hav $(24^{\text{h}}-h)$.

Hence in all cases

Hav apparent time$=$

$$\sec d\ .\ \sec l\ \sqrt{\text{hav }\{z+(l\pm d)\}\ \text{hav }\{z-(l\pm d)\}}\ \text{in logs.}$$

L hav apparent time$=$
L sec $d+$L sec $l+\frac{1}{2}$ L hav $\{z+(l\pm d)\}+\frac{1}{2}$ L hav $\{z-(l\pm d)\}-20$.

Hence the rule:

Put down the latitude and under it the declination. Take the difference if the names be alike, but the sum if unlike. Under the result put the zenith distance, and find the sum, also the difference.

Add together L sec lat., L sec dec., half the L hav of the sum and half the L hav of the difference. Reject 20, and we get the L hav of the apparent time to be taken out at the top of the page if the sun be west of the meridian, but at the bottom of the page if the sun be east of the meridian.

If using the alternative formula on page 204 the rule will be: Take (S), half the sum of altitude, latitude, and polar distance. From this subtract the altitude $(S-a)$. Add together L sec lat., L cosec polar dist., L cos S and L sin $(S-a)$; reject 20, and half the sum will be L sin half the apparent time.

Fig. 142.

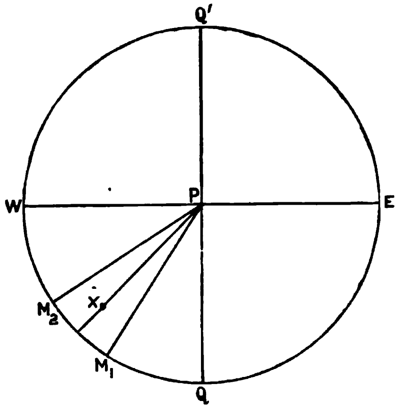
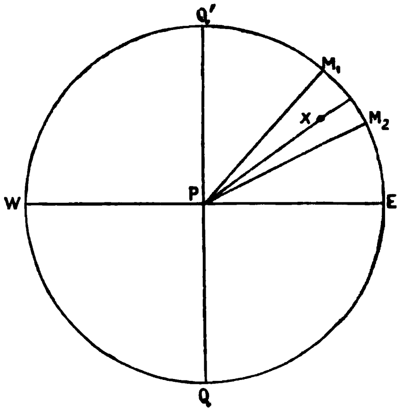


Fig. 143.



2. To prove that mean time = apparent time ± equation of time.

In fig. 142, 143, let E Q W Q′ represent the celestial equator.

P „ the pole.

Q P Q′ „ the celestial meridian.

Let X be the place of the true sun,

M_1, M_2 be two positions of the mean sun.

Then

QPM_1 (westward) $= QPX$ (westward) $- XPM_1$

i.e. mean time = apparent time − equation of time,

or

QPM_2 (westward) $= QPX$ (westward) $+ XPM_2$

i.e. mean time = apparent time + equation of time.

3. To prove that longitude in time equals the difference between Greenwich mean time and mean time at place. West if Greenwich time is the greater, East if Greenwich time is the less.

In fig. 144, let P G P′, P A P′, P B P′ be meridians through Greenwich, and through two places A and B, the former in west longitude, the latter in east longitude.

Let P M P′ be the declination circle through the mean sun.

Then G P M = Greenwich mean time, A P M = mean time at A, B R M = mean time at B, G P A = longitude of A (west), G P B = longitude of B (east). Then

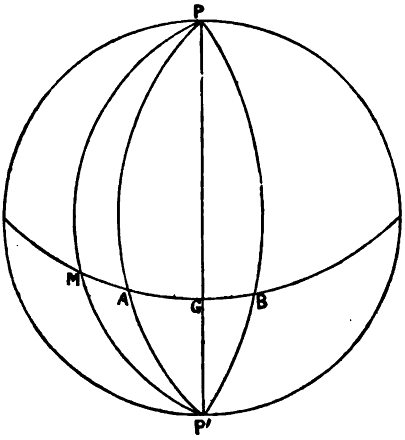
G P A = G P M − A P M,

or longitude of A (west) = Greenwich mean time − mean time at A; and

G P B = B P M − G P M,

or longitude of B (east) = mean time at B − Greenwich mean time.

Fig. 144.



Example. Sun west of meridian

Sept. 23, at $4^h\ 42^m$ P.M. mean time nearly, in latitude 50° 30′ N., and longitude by account 110° 0′ W., when a chronometer showed $11^h\ 59^m\ 30^s$, the observed altitude of the sun's lower limb[1] was 11° 0′ 50″, the index correction −3′ 20″, and the height of the eye above the sea 20 feet. Required the longitude. On August 21 at Greenwich mean noon the chronometer was fast on Greenwich mean time $0^m\ 45{\cdot}5^s$, and its daily rate was $5^s{\cdot}7$ losing.

		Sun's declination.	Eq. of Time	Semi.
Ship, Sept. 23	4^h 42^m	Sept. 23 0° 6′ 57″ S.	Sept. 23 7^m $41{\cdot}8^s$ sub. from A T	15′ 58′
long. in time	7 20 W.	corr. . . 11 43	corr. . . 10·4	
Gr., Sept. 23	. 12 2	0 18 40	7 52·2	
		Var. in 1 hour 58·47″	$0{\cdot}868^s$	
Interval of time		Hours since noon 12·03	12·03	
from Aug. 21		17541	2604	
to Gr. date . 33^d 12^h		11694	1736	
		5847	868	
		703·3941 or 11′ 43″	10·44204	

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[1] Usually the mean of three or five observations.

Daily rate		5·7^{s} losing
		33
		171
		171
		188·1
12^{h}	½	2·8
	60)	190·9
accumulated loss ...		3^{m} 10·9
chron. showed		11^{h} 59 30·0
		12 2 40·9
original error		0 45·5 fast.

Obs. alt.	11°	0′	50″
index corr.		3	20 –
	10	57	30
dip		4	24 –
	10	53	6
corr. in alt.		4	45 –
	10	48	21
semi.		15	58 +
	11	4	19
	90		
∴ zenith distance	78	55	41

∴ Gr., Sept. 23 ... 12 1 55·4 or 12·03 hrs.

Latitude	50°	30′	0″ N.	log. sec.	0·196490
declination	0	18	40 S.	,, sec.	0·000006
	50	48	40	½ L. hav (S)	4·956810
zenith distance ...	78	55	41	½ ,, hav (D)	4·385445
sum	126	44	21 (S)	L. hav	9·538751
difference	28	7	1 (D)		

ship app. time	4^{h}	48^{m}	7^{s}
equation of time		7	52·2 –
ship mean time	4	40	14·8
Greenwich mean time	12	1	55·4
longitude in time	7	21	40·6
∴ LONGITUDE	110°	25′	9″ W.

If using the alternate method given on page 204 the work will be as follows:

Alt.	11°	4′	19″		
Lat.	50	30	00	sec ...	0·196490
P. D.	90	18	40	cosec	0·000006
2)	151	52	59		
S =	75	56	29	cos. ...	9·385442
S – a =	64	52	10	sin. ...	9·956813
				2)	19·538751
				sin. $\frac{h}{2}$ =	9·769375
				∴ $\frac{h}{2}$ =	2^{h} 24^{m} 3·5^{s}
				and h as before =	4^{h} 48^{m} 7^{s}

Example. Sun east of meridian

April 18, at 9^{h} 18^{m} A.M. mean time nearly, in latitude 50° 48′ N., and longitude by account 1° 0′ W., when a chronometer showed 9^{h} 27^{m} 48^{s}, the observed altitude of the sun's lower limb was 38° 15′ 15″, index correction 3′ 40′ –, and the height of the eye above the sea 20 feet. Required the longitude. On April 1, at Greenwich mean noon, the chronometer was fast 1^{m} 58·7^{s} on Greenwich mean time, and its mean daily rate was 11·2^{s} gaining.

		Sun's Declination.		Eq. of Time.	Semi.
Ship, April 17	21^h 18^m	April 18 ...	10° 57′ 47″ N.	4 0^m $45{\cdot}1^s$ sub.	15′ 56″
long. in time	4 W.	corr.	2 19	1·5	
Gr., April 17	21 22		10 55 28	0 43·6	
		Var. in 1 hour	52·68″	$0{\cdot}582^s$	
Interval from		Hours from noon	2·64	2·64	
Ap. 1 at noon			21072	2328	
to Greenwich			31608	3492	
date	16^d $21\frac{1}{2}^h$		10536	1164	
or	$16{\cdot}89^d$		139·0752	1·53648	

Daily rate	$11{\cdot}2^s$ gaining	Obs. alt.	38° 15′ 15″
No. of days	16·89	index corr.	8 40 −
	3378		38 11 35
	1689	dip	4 24 −
	1689		38 7 11
	189·168 sec.	corr. in alt.	1 48 −
	or 3^m $9{\cdot}2^s$		38 5 23
accum. gain ...	3^m $9{\cdot}2^s$	semi.	15 56 +
chro. showed ...	9 27 48·0	true alt	38 21 19
	9 24 38·8		90
original error ...	1 58·7 fast.	∴ zenith distance	51 38 41
	9 22 40·1 A.M.		
	12		
Gr., April 17 ...	21 22 40·1 or 21·36 hrs.		

Latitude	50° 48′ 0″N.	... log. sec	0·199263 .	+15
declination	10 55 28 N.	... „ sec	0·007943	
	39 52 32	½ L. hav (S)	4·855173 .	+061
zenith distance ...	51 38 41	½ „ hav (D)	4·010890 .	−613
sum	91 31 13 (S)	L. hav	9·073269 .	−397 = 4·6 sec
difference	11 46 9 (D)			

ship apparent time	21^h 19^m 0^s
equation of time	43·6 −
ship mean time	21 18 16·4
Greenwich mean time	21 22 40·1
longitude in time	4 23·7
LONGITUDE	1° 6′ W.

NOTE.—As a safeguard in calculating the hour angle by this method, it may be observed that the zenith distance is *always* greater than the line above it.

The longitude thus found is that corresponding to the time of observation, and the latitude employed in the calculation is generally that obtained by dead reckoning from some previous determination, and may be considerably in error. This error is ascertained when observations are subsequently taken at noon, and the chronometer may be reworked with the correct latitude.

In practice, however, the error in the hour angle (and therefore in longitude), arising from an error in latitude of *one mile more*, is found by taking by inspection from the tables the corresponding changes in the logarithms, and writing them as shown in the preceding example, whence it is seen that the above observations worked with latitude 50° 49′ would give an hour angle of 4·6 sec. more, and therefore in this case a longitude 1¼′ *less* than is obtained by working with latitude 50° 48′; that is, an error of 1 mile of latitude occasions in this case an error of 1¼′ of longitude of an opposite sign, more latitude giving less longitude and *vice versâ* (*see* footnote, page 368).

The longitude at time of sights (1° 6′ W.) is corrected to noon for the ship's run in the interval, and then if it is found from observations of the sun that the supposed latitude is 5 miles different from the truth a correction of 5 × 1¼′ is applied with the proper sign to the supposed longitude at noon.[1]

Examples

1. Sept. 25, at $4^h\ 20^m$ P.M. mean time nearly, in latitude 59° 30′ N., and longitude by account 112° 30′ W., when a chronometer showed $11^h\ 44^m\ 20^s$, the observed altitude of the sun's lower limb was 10° 50′ 10″, the index correction + 5′ 10″, and height of eye above the sea 18 feet. Required the longitude. On Sept. 20, at noon G. M. T., the chronometer was fast on Greenwich mean time $0^m\ 30{\cdot}7^s$, and its daily rate was $10{\cdot}5^s$ losing.

Ans. 112° 33′ W.

2. May 30, at $3^h\ 10^m$ P.M. mean time nearly, in latitude 30° 12′ 0″ S., and longitude by account 156° 0′ E., the observed altitude of the sun's lower limb was 21° 8′ 40″, when a chronometer showed $4^h\ 44^m\ 56^s$, the index correction − 1′ 10″, and the height of eye above the sea 30 feet. Required the longitude. On May 19, at noon G. M. T., the chronometer was fast $5^m\ 16^s$ on Greenwich mean time, and its daily rate was $3{\cdot}5^s$ gaining.

Ans. 156° 23′ E.

3. July 8, at $1^h\ 40^m$ P.M. mean time nearly, in latitude 50° 48′ N., and longitude by account 1° 1′ W., the observed altitude of the sun's lower limb was 55° 2′ 10″, the chronometer showed $1^h\ 44^m\ 14^s$, the index correction + 1′ 25″, and the height of eye above the sea 24 feet. Required the longitude. On July 1, at noon

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[1] This correction may be obtained by inspection from a table published by Mr. A. C. Johnson, R.N., or from Table 35 of Inman's Tables; the earliest of such useful tables is probably that by Lieut. L. G. Heath, R.N., given in the *Nautical Magazine* for 1846.

Some observers take out the difference of hour angle occasioned by *ten* or more miles difference of latitude, and allow by proportion for the actual error when found. The correction may also be found by substitution in the formula:

Error in hour angle = error in lat. × cot azimuth × sec lat. (*see* page 367).

G. M. T., the chronometer was slow on Greenwich mean time 8^m $18{\cdot}4^s$, and its daily rate was $3{\cdot}5^s$ losing. *Ans.* 1° 7′ 30″ W.

4. January 20, at 6^h 40^m A.M. mean time nearly, in latitude 56° 20′ S., and longitude by account 83° 10′ W., when a chronometer showed 0^h 14^m 50^s, the observed altitude of the sun's lower limb was 20° 20′ 30″, the index correction − 1′ 30″, and the height of the eye above the sea 20 feet. Required the longitude. On Jan. 2, at noon G. M. T., the chronometer was fast on Greenwich mean time 5^m 20^s, and on Jan. 6, at noon G. M. T., it was fast 4^m 52^s, from which may be found its mean daily rate. *Ans.* 83° 5′ 30″ W.

5. Feb. 10, at 7^h 50^m A.M. mean time nearly, in latitude 50° 48′ N., and longitude by account 170° 30′ E., when a chronometer showed 9^h 59^m 25^s, the observed altitude of the sun's lower limb was 51° 9′ 10″, the index correction − 3′ 20″, and the height of eye above the sea 16 feet. On Jan. 31, at Greenwich noon, the chronometer was fast 1^h 34^m 43^s, and its daily rate was $20{\cdot}6^s$ losing. The run from time of sights to noon was S.E. (true) 45 miles, when the D. R. lat. was found to be 5′ north of that by observation. Required the longitude at noon. *Ans.* 171° 26′ E.

6. May 14, at 7^h 20^m A.M. mean time nearly, in lat. 50° 50′ N., and longitude by account 4° 10′ E., when a chronometer showed 7^h 0^m 20^s, the observed altitude of the sun's lower limb was 27° 20′ 10″, the index correction − 7′ 20″, and the height of the eye above the sea 19 feet. On April 24, at noon G. M. T., the chronometer was slow on Greenwich mean time 1^m $10{\cdot}5^s$, and its daily rate was $5{\cdot}7^s$ losing. The run from time of sights to noon was N. 35° W. (true) 51 miles, when observation showed the ship to be 6′ north of her reckoning. Find the longitude at noon. *Ans.* 3° 18′ E.

Elements from 'Nautical Almanac.'

Sun's Declination.		Equation of Time.	Semi.
Sept. 25	0° 53′ 47″ S.	8^m $23{\cdot}0^s$ sub.	15′ 59″
Var. in 1 hour	58·49″ increasing	Var. in 1 hour	$0{\cdot}857^s$ increasing
May 29	21° 38′ 43″ N.	2^m $56{\cdot}4^s$ sub.	15′ 47″
Var. in 1 hour	23·10″ increasing	Var. in 1 hour	$0{\cdot}320^s$ decreasing
July 8	22° 29′ 9″ N.	4^m $40{\cdot}9^s$ add	15′ 45″
Var. in 1 hour	17·15″ decreasing	Var. in 1 hour	$0{\cdot}388^s$ increasing
Jan. 20	20° 8′ 22″ S.	11^m $19{\cdot}2^s$ add	16′ 16″
Var. in 1 hour	32·74″ decreasing	Var. in 1 hour	$0{\cdot}728^s$ increasing
Feb. 9	14° 41′ 33″ S.	14^m $31{\cdot}0^s$ add	16′ 13″
Var. in 1 hour	48·29″ decreasing	Var. in 1 hour	$0{\cdot}052^s$ increasing
May 13	18° 19′ 29″ N.	3^m $52{\cdot}4^s$ sub.	15′ 51″
Var. in 1 hour	36·87″ increasing	Var. in 1 hour	$0{\cdot}026^s$ increasing

LONGITUDE BY STAR CHRONOMETER.

Example. Star west of meridian

Sept. 10, at 7^h 15^m P.M. mean time nearly, in latitude 48° 20′ N., and longitude by account 32° E., when a chronometer showed 5^h 1^m 28^s, the observed altitude of *a* Böotis (Arcturus) W. of meridian was 31° 5′ 40″, the index correction −4′ 10″, and height of eye above the sea 20 feet. Required the longitude. On Aug. 25, at noon of G. M. T., the chronometer was slow on Greenwich mean time 2^m 40^s, and its daily rate was $4{\cdot}3^s$ gaining.

Ship, Sept. 10 ...	7^h	15^m	
long. in time ...	2	8 E.	
Gr., Sept. 10 ...	5	7	
star's R. A.	14^h	8^m	35^s
star's decl. .	19°	59′	44″ N.
R. A. Mean Sun.			
Sep. 10 sid. time at Gr. mean noon	11^h	18^m	$28{\cdot}15^s$
corr. for 5^h			49·28
3^m .			0·49
R. A. mean sun at time of observ.	11	19	17·92

Daily rate . .	$4{\cdot}3^s$ gaining.		
interval 16^d 5^h.	16		
	258		
	43		
	68·8		
4 ... $\frac{1}{8}$	0·7		
1 ... $\frac{1}{4}$	0·2		
60	69·7		
accum. rate		1^m	$9{\cdot}7^s$
chro. shows	5	1	28·0
	5	0	18·3
orig. error .		2	40·0 slow.
Gr. M. T. .	5	2	58·3

Obs. alt. ...	31°	5′	40″
in corr. ...		4	10−
	31	1	30
dip		4	24−
	30	57	6
ref.		1	37−
	30	55	29
	90		
zen. dist. .	59	4	31

Latitude	48°	20′	0″ N.	...
declination	19	59	44 N.	...
	28	20	16	
zenith distance .	59	4	31	
sum	87	24	47 (S)	
diff.	30	44	15 (D)	

log. sec	0·177312
„ sec	0·027003
$\frac{1}{2}$ L. hav (S) ...	4·839453
$\frac{1}{2}$ „ hav (D) ...	4·423295
L. hav	9·467063

∴ hour angle ...	4^h	22^m	15^s
star's R. A.	14	8	35
sidereal time	18	30	50
R. A. mean sun .	11	19	18
ship mean time .	7	11	32[1]
Gr. mean time .	5	2	58
long. in time	2	8	34
∴ long.	32°	8′	30″ E.

Example. Star east of meridian

May 24, at 11^h 11^m P.M. mean time nearly, in latitude 50° 48′ N., and longitude by account 1° 0′ W., when a chronometer showed 11^h 12^m $11{\cdot}8^s$, the observed altitude of *a* Lyræ (Vega) E. of meridian was 54° 51′ 30″, the index correction −3′ 46″, and height of eye above the sea 25 feet. Required the longitude. On May 14, at Greenwich mean noon, the chronometer was slow 1^m $15{\cdot}8^s$, and its mean daily rate was $7{\cdot}4^s$ losing.

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[1] Mean time at ship may also be found as follows:

Ship sidereal time (S. S. T.) = star's hour angle + star's R. A.;

Greenwich sidereal time (G. S .T.) = ship sidereal time ± longitude in time.

Then ship mean time = G. M. T. of preceding transit of first point of Aries − retardation for G. S. D. + S. S. T.

Ship, May 24. …	11^h	11^m	
long. in time …		4	
Gr., May 24 …	11	15	
star's R. A.	18^h	31^m	42^s
star's decl. .	38°	38′	24″ N.

Daily rate ………			$7{\cdot}4^s$
interval, 10^d 11^h 15^m			10
			74·0
8^h is ⅓			2·5
2 „ ¼			0·6
1 „ ½			0·3
15^m „ ¼			0·1
		60)	77·5
accum. rate .		1^m	$17{\cdot}5^s$
chro. showed	11^h	12	11·8
	11	13	29·3
original error		1	15·8
G.M. T. ……	11	14	45·1

Obs. alt. …	54°	51′	30″
index corr.		3	45 −
	54	47	45
dip ………		4	55 −
	54	42	50
ref. ………			45 −
	54	42	5
	90		
zen. dist. …	35	17	55

Right Ascension Mean Sun.

May 24 sid. time at Gr. mean noon	4^h	8^m	$43{\cdot}56^s$
Corr. for 11^h…		1	48·42
„ 14^m…			2·30
„ 45^s…			0·13
	4	10	34.41

Latitude …………	50°	48′	0·0″	N.	… log. sec ………	0·199263
declination ………	38	38	24	N.	… „ sec ………	0·107300
	12	9	36		½ L. hav (S) …	4·604673
zenith distance …	35	17	55		½ „ hav (D) …	4·302209
sum ………………	47	27	31	(S)	L. hav ………	9·213445
difference …………	23	8	19	(D)		

Hour angle ……………………	20^h	49^m	13^s
star's right ascension …………	18	31	42
sidereal time ……………………	39	20	55
right ascension mean sun ……	4	10	34
ship mean time ………………	11	10	21
Greenwich mean time ………	11	14	45
longitude in time ………………		4	24 = 1° 6′ W.

7. Aug. 20, at 0^h 30^m A.M. mean time nearly, in lat. 50° 20′ N., and long. by account 142° 0′ E., when a chronometer showed 2^h 41^m 12^s, the observed altitude of the star *a* Aquilæ (Altair) was 36° 59′ 50″ west of the meridian, the index correction + 6′ 30″, and height of eye above the sea 20 feet. Required the longitude. On August 1, at noon G. M. T., the chronometer was slow on Greenwich mean time 17^m 45^s, and it daily rate was $4{\cdot}3^s$ losing.

Ans. 142° 14′ 15″ E.

8. Sept. 10, at 4^h 21^m A.M. mean time nearly, in lat. 40° 36′ N., and longitude by account 73° E., when a chronometer showed 11^h 21^m 56^s, the observed altitude of *β* Geminorum (Pollux) was 39° 0′ 10″ east of meridian, the index correction − 4′ 10″, and height of eye above the sea 20 feet. Required the longitude. On Aug. 20, at noon G. M. T., the chronometer was slow on

Greenwich mean time 3^{m} $19·9^{s}$, and its daily rate was $9·3^{s}$ gaining. *Ans.* 72° 41′ 15″ E.

9. January 16, at 8^{h} 0^{m} P.M. mean time nearly, in lat. 49° 56′ 50″ N., and longitude by account 90° 30′ E., when a chronometer showed 2^{h} 24^{m} 30^{s}, the observed altitude of *α* Leonis (Regulus) was 8° 4′ 20″ east of meridian, the index correction −4′ 20″, and height of eye above the sea 25 feet. Required the longitude. On January 1, at noon G. M. T., the chronometer was fast on Greenwich mean time 5^{m} $30·5^{s}$, and its daily rate was $5·5^{s}$ losing.

Ans. 86° 5′ E.

10. January 20, at 8^{h} 30^{m} P.M. mean time nearly, in latitude 50° 48′ N., and long. by account 7° 10′ E., when a chronometer showed 8^{h} 32^{m} 50^{s}, the observed altitude of *ε* Leonis was 28° 0′ 10″ east of meridian, the index correction −6′ 20″, and height of eye above the sea 20 feet. Required the longitude. On January 2, at noon G. M. T., the chronometer was fast on Greenwich mean time 30^{m} 30^{s}, and its mean daily rate was $15·5^{s}$ losing.

Ans. 7° 19′ E.

Elements from 'Nautical Almanac.'

	Sidereal Time at Gr. Mean Noon.	Star's Right Ascen.	Star's Decln.
Aug. 19	9^{h} 50^{m} $46·5^{s}$	19^{h} 43^{m} $17·0^{s}$	9° 28′ 7″ N.
Sept. 9	11 14 31·6	7 35 48·6	28 23 40 N.
Jan. 16	19 43 7·2	10 0 8·8	12 43 6 N.
Jan. 20	19 57 55·9	9 37 8·3	24 28 33 N.

Remarks

The longitude is obtained by *planet* or *moon* chronometer in precisely the same way as that by star chronometer, except that the declination and right ascension are corrected for the Greenwich date. Moon, planet, or star chronometers, when taken by day or at twilight, are considered as trustworthy as those calculated from sun observations, but the rapid changes in position of the moon necessitate careful correction of the declination and right ascension, the former at times changing 2¼′ and the latter 26 seconds in ten minutes of time.

Unless the rate of chronometer is *large* it is usual to consider the chronometer fast or slow on G. M. T. at *ship mean noon*, and to consider from one noon to the next as a *whole* day of 24 hours, thus disregarding change of longitude in the interval.

LONGITUDE BY LUNAR OBSERVATIONS.

The moon in its motion round the earth is seen continually to change its distance from the heavenly bodies lying near its path. The angular distances of some of these (9 fixed stars, 4 planets, and the sun) have been computed and recorded in the *Nautical Almanac* for intervals of every three hours—namely, at III.$^{\text{h}}$, VI.$^{\text{h}}$, IX.$^{\text{h}}$, &c., Greenwich mean time.[1] The quantities thus inserted in the *Nautical Almanac* are the *true* distances; that is, the angular distances as supposed to be measured from the *centre* of the earth and unaffected by refraction. An observer at any place on the *surface* of the earth can also measure the angular distance between the moon and one of these stars by means of a sextant; but the distance so observed is the *apparent* distance, and before it can be compared with the true distance in the *Nautical Almanac*, it must be corrected for the effects of parallax and refraction, so as to obtain the true distance between the two bodies at the moment of observation.

The formula about to be investigated will enable us to find the true distance from the apparent or observed distance. This is technically termed *clearing the distance.* If the true distance thus computed is exactly the same as that inserted in the *Nautical Almanac* for some given hour at Greenwich, we have found *Greenwich mean time* when the observation was taken. But this coincidence must rarely happen; the computed true distance will generally come out between two of the recorded distances: to obtain the Greenwich mean time corresponding to that distance a simple proportion is generally only necessary, as will be seen hereafter.

The *mean time at place* at the instant the observed distance is taken can be readily found in the same manner as in the rule for finding the longitude by chronometer (page 284)—namely, by observing the altitude of a heavenly body [2] and computing its hour angle, and thence ship mean time.

The method of finding the longitude by a lunar observation, therefore, divides itself into two distinct parts.

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[1] The nine lunar distance stars are α Arietis, α Tauri (*Aldebaran*), β Geminorum (*Pollux*), α Leonis (*Regulus*), α Virginis (*Spica*), α Scorpii (*Antares*), α Aquilæ (*Altair*), α Piscis Australis (*Fomalhaut*), and α Pegasi (*Markab*); the four planets are Venus, Mars, Jupiter, and Saturn.

A body is said to be '*in distance*' when its angular distance from the moon lies within the limits given in the *Nautical Almanac*, these limits being from about 20° to 140°.

The path of the moon among these heavenly bodies is well shown on a chart in Harbord's *Glossary of Navigation.*

[2] This need not be the heavenly body whose distance from the moon is measured, and circumstances may necessitate the ship mean time being found at some time before or after the observations for distance; in these cases a correction for run must be applied (*see* page 224). The observations for *longitude* by lunar with altitudes *observed* can also often be made available as a double altitude for *latitude.*

(1) To find Greenwich mean time.

(2) To find ship mean time.

The former is deduced from the observed distance; the latter, in the usual way, from an observed altitude of some heavenly body.

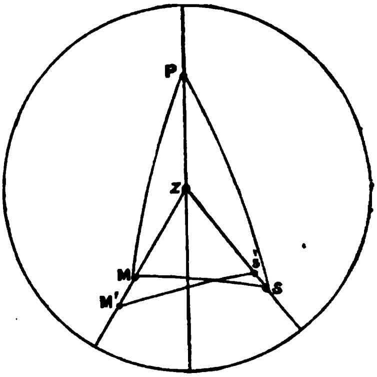
Lunars are very rarely employed in the *practice* of navigation, but are rightly retained as an important feature in the *instruction* of junior officers, nor is it difficult to imagine in time of war many circumstances which would probably arise (such as the navigation of prizes) when this method of finding the longitude would be the only one available.

In a paper published in the *Philosophical Transactions* for 1797 Mendoza Rios investigated no less than forty different formulæ for clearing lunar distances.

To clear the distance

Given the apparent distance, and the apparent and true altitudes of the two bodies; to investigate a formula for computing the *true* distance.

Fig. 145.



Let S′ be the apparent place of the heavenly body, as the sun or star; then, in consequence of refraction and parallax, the former exceeding the latter, its true place will be below S′, as at S, in a vertical plane (if we suppose the effect of parallax to take place in a plane passing through the true zenith Z, which it does very nearly). Let M′ be the apparent place of the moon; then its true place will be above M′, since the moon is depressed more by parallax than it is raised by refraction: let, therefore, its true place be supposed to be at M, in a plane passing through Z. Through M′ and S′ draw the arc of a great circle M′ S′; this arc will be the *apparent* distance found by observation. Through M and S, the true places of the heavenly bodies, draw the arc of a great circle M S. The arc M S is the *true distance* to be computed. The true distance M S may be found by the common rules of spherical trigonometry;[1] but it will be more readily obtained by means of a special formula or rule, which will determine the true distance in *terms of the versines*, since the table of versines is calculated to the nearest second.

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[1] In the triangle M′ Z S′ we know all three sides, viz. the two *apparent* zenith distances Z M′ and Z S′ and the *apparent* distance M′ S′ between the moon and heavenly body from which the angle M′ Z S′ can be found. Then in the triangle M Z S we know Z M, Z S the *true* zenith distances of the moon and heavenly body and also the included

Investigation of Formula for Clearing the Distance.[1]

Let Z be the zenith of the observer; then, if we suppose the effect of parallax to take place in a vertical circle,[2] $\mathrm{Z\,M'}$ and Z S are circles of altitude.

Let $z = \mathrm{Z\,M}$, the *true* zenith distance of the moon,
$z_1 = \mathrm{Z\,S}$, the *true* zenith distance of the sun or star,
$a =$ the apparent altitude of the moon,
$a_1 =$ the apparent altitude of sun or star.

Then $\mathrm{Z\,M'} = 90° - a$, and $\mathrm{Z\,S'} = 90° - a_1$.

Let $d = \mathrm{M'\,S'}$, the apparent distance of the centres of the two bodies,
and $\mathrm{D} = \mathrm{M\,S}$, the true distance of the centres;

it is required to investigate a formula for computing D.

$$\text{In triangle ZMS, } \cos \mathrm{Z} = \frac{\cos \mathrm{D} - \cos z \,.\, \cos z_1}{\sin z \,.\, \sin z_1}$$

$$\text{,,} \qquad \mathrm{Z\,M'\,S'}, \cos \mathrm{Z} = \frac{\cos d - \sin a \,.\, \sin a_1}{\cos a \,.\, \cos a_1}$$

$$\therefore \frac{\cos \mathrm{D} - \cos z \,.\, \cos z_1}{\sin z \,.\, \sin z_1} = \frac{\cos d - \sin a \,.\, \sin a_1}{\cos a \,.\, \cos a_1}$$

$$\therefore \frac{\cos \mathrm{D} - \cos z \,.\, \cos z_1}{\sin z \,.\, \sin z_1} + 1 = \frac{\cos d - \sin a \,.\, \sin a_1}{\cos a \,.\, \cos a_1} + 1,$$

$$\text{or } \frac{\cos \mathrm{D} - (\cos z \,.\, \cos z_1 - \sin z \,.\, \sin z_1)}{\sin z \,.\, \sin z_1} = \frac{\cos d + (\cos a \,.\, \cos a_1 - \sin a \,.\, \sin a_1)}{\cos a \,.\, \cos a_1};$$

---

angle M Z S, from which data M S, or the *true* distance between M and S, can be calculated.

The principal objection to this direct form of calculation arises from the trouble of interpolating for seconds in the logarithmic tables. This, however, may be partially obviated by taking the apparent altitudes and apparent distance to the nearest quarter of a minute, and altering the true altitudes in such a way that the differences between the apparent and true altitudes remain as before. Then, after the true distance is calculated, the seconds omitted from the apparent distance are applied to it. Thus in the example on page 295 the *apparent* altitudes of the sun and moon would be taken as 34° 21′ 30″ and 57° 11′ 30″, while the *true* altitudes would be changed to 34° 20′ 12″ and 57° 40′ 16″; the apparent distance would be taken as 35° 47′ 30″ and 6″ subtracted from the calculated true distance.

[1] First given in this form by Krafft in 1791.

[2] This is not rigidly correct (*see* page 378).

$$\therefore \frac{\cos \text{D} - \cos (z+z_1)}{\sin z \cdot \sin z_1} = \frac{\cos d + \cos (a+a_1)}{\cos a \cdot \cos a_1};$$

$$\text{or } \cos \text{D} - \cos (z+z_1) = \{\cos d + \cos (a+a_1)\} \cdot \frac{\sin z \cdot \sin z_1}{\cos a \cdot \cos a_1}.$$

Assume $\frac{\sin z \cdot \sin z_1}{\cos a \cdot \cos a_1} = 2 \cos \text{A}$ (A being an auxiliary angle);

$$\begin{aligned} \therefore \cos \text{D} - \cos (z+z_1) &= 2 \cos \text{A} \cdot \{\cos d + \cos (a+a_1)\} \\ &= 2 \cos \text{A} \cdot \cos d + 2 \cos \text{A} \cdot \cos (a+a_1) \\ &= \cos (d+\text{A}) + \cos (d \sim \text{A}) + \cos (a+a_1+\text{A}) + \cos (a+a_1 \sim \text{A}) \end{aligned}$$

$$\begin{aligned} \therefore 1 - \cos \text{D} = 1 - \cos (z+z_1) + 1 - \cos (d+\text{A}) + 1 - \cos (d \sim \text{A}) + 1 \\ - \cos (a+a_1+\text{A}) + 1 - \cos (a+a_1 \sim \text{A}) - 4, \end{aligned}$$

$$\begin{aligned} \therefore \text{vers D} = \text{vers } (z+z_1) + \text{vers } (d+\text{A}) + \text{vers } (d \sim \text{A}) + \text{vers } (a+a_1+\text{A}) \\ + \text{vers } (a+a_1 \sim \text{A}) - 4. \end{aligned}$$

But tabular versine=1000000 × versine. Reducing the formula to tabular versines and clearing of fractions, we shall see, since

$$\text{vers D} = \frac{\text{tab vers D}}{1000000} \text{ and vers } (z+z_1) = \frac{\text{tab vers } (z+z_1)}{1000000} \text{ \&c.}$$

$$\begin{aligned} \text{tab vers D} = \text{tab vers } (z+z_1) + \text{tab vers } (d+\text{A}) + \text{tab vers } (d \sim \text{A}) \\ + \text{tab vers } (a+a_1+\text{A}) + \text{tab vers } (a+a_1 \sim \text{A}) - 4000000; \end{aligned}$$

or, suppressing the word tabular, it being understood that the formula is expressed in terms of tabular versines, we have

$$\begin{aligned} \text{vers D} = \text{vers } (z+z_1) + \text{vers } (d+\text{A}) + \text{vers } (d \sim \text{A}) + \text{vers } (a+a_1+\text{A}) \\ + \text{vers } (a+a_1 \sim \text{A}) - 4000000. \end{aligned}$$

From which expression the true distance D may be computed, when the true and apparent altitudes and the apparent distance are known.

Before, however, this formula can be used, the value of the auxiliary angle A must be computed, and formed into a table; this has been done, and may be found in Inman's Tables, No. 34.

The auxiliary angle A may be computed as follows:

Construction of Table of Auxiliary Angle A.

Let it be required to compute the value of A, when the true zenith distance of the moon $z=32° 19' 50''$, the apparent altitude of the moon $a=57° 11' 25''$, true zenith distance of sun $z_1 = 55° 39' 46''$, and apparent altitude of sun $a_1 = 34° 21' 32''$.

Since $2 \cos \text{A} = \frac{\sin z \cdot \sin z_1}{\cos a \cdot \cos a_1} = \sin z \cdot \sin z_1 \cdot \sec a \cdot \sec a_1$,

$\therefore$ in logarithms,

$$\text{L} \cos \text{A} = \text{L} \sin z + \text{L} \sec a + \text{L} \sin z_1 + \text{L} \sec a_1 - 30{\cdot}301030.$$

Calculation.

L sin z	9·728194
„ sec a	10·266120
„ sin z_1	9·916839
„ sec a_1	10·083273
	39·994426
	30·301030
„ cos A	2·693306 ∴ A=60° 25′ 15″.

And in the same manner may other values of A be computed for any given apparent altitude of the moon, the argument of the table.

NOTE.—After adding to the value of the auxiliary angle A taken from Table No. 34 (Inman's Tables) the correction for *seconds* of hor. par. given at the side, a further *addition* should be made depending upon the altitude of the body observed with the moon, and which for the sun (☉) and stars (✱) is found in Table No. 34, and for planets in Table No. 52 (*see* Explanation of those tables).

The value of the auxiliary angle A always lies between 60° and 61° since if there were no refraction or parallax the value of $\frac{\sin z \sin z'}{\cos a \cos a'}$ would equal 1, and A would be 60°, the values of refraction and parallax can never make A equal 61°.

Required the true distance of the moon from the sun, having given Example

App. dist. of the centres	35°	47′	24″	True alt. sun	34°	20′	14″
app. alt. sun	34	21	32	true alt. moon	57	40	10
app. alt. moon	57	11	25	auxiliary angle A	60	25	15

Sun's true zen. dist. z_1	55°	39′	46″	
moon's true zen. dist. z	32	19	50	
$z+z_1$	87	59	36	vers
apparent distance d	35	47	24	
aux. angle A	60	25	15	
$d+$A	96	12	39	vers
$d\sim$A	24	37	51	vers
moon's app. alt. a	57	11	25	
sun's app. alt. a_1	34	21	32	
	91	32	57	
auxiliary angle A	60	25	15	
$a+a_1+$A	151	58	12	vers
$a+a_1\sim$A	31	7	42	vers

	Parts for Seconds.
964810	174
1107999	188
90885	103
1882674	27
143883	105
4190251	597
597	
4190848	
4000000	
vers 190848	

∴ true dist. 35° 59′ 13″

In practice it is not necessary to take from the table of versines more than the last five figures, rejecting also all but these last five in the sum, since the true distance will be always either in the same column with the apparent distance, or the adjacent one.[1]

Find the TRUE DISTANCE in the following examples:

11. Sun's app. alt. is 30° 29′ 48″; moon's app. alt., 50° 54′ 40″; moon's true zen. dist., 38° 30′ 40″; sun's true zen. dist., 59° 31′ 45″; the auxiliary angle A, 60° 24′ 10″; and app. dist., 88° 49′ 58″. *Ans.* True distance, 88° 24′ 19″.

12. Sun's app. alt. is 54° 29′ 30″; moon's app. alt., 5° 26′ 00″; moon's true zen. dist., 83° 48′ 30″; sun's true zen. dist., 35° 31′ 0″; auxiliary angle A, 60° 2′ 16″; and app. dist., 105° 5′ 47″. *Ans.* True distance, 104° 26′ 20″.

13. Sun's app. alt. is 17° 39′ 30″; moon's app. alt., 24° 13′ 45″; moon's zen. dist., 64° 56′ 45″; sun's zen. dist., 72° 23′ 20″; auxiliary angle A, 60° 12′ 32″; and app. dist., 111° 20′ 45″. *Ans.* True distance, 110° 55′ 57″.

14. Sun's app. alt. is 54° 47′ 5″; moon's app. alt., 21° 20′ 0″; moon's zen. dist., 67° 51′ 5″; sun's zen. dist., 35° 13′ 30″; auxiliary angle A, 60° 10′ 45″; and app. dist., 71° 16′ 44″. *Ans.* True distance, 70° 38′ 4″.

15. Sun's app. alt. is 12° 19′ 30″; moon's app. alt., 20° 40′ 20″; moon's zen. dist., 68° 28′ 19″; sun's zen. dist., 77° 44′ 40″; auxiliary angle A, 60° 10′ 52″; and app. dist., 124° 44′ 32″. *Ans.* True distance, 124° 19′ 10″.

16. Sun's app. alt. is 57° 53′ 50″; moon's app. alt., 35° 3′ 0″; moon's zen. dist., 54° 11′ 56″; sun's zen. dist., 32° 6′ 40″; auxiliary angle A, 60° 17′ 56″; and app. dist., 65° 34′ 42″. *Ans.* True distance, 64° 58′ 7″.

17. Sun's app. alt. is 15° 43′ 50″; moon's app. alt., 16° 5′ 5″; moon's zen. dist., 73° 1′ 30″; sun's zen. dist., 74° 19′ 30″; auxiliary angle A, 60° 8′ 35″; and app. dist., 119° 44′ 31″. *Ans.* True distance, 119° 19′ 52″.

18. App. alt. of a star is 20° 13′ 25″; moon's app. alt., 31° 17′ 20″; star's zen. dist., 69° 49′ 10″; moon's zen. dist., 57° 57′ 45″; auxiliary angle A, 60° 15′ 22″; and app. dist., 72° 42′ 16″. *Ans.* True distance, 72° 33′ 3″.

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[1] The true distance may be either greater or less than the observed distance according to the positions of the bodies observed, but, under all *practical* circumstances of observation, they never differ by more than one degree—that is, the difference cannot under *any* circumstances exceed the sum of the two corrections for altitude. Thus with both bodies on the same vertical circle, and on *opposite* sides of the meridian, the *true* distance is found by adding to the *observed* distance the *difference* between the corrections in altitude of the bodies. If the bodies are on the *same* side of the meridian the *sum* of the corrections added to the observed distance gives the true distance.

19. App. alt. of a star is 29° 59′ 15″; moon's app. alt., 32° 30′ 10″; star's zen. dist., 60° 2′ 25″; moon's zen. dist., 56° 41′ 30″; auxiliary angle A, 60° 17′ 24″; and app. dist., 58° 44′ 19″.

Ans. True distance, 58° 30′ 21″.

Greenwich mean time

The true distance D being thus calculated by means of the preceding formula, the mean time at Greenwich corresponding thereto may be found as follows:

As the difference of distances in the *Nautical Almanac* for the three hours between which the computed true distance lies: 3 hours :: difference between the distance in *Nautical Almanac* at the beginning of the 3 hours and the computed distance: *t*; *t* being the hours, minutes, and seconds elapsed from the beginning of the 3 hours.

The quantity *t* is, however, usually found by means of proportional logarithms.

Proportional logarithms

The logarithm of 10800^s (or 3 hours), diminished by the logarithm of any other number of seconds (less than 3 hours), is termed the proportional logarithm of that number.[1]

Thus prop. log $2^h\ 42^m = \log 10800 - \log 9720 = 0{\cdot}045757$.

It will be remarked that while *common* logarithms *increase* with the numbers of which they are the logarithms, proportional logarithms *decrease*.

Let d = difference of distances in 3 hours (taken from *Nautical Almanac*), d' = difference between first distance taken out and the calculated distance, t = time elapsed from the hour corresponding to the first distance taken out.

Then $d : d' :: 3$ hours $: t, \therefore t\,d = 3\,d'$;

$\therefore \log t + \log d = \log 3^h + \log d'$, or $\log t = \log 3^h - \log d + \log d'$;

$\therefore \log 3^h - \log t = -(\log 3^h - \log d) + (\log 3^h - \log d')$,

or prop. log t = prop. log d' − prop. log d.

Then the time t thus found, added to the hour corresponding to the first distance in the *Nautical Almanac* taken out, will be the Greenwich mean time at the instant when the observation was taken.

Given the sun's true distance (calculated from an observed distance) = 73° 51′ 58″, and two distances taken out of the *Nautical Almanac* as follows, namely, that at $\text{vi}^h = 73°\ 23'\ 44''$, and at $\text{ix}^h = 74°\ 54'\ 7''$; to find Greenwich mean time when the observation was taken.

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[1] **Proportional logarithms were first calculated by Maskelyne and given in the *Nautical Almanac* for 1767. *See* Inman's Tables, No. 54.**

	. prop. log t=prop. log d' − prop. log d	
$d'=28'\ 14''$	prop. log d'	8045
$d=1^\circ\ 30'\ 23''$	prop. log d	2992 (from *Nautical Almanac*).
	∴ prop. log t	5053 ∴ $t=0^h\ 56^m\ 14^s$
Time at first distance in *Nautical Almanac* ...6		

∴ Greenwich mean time =6 56 14, when the sun's true distance was 73° 51′ 58″.

Examples November 22, the true distance of Saturn from the moon was found to be 77° 52′ 45″. Required Greenwich mean time when the observation was taken.

True distance at observation...........		77° 52′ 45″
in *Nautical Almanac* dist. at iii^h		77 14 40
vi^h		78 47 24
prop. logarithm 6745....................		38 5
2880..........		1 32 44
3865	corr.	$1^h\ 13^m\ 55^s$
	adding	3
Greenwich mean time Nov. 22.........		4 13 55

Find mean time at Greenwich from each of the following observations:

20. November 24, when true distance of Aldebaran was 93° 38′ 45″. *Ans.* $3^h\ 57^m\ 18^s$.

21. Sep. 24, when true distance of Regulus was 58° 45′ 8″. *Ans.* $16^h\ 3^m\ 6^s$.

22. May 27, when true distance of the sun was 110° 8′ 50″. *Ans.* $14^h\ 2^m\ 22^s$.

Distances from 'Nautical Almanac.'

Dist. Aldebaran, Nov. 24,	at iii^h	...93° 7′ 57″	...at	vi^h	...94° 44′ 42″
„ Regulus, Sept. 24...	„ xv	...59 16 16	... „	xviii	...57 47 27
„ Sun, May 27	„ mid.	...111 12 57	... „	xv	...109 38 38

If the distance between the moon and a heavenly body increased or decreased *uniformly*, the above method would be strictly accurate, but a further correction for this want of uniformity in motion should sometimes in practice be applied. This connection for second differences, as it is termed, forms No. 55 of Inman's tables, and is of especial importance in the case of *star* or *planet* lunars.

This table is used as follows:

1. Find the approximate interval elapsed since the nearest preceding hour given in the *Nautical Almanac* by the preceding rule.

2. Take the differences between the proportional logarithm following the distance in the Almanac, and the proportional logarithms immediately to the left and right, and note the *mean* of these differences.

3. With the approximate interval and this *mean* difference, as arguments, take out the correction from the table.

4. If the proportional logarithms are *decreasing*, *add* the correction to the approximate time; but if *increasing*, *subtract* it: the result will be the Greenwich mean time.

Example.—Suppose it were required to find the Greenwich mean astronomical time at which the *true* distance between the moon and Antares would be 45° 40′ 8″ about July 8 on a certain year, and that it appears, by inspecting the distances, that the *nearest* distance *preceding* it, in order of time, is that on July 8 at *midnight*, therefore,

Distance at *midnight*	46° 30′ 56″	and P. L.	2589
True distance	45 40 8		
Difference	0 50 48	...... P. L.	5494
Approximate interval	$1^h\ 32^m\ 12^s$	... P. L.	2905

The proportional logarithm following the distance in the Almanac is 2589, those immediately to the left and right are respectively 2570 and 2609; the differences are therefore 19 and 20, and the *mean* of them, 20.

Opposite to $1^h\ 32^m\ 12^s$ (or the quantity nearest to it, $1^h\ 30^m$), and under 20 in the table, we have for the correction 6^s, which, *subtracted* from the approximate interval, $1^h\ 32^m\ 12^s$, because the proportional logarithms are *increasing*, gives $1^h\ 32^m\ 6^s$ for the true interval after *midnight*; the Greenwich mean astronomical time is, therefore, July 8, $13^h\ 32^m\ 6^s$. The omission of this correction would in this case produce an error of 1·5′ in the longitude.

It will sometimes happen that the *mean* difference of the proportional logarithms will exceed the limit of the table of correction; in this case the table may be entered with the approximate interval, and *any fraction* of the difference and the corresponding correction *increased in like proportion*.

The following is a fully calculated example of a lunar, followed by others for practice. **Example**

Mar. 13, at $3^h\ 15^m$ P.M. mean time nearly, in lat. 29° 50′ N., and long. by account 40° 00′ W., the following lunar observation was taken:

	Obs. Alt. Sun's L. L.	Obs. Alt. Moon's L. L.	Obs. Dist. N. L.[1]
	34° 10′ 40″	57° 2′ 11″	35° 15′ 40″
Index corr.	1 10−	1 40−	0 40+

The height of the eye above the sea was 17 feet. Required the longitude.

Ship, March 13	3ʰ 15ᵐ
Longitude in time	2 40 W.
Greenwich, March 13	5 55 or 1·9 hrs.

Elements from *Nautical Almanac* corrected for Greenwich date sun's declination 2° 53′ 52″ S.; sun's semi. 16′ 6″; eq. of time, 9ᵐ 41ˢ+ ; moon's semi. 14′ 58″ ; moon's hor. par. 54′ 13.″

Sun's Alt.				Moon's Alt.				Distance N. L.			
Obs. alt. ...	34°	10′	40″	Obs. alt. ...	57°	2′	11″		35°	15′	40″
index corr.		1	10−	index corr.		1	40−	index corr.		0	40+
	34	9	30		57	0	31		35	16	20
dip		4	4−	dip		4	4−	sun's semi.		16	6+
	34	5	26		56	56	27	moon's semi.		14	58+
semi.		16	6+	semi.		14	58+	app. dist.	35	47	24
app. alt. ...	34	21	32	app. alt.	57	11	25	Aux. Angle.			
corr. in alt.		1	18−	corr. in alt.		28	38+		60°	25′	7″
true alt. ...	34	20	14				7+	corr. for sec H. P.			6
sun's Z. D.	55	39	46	true alt.	57	40	10	„ ⊙			3
									60	25	16
				moon's Z. D.	32	19	50				

To find Ship Mean Time.

Latitude	29°	50′	0″ N. ...	log. sec	0·061742
sun's declination	2	53	52 S.	„ sec	0·000555
	32	43	52		
sun's zenith distance ...	55	39	46		
sum	88	23	38 ...	½ L hav ...	4·843311
difference	22	55	54 ...	½ „ hav	4·298380
				L hav	9·203988

Apparent time	3ʰ 8ᵐ 36ˢ
equation of time	9 41+
ship mean time	3 18 17

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[1] N. L. and F. L. are used as abbreviations for nearer and farther limbs. The bright limb of the moon being always turned to the sun, the distance in a *sun* lunar is always measured to N. L.

When there is only *one* observer, the altitudes at the *time of observing the distance* are obtained by proportion between altitudes observed before and after that time. It is recommended to observe first a single altitude of the body farthest from the meridian, then one of the other body, and then the distance, concluding with single altitudes taken again in reverse order.

The observation of a lunar distance with the altitudes observed furnishes data not only for the longitude but for the latitude by double altitude as well.

To find Greenwich Mean Time.

Moon's zenith distance ……	32°	19′	50″			
sun's zenith distance ………	55	39	46			
	87	59	36 vers			
					Versines	
moon's app. alt. ……………	57	11	25		64810	174
sun's app. alt. ………………	34	21	32		82674	030
sum …………………………	91	32	57		43888	102
auxiliary angle A ……………	60	25	16		07999	195
sum ……………… ………	151	58	13 vers		90885	104
difference ……………………	31	7	41 vers		90251	605
					605	
apparent distance …………	35	47	24·4		90856	
auxiliary angle A …………	60	25	16	35	59	16 True Dist.
sum …………………………	96	12	40·4 vers	35	56	58 at vi. hrs.
difference ……………………	24	37	51·6 vers	37	18	0 „ ix.
					2	18
				1	21	2
	0^{h}	5^{m}	7^{s}		1·8935	
	6				·3466	Prop. logs
Greenwich mean time ……	6	5	7 ……………		1·5469	
ship mean time ……………	3	18	17			
longitude in time …………	2	46	50			
longitude ………………	41	42	30 W.			

Note.—At night, when circumstances admit of observing *altitudes*, a lunar taken with the moon and a star or planet is calculated in a precisely similar way to the above.

If any of the arcs marked (*vers*) be greater than 180°, it must be subtracted from 360°, and the remainder used instead of it.

23. March 25, at 3^{h} 30^{m} P.M. mean time nearly, in lat. 52° N., and long. by account 33° W., the following lunar was taken:

	Obs. Alt. Sun's L. L.	Obs. Alt. Moon's L. L.	Obs. Dist. N. L.
	23° 10′ 20″	23° 50′ 10″	112° 56′ 30″
Index cor.	6 10 −	5 0 +	4 20 −

The height of the eye above the sea was 20 feet. Required the longitude. *Ans.* Hour angle, 3^{h} 32^{m} 9^{s}; true dist., 112° 52′ 49″; longitude, 33° 1′ 45″ W.

24. April 20, at 2^{h} 0^{m} P.M. mean time nearly, in lat. 50° 50′ N., and long. by account 1° 40′ E., the following lunar was taken:

	Obs. Alt. Sun's L. L.	Obs. Alt. Moon's L. L.	Obs. Dist. N. L.
	43° 16′ 30″	24° 39′ 20″	69° 12′ 0″
Index corr.	0 10 +	0 20 −	0 50 −

The height of the eye above the sea was 20 feet. Required the longitude. *Ans.* Hour angle, 2^{h} 1^{m} 25^{s}; true dist., 69° 11′ 40″; longitude, 1° 19′ 9″ E.

25. May 19, at $2^h\ 50^m$ P.M. mean time nearly, in lat. 51° 30′ N., and long. by account 20° 40′ E., the following lunar was taken:

	Obs. Alt. Sun's L. L.	Obs. Alt. Moon's L. L.	Obs. Dist. N. L.
	42° 50′ 30″	25° 10′ 20″	61° 40′ 20″
Index corr.	4 10+	6 10−	2 10+

The height of the eye above the sea was 20 feet. Required the longitude. *Ans.* Hour angle, $2^h\ 57^m\ 20^s$; true dist., 61° 44′ 40″; longitude, 20° 40′ 24″ E.

26. Feb. 20, at $3^h\ 50^m$ P.M. mean time nearly, in lat. 10° 20′ N., and long. by account 7° W., the following lunar was taken:

	Obs. Alt. Sun's L. L.	Obs. Alt. Moon's L. L.	Obs. Dist. N. L.
	30° 15′ 40″	24° 10′ 10″	100° 55′ 10″
Index corr.	3 10−	1 10+	0 30+

The height of the eye above the sea was 20 feet. Required the longitude. *Ans.* Hour angle, $3^h\ 44^m\ 12^s$; true dist., 100° 54′ 54″; longitude, 7° 3′ 31″ W.

27. July 16, at $4^h\ 15^m$ P.M. mean time nearly, in lat. 25° 30′ N., and long. by account 48° 18′ E., the following lunar was taken:

	Obs. Alt. Sun's L. L.	Obs. Alt. Moon's L. L.	Obs. Dist. N. L.
	34° 9′ 25″	45° 42′ 40″	93° 21′ 52″
Index corr.	2 10−	2 10+	1 15+

The height of the eye above the sea was 20 feet. Required the longitude. *Ans.* 48° 13′ 30″ E.

Elements from 'Nautical Almanac.'

	Sun's Declin.	Eq. of Time.	Moon's Semi.	Hor. Par.	Sun's Semi.
Mar. 25	1° 40′ 56″ N.	$6^m\ 13{\cdot}8^s$ add	Noon 14′ 59″	54′ 59″	16′ 3″
Var. in 1 hour,	58·85″ incr.	$\cdot769^s$ decr.	Mid. 14 55	54 44	
Distance at iii. hours, 111° 33′ 34″; at vi. hours, 112° 57′ 16″.					
Apr. 20	11° 23′ 54″ N.	$1^m\ 3{\cdot}4^s$ sub.	Noon 15′ 23·7″	56′ 29·8″	15′ 56″
Var. in 1 hour,	51·18″ incr.	$\cdot529^s$ incr.	Mid. 15 17·0	56 5·3	
Distance at noon, 68° 14′ 43″; at iii. hours, 69° 43′ 48″.					
May 19	19° 41′ 37″ N.	$3^m\ 49{\cdot}58^s$ sub.	Noon 15′ 11·6″	55′ 45·5″	15′ 49″
Var. in 1 hour,	31·79″ incr.	$\cdot120^s$ decr.	Mid. 15 6·3	55 25·7	
Distance at noon, 61° 0′ 58″; at iii. hours, 62° 27′ 35″.					
Feb. 20	10° 55′ 53″ S.	$14^m\ 1{\cdot}7^s$ add	Noon 16′ 4·4″	58′ 59·2″	16′ 11″
Var. in 1 hour,	54·05″ decr.	$\cdot285^s$ decr.	Mid. 16 9·6	59 18·0	
Distance at iii. hours, 100° 7′ 50″; at vi. hours, 101° 45′ 50″.					
July 16	21° 20′ 13″ N.	$5^m\ 45{\cdot}0^s$ add	Noon 16′ 5·1″	58′ 55·7″	15′ 46″
Var. in 1 hour,	25·0″ decr.	$\cdot24^s$ incr.	Mid. 16 1·4	58 42·2	
Distance at noon, 92° 49^h 59″; at iii. hours, 94° 27′ 44″.					

28. Jan. 8, at $7^h\ 0^m$ P.M. mean time nearly, in lat. 50° 40′ N., and long. by account 4° E., the following lunar was taken:

	Obs. Alt. Aldebaran E. of Meridian	Obs. Alt. Moon's L. L.	Obs. Dist. F. L.
	45° 20′ 10″	30° 30′ 0″	71° 31′ 10″
Index corr.	2 20−	2 10+	3 30−

The height of eye above the sea was 18 feet. Required the longitude. *Ans.* Hour angle, 21^h 36^m 57^s; true dist., 70° 42′ 56″; longitude, 3° 49′ 16″ E.

29. Jan. 9, at 7^h 50^m P.M. mean time nearly, in lat. 49° 40′ N., and long. by account 10° E., the following lunar was taken:

	Obs. Alt. Pollux. E. of Meridian.	Obs. Alt. Moon's L. L.	Obs. Dist. F. L.
	37° 10′ 10″	31° 50′ 10″	103° 20′ 0″
Index corr.	1 10−	1 20+	1 30+

The height of eye above the sea was 18 feet. Required the longitude. *Ans.* Hour angle, 19^h 39^m 51^s; true dist., 102° 28′ 7″; longitude, 10° 21′ 54″ E.

30. April 18, at 9^h 40^m P.M. mean time nearly, in lat. 56° 10′ N., and long. by account 23° E., the following lunar was taken:

	Obs. Alt. α Virginis. E. of Meridian.	Obs. Alt. Moon's L. L.	Obs. Dist. F. L.
	19° 40′ 0″	48° 40′ 20″	91° 30′ 20″
Index corr.	1 10+	1 20+	0 30+

The height of eye above the sea was 20 feet. Required the longitude. *Ans.* Hour angle, 22^h 8^m 54^s; true dist., 90° 55′ 19″; longitude, 23° 5′ 3″ E.

31. April 17, at 8^h 45^m P.M. mean time nearly, in lat. 51° 20′ N., and long. by account 5° 10′ E., the following lunar was taken:

	Obs. Alt. α Virginis. E. of Meridian.	Obs. Alt. Moon's L. L.	Obs. Dist. F. L.
	18° 15′ 32″	36° 25′ 10″	105° 33′ 28″
Index corr.	1 10+	0 20−	0 20+

The height of eye above the sea was 20 feet. Required the longitude. *Ans.* 5° 8′ E.

32. Jan. 11, at 5^h 40^m P.M. mean time nearly, in lat. 50° 48′ N., and long. by account 70° W., the following lunar was taken:

	Obs. Alt. α Tauri. E. of Meridian.	Obs. Alt. Moon's L. L.	Obs. Dist. F. L.
	36° 25′ 50″	30° 39′ 30″	71° 50′ 55″
Index corr.	5 30+	2 10−	2 15−

The height of eye above the sea was 20 feet. Required the longitude. *Ans.* 69° 59′ 53″ W.

33. March 20, at 7^{h} 50^{m} P.M. mean time nearly, in lat. 49° 50′ N., and long. by account 1° 30′ E., the following lunar was taken:

	Obs. Alt. Aldebaran. W. of Meridian.	Obs. Alt. Moon's L. L.	Obs. Dist. N. L.
	37° 4′ 30″	38° 17′ 20″	75° 15′ 10″
Index corr.	2 4−	1 5+	0 20+

The height of eye above the sea was 20 feet. Required the longitude. *Ans.* 1° 34′ 30″ E.

Elements from 'Nautical Almanac.'

Sid. Time at Gr. Mean Noon.	Moon's Semi.	Hor. Par.		
Jan. 8, 19^{h} 9^{m} $45{\cdot}72^{s}$	Noon, 14′ 48·8″	54 21·8″	Star's R. A.	4^{h} 27^{m} $22{\cdot}7^{s}$
	Mid. 14 52·0	54 33·3	Star's decl.	16° 12′ 13″ N.
Distance at vi. hours, 71° 1′ 57″; at ix. hours, 69° 32′ 16″.				
Jan. 9, 19^{h} 13^{m} $42{\cdot}27^{s}$	Noon, 14′ 55·8″	54′ 47·2″	Star's R. A.	7^{h} 36^{m} 12^{s}
	Mid. 15 0·2	55 3·6	Star's decl.	28° 22′ 46″ N.
Distance at vi. hours, 103° 8′ 4″; at ix. hours, 101° 37′ 51″.				
Ap. 18, 1^{h} 44^{m} $58{\cdot}62^{s}$	Noon, 16′ 8·8″	59′ 15·4″	Star's R. A.	13^{h} 17^{m} 19^{s}
	Mid. 16 8·7	59 14·8	Star's decl.	10° 22′ 43″ S.
Distance at vi. hours, 92° 10′ 13″; at ix. hours, 90° 24′ 32″.				
Ap. 17, 1^{h} 41^{m} $2{\cdot}07^{s}$	Noon, 16′ 8·2″	59′ 13·1″	Star's R. A.	13^{h} 17^{m} 19^{s}
	Mid. 16 8·7	59 14·9	Star's decl.	10° 22′ 43″ S.
Distance at vi. hours, 106° 16′ 11″; at ix. hours, 104° 30′ 26″.				
Jan. 11, 19^{h} 21^{m} $8{\cdot}50^{s}$	Noon, 16′ 41·6″	61′ 9·8″	Star's R. A.	4^{h} 28^{m} 21^{s}
	Mid. 16 37·2	60 53·6	Star's decl.	16° 14′ 21″ N.
Distance at ix. hours, 70° 16′ 10″; at Midnight, 72° 8′ 51″.				
Mar. 20, 23^{h} 51^{m} $30{\cdot}11^{s}$	Noon, 15′ 12·2″	55′ 47·6″	Star's R. A.	4^{h} 27^{m} 3^{s}
	Mid. 15 17·4	56 6·7	Star's decl.	16° 11′ 30″ N.
Distance at vi. hours, 74° 7′ 12″; at ix. hours, 75° 42′ 9″.				

Lunar observation—altitudes calculated

At night, in the event of not being able to measure either or both of the altitudes, the *true* altitudes are *calculated,* and thence the *apparent* altitudes deduced by applying the corrections for refraction and parallax with their signs reversed.[1]

Having found the apparent altitudes, the problem is solved in the same way as though they had *observed,* but they will not, of course, be available for determining ship mean time, which is found by the altitude of a heavenly body near to the prime vertical, taken at some preceding or following time, as near to that when the distance is measured as possible.

The successive steps of this long calculation are as follows:

(1) *To find star's hour angle.*

Practical rule

To the right ascension of mean sun (corrected for approximate Greenwich date) add mean time at ship, and from the sum

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[1] In practice it is sufficiently accurate to calculate these true altitudes only to the nearest *minute* of arc.

(increased if necessary by 24 hours) subtract the star's right ascension: the remainder is the hour angle of the star.

(2) *To calculate the star's true altitude.*

On referring to fig. 103, page 202, we see that in the triangle Z P X are known P Z = colat; P X = star's pol. dist., and Z P X = star's hour angle; hence the value of Z X can be found, and from that the star's true altitude (90° − Z X). *See* also page 215.

(3) *To find star's apparent altitude.*

To the true altitude just found add the refraction: the result is the star's apparent altitude.[1]

(4) *To find the moon's hour angle.*

To the right ascension of mean sun (corrected for approximate Greenwich date) add ship mean time, and from the sum (increased if necessary by 24 hours) subtract the moon's right ascension (also corrected for Greenwich date): the result is the moon's hour angle (rejecting 24 hours if greater than 24 hours).

(5) *To find the moon's true altitude.*

This is found from the triangle Z P M in the same way as given before for the star's true altitude.

(6) *To find the moon's apparent altitude.*

Consider the moon's true altitude just found as the apparent altitude; enter the table with it, and take out the correction in altitude approximately; subtract this correction from the moon's true altitude, and thus find the moon's apparent altitude nearly. Then enter the table again with this *corrected altitude*, and thus take out the correction in altitude more exactly; subtract this correction from the moon's *true* altitude, and the result will be the moon's apparent altitude very nearly.

Take out at the same opening of the table the auxiliary angle A.

After this the successive steps are similar to those of the preceding example on page 300.

Note.—The true altitudes of both star and moon may be calculated from the equation cosec alt. = sec ϕ sec $(l-d)$ where hav $\phi = \cos l \cos d \sec (l-d)$.

Example May 16, in lat. 50° 37′ 30″ N., and long. by account 1° 6′ W., when a chronometer showed $10^h\ 39^m\ 26^s$ P.M., the observed distance of the star α Virginis from the moon's farthest limb was 64° 1′ 50″, index correction + 0′ 40″, the error of the chronometer on ship mean time being fast $4^m\ 14^s$. Required the longitude.

---

[1] In strictness, the table for correction of altitude ought to have been entered with the *apparent* altitude, instead of the *true*, to get the correction in altitude; but in the case of the star's altitude the above is sufficiently correct. A more exact method is followed in finding the *moon's* apparent altitude from the true.

		Sid. Time at Gr. Mean Noon.	R. A. M. S.	Star's R. A. and Decl.
Time by chron.	10^h 39^m 26^s			
error fast ……	4 14	May 16……	3^h 35^m $9{\cdot}00^s$	13^h 16^m 56^s
ship mean time	10 35 12	corr. for 10^h	1 38·6	10° 20′ 25″ S.
long. in time …	4 24	„ 40 min.	6·4	
Gr., May 16 …	10 40 or 10·7 hrs.		3 36 54	
		Ship mean time………	10 35 12	
			14 12 6	…………14^h 12^m 6^s
		star's right ascen. ……	13 16 56	.. ……… 9 19 40
		∴ star's hour angle……	0 55 10	☾'s H.A. 4 52 26

	Moon's R. A.	Moon's Decl.	Moon's Semi.	M.'s H. P.
At 10^h …	9^h 18^m 10^s …	13° 25′ 13″ N. …	Noon …… 16′ 7·0″ …	59′ 1·0″
corr……	1 30 …	8 56 ………	Mid. …… 16 8·1 …	59 5·6
	9 19 40 …	13 16 17 N.	1·1	4·6
var. in 10 min. ……	$22{\cdot}5^s$ ………	2′ 14″ decr.	corr. by $insp^n$. 1……	4·1
	4	4	16 8	59 5·1
„ 40 min. ……	90·0	8 56	aug. …… 6	
			16 14	

To calculate Star's Apparent Altitude.		*To calculate Moon's Apparent Altitude.*		
Lat. ……………	50° 37′ 30″ N.	Lat. ……………	50° 37′ 30″ N.	
decl. ……………	10 20 25 S.	decl. ……………	13 16 17 N.	
	60 57 55 v.		37 21 13 $v_,$.	
L cos lat. ………	9·802359	L cos lat. ……	9·802359	
„ cos star's decl.	9·992887	„ cos moon's decl.	9·988245	
„ hav star's H. A.	8·158830	„ hav moon's H. A.	9·549884	
„ θ……. ….	7·954076	„ θ ………..	9·340488	
ver θ ……………	17993	ver θ ……………	438044	
ver v ……………	514427	ver $v_,$ ……………	205056	
	232		38	
ver zen. dist. …	532652	ver zen. dist. …	643138	
	584		2990	
62° 8′ 16″	68		640148	
∴ star's zen. dist.	62° 8′ 16″	∴ moon's zen. dist.	69° 5′ 33″	
	90		90	
∴ star's true alt.	27 51 44	∴ moon's true alt.	20 54 27	
refr. ……………	+1 50	corr. in alt. (nearly)	52 0 –	
∴ star's app. alt.	27 53 34	app. alt. (nearly)	20 2 0	**Aux. Angle A**
		corr. in alt. ……	52 48 5	60° 10′ 47″ 1
		true corr. in alt.	52 53	0
		moon's true alt.	20 54 27	60 10 48
		moon's app. alt.	20 1 34	

The apparent altitudes having been found, the remainder of the calculation is similar to that on page 301.

The reader will remark that sun lunars *might* also be worked from *calculated* instead of *observed* altitudes.

34. April 20, in lat. 50° 37′ 12″ N., and long. by account 1° 6′ W., when a chronometer showed 8^h 58^m 45^s P.M., the observed distance of the star *a* Leonis from the moon's farthest limb was 46° 2′ 12″, index correction +0′ 30″, the error of chronometer on ship mean time being fast 3^m 22^s. Required the longitude. *Ans.* 0° 54′ 30″ W.

35. December 10, in lat. 50° 37′ 30″ N., and long. by account 1° 6′ W., when a chronometer showed 9^h 24^m $48{\cdot}3^s$ P.M., the observed distance of the star Pollux from the moon's farthest limb was 65° 28′ 30″, index correction + 0′ 30″, the error of the chronometer on ship mean time being fast 12^m $50{\cdot}8^s$. Required the longitude. *Ans.* 0° 29′ 37″ W.

36. April 19, in lat. 50° 48′ N., and long. by account 1° 6′ W., when a chronometer showed 8^h 40^m 18^s P.M., the observed distance of the star Regulus from the moon's farthest limb was 59° 11′ 2″, index correction + 30″, the error of chronometer on ship mean time being fast 9^m $30{\cdot}4^s$. Required the longitude.
Ans. 1° 5′ 6″ W.

37. September 1, in lat. 50° 37′ 30″ N., and long. by account 1° 6′ W., when a chronometer showed 8^h 2^m $54{\cdot}4^s$ P.M., the observed distance of the planet Jupiter from the moon's farthest limb was 64° 19′ 57″, index correction + 1′ 50″, the error of the chronometer on ship mean time being fast 2^m $2{\cdot}6^s$. Required the longitude. *Ans.* 0° 35′ 15″ W.

38. September 5, in lat. 50′ 48″ N., and long. by account 1° 6′ W., when a chronometer showed 8^h 52^m 39^s P.M., the observed distance of the planet Mars from the moon's nearest limb was 45° 11′ 23·3″, index correction + 1′ 50″, the error of the chronometer on ship mean time being fast 4^m $47{\cdot}4^s$. Required the longitude. *Ans.* 1° 16′ 10″ W.

Elements from 'Nautical Almanac.'

	Star's Declin.	**Star's Right Ascen.**	**Sid. time at Gr. Mean Noon.**
April 20...	12° 42′ 30″ N. ...	10^h 0^m $15{\cdot}35^s$	1^h 51^m $48{\cdot}09^s$

	Moon's Right Ascen.	**Moon's Declin.**	**Moon's Semi.**	**Hor. Par.**
20th, at 8^h...	6^h 51^m $3{\cdot}8^s$ ...	17° 39′ 51″ N....	Noon 15′ 23·7″...	56′ 29·8″
„ 9^h...	6 53 16·8 ...	17 36 40 N....	Mid. 15 17·0 ...	56 5·3

Distance at vi. hours, 46° 51′ 27″; at ix. hours, 45° 16′ 23″.

	Star's Declin.	Star's Right Ascen.	Sid. time at Gr. Mean Noon.
Dec. 10th	28° 23′ 22″ N.	7^h 35^m 54·9^s	17^h 16^m 17·09^s

	Moon's Right Ascen.	Moon's Decl.	Moon's Semi.	Hor. Par.
10th, at 9^h	2^h 54^m 19·38^s	16° 40′ 52″ N.	Noon 15′ 11·5″	55′ 45·0″
„ 10^h	2 56 27·74	16 47 6 N.	Mid. 15 7·7	55 30·9

Distance at ix. hours, 65° 12′ 56″; at mid., 63° 41′ 12″.

	Star's Declin.	Star's Right Ascen.	Sid. time at Gr. Mean Noon.
April 19th	12° 42′ 30″ N.	10^h 0^m 15·35^s	1^h 47^m 51·54^s

	Moon's Right Ascen.	Moon's Declin.	Moon's Semi.	Hor. Par.
19th, at 8^h	5^h 56^m 41·12^s	18° 28′ 9·6″ N.	Noon 15′ 38·4″	57′ 23·5″
„ 9^h	5 58 59·68	18 27 17·2 N.	Mid. 15 30·9	56 56·1

Distance at vi. hours, 59° 46′ 25″; at ix. hours, 58° 8′ 6″.

	Planet's Declin.	Planet's Right Ascen.	Sid. time at Gr. Mean Noon.
Sept. 1st	15° 46′ 6·8″ S.	21^h 33^m 3·13^s	10^h 39^m 59·98^s
2nd	15 48 22·9 S.	21 32 35·33	

	Moon's Right Ascen.	Moon's Declin.	Moon's Semi.	Hor. Par.
1st, at 8^h	17^h 0^m 32·9^s	23° 56 5·7″ S.	Noon 15′ 54·8″	58′ 24·0″
„ 9^h	17 3 2·2	23 56 34·4 S.	Mid. 15 49·5	58 4·5

Distance at vi. hours, 65° 7′ 44″; at ix. hours, 63° 24′ 57″.

	Planet's Declin.	Planet's Right Ascen.	Sid. time at Gr. Mean Noon.
Sept. 5th	26° 34′ 9·5″ S.	17^h 32^m 5·8^s	10^h 55^m 46·19^s
6th	26 34 29·5 S.	17 34 27·8	

	Moon's Right Ascen.	Moon's Declin.	Moon's Semi.	Hor. Par.
5th, at 8^h	20^h 41^m 16·9^s	15° 1′ 2″ S.	Noon 15′ 15·7″	56′ 0·4″
„ 9^h	20 43 20·7	14 50 47 S.	Mid. 15 11·7	55 45·5

Distance at vi. hours, 44° 14′ 50″; at ix. hours, 45° 45′ 38″.

NOTE.—The celebrated problem of finding the longitude by lunars was originally suggested by Werner, an astronomer of Nuremberg, in 1514, and a method of graphically solving the problem on a globe was given by Gemma Frisius in 1350.

Notes on Lunars

The first solution by *calculation* and reduction of an observed distance to the centre of the earth appears due to Morin, a French astronomer, in 1633.

The method could not, however, be practically employed at sea, (1) partly from the imperfection of the rough nautical instruments then in use, and (2) partly from the very imperfect knowledge which then prevailed as to the apparent path of the moon in the heavens; this, together with some uncertainty as to the astronomical positions of the stars lying near that path, prevented any reliable calculations being made which would furnish the mariner *beforehand* with the lunar distances at certain hours at Greenwich with which to compare his own observations. The first of these difficulties was removed by the invention and subsequent improvement of the sextant, and the Royal Observatory was established in 1675[1] expressly to obtain large numbers of observations of the positions of the moon and stars near her path, so that predictions could be made of their distances apart at certain hours of Greenwich mean time.

From time to time improvements have been made in the theory which explains the moon's very devious path among the stars, and the last great corrections by Hansen in 1862 appear, on comparing her predicted positions with those actually observed at the Royal Observatory, to leave but little practically to be desired.

The uncertainty of position of the moon and heavenly bodies near her path may now be considered as practically removed, but the measurement of distances on board ship requires great care and practice.

The greatest precision will be attained by selecting from the *Nautical Almanac* that body towards which the moon is moving most rapidly: if the sun or a fixed star, this will be that which lies in her direct path; but since the planets appear at times to move in a direction *contrary* to that of the moon, their distances from her will change at such times more rapidly than those of the sun or fixed stars. In either case that object of which the proportional log. given in the *Almanac* is *least* is the best to select (*see* page 297).

The brighter object should always be that *reflected* in the sextant, the relative brilliancy of the two being regulated by the up and down piece.

Every instrumental error should be previously examined and allowed for (including the error of centering), and the contact with the limb of the moon made in the centre of the field of the telescope; the altitudes are not required to be taken with extreme precision.[2]

Lunar distances should be measured with stars both east and west of the moon, thus obviating errors in determining the index error; also, if such distances are nearly equal, the effect of the error of centering is avoided.

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[1] The cost of the original building of the Royal Observatory was 520*l.*, and the salary of the first Astronomer Royal (Flamsteed) was 100*l.* per annum, for which sum he had to provide his own instruments.

[2] Except when intending to obtain the ship mean time from either or both.

The necessity for extreme care in the measurement of a lunar distance arises from the fact that, apart from other sources of error in the calculation, an error in the observed distance produces 30 times that error (approximately) in the longitude.

The moon, at her average rate of revolution round the earth, traverses 13° 11′ of arc in one day,[1] moving more or less rapidly when in those parts of her orbit nearest and farthest from the earth.

Hence an error of 1′ in an observed lunar distance to a fixed star lying directly in the moon's path would occasion an error in the Greenwich mean time found from it of 2 minutes, equivalent to an error in longitude of 30′.[2]

At the time when lunars formed the *only* means of obtaining longitude at sea it may be interesting to record the experience of a good observer.

Maskelyne (afterwards Astronomer Royal), in his voyage to St. Helena to observe the transit of Venus in 1761, took a great number of lunars, and on his return published an interesting work styled the 'British Mariners' Guide,' in which he expresses the opinion that even in the then imperfect state of the lunar theory a good observer could obtain the longitude at sea within 1½° of the truth, an opinion justified by his own published observations.[3]

When the lunar observation afforded the *only* means of obtaining the longitude at sea the frequent recurrence of such

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[1] Approximately moving the breadth of her diameter to the eastward every hour, or passing a fixed star at the rate of 33″ per minute; the greatest change of right ascension is $2^m\ 55^s$ in an hour, and the least $1^m\ 45^s$, depending upon the distance of the moon from the earth.

[2] It may be shown that if a and b represent the *apparent*, and a', b' the *true* altitudes of the moon and heavenly body to which the distance d is measured, then

$$\text{Error in true distance } D = \text{error in observed distance } d \times \frac{\sin d \cos a' \cos b'}{\sin D \cos a \cos b}.$$

Now, since $\dfrac{\sin d \cos a' \cos b'}{\sin D \cos a \cos b}$ always nearly equals unity,

∴ error in true distance always approximately equals error in observed distance, and particularly so if $d = 90°$; the errors have also the same sign.

The error in the resulting longitude in time for a given error in the true distance will evidently be the same fraction of three hours as the error in distance is of the arc passed over by the moon in three hours with regard to the heavenly body considered.

In the case of the sun this arc varies from 1° 55′ to 1° 17′, and the resulting error in longitude varies from 23′ to 35′ for an error in distance of 1′.

The maximum error of the lunar tables may now be considered to be 10″, while the maximum error of measurement of distance under favourable circumstances may be considered as 20″. A good observer can depend on longitude by lunar within 8′ or 10′.

[3] In the account of this voyage to St. Helena it is mentioned that the longitude by dead reckoning on arrival differed 7° 12′ from the truth.

long calculations led to the introduction of many ingenious *short* methods[1] by which, without any great sacrifice of accuracy, much time and trouble was saved. The best among these are probably those methods the distinctive feature of which is that the effects of refraction and parallax are separately dealt with.

Since, however, the introduction of chronometers the lunar problem has ceased to be of great practical importance under ordinary circumstances, and only one method of solution has been given in this work, which is that proposed by Krafft in 1791.

NOTE.—The chief among the refinements usually omitted in sea observations are (1) the reduction of the moon's equatorial horizontal parallax (Inman's Tables, No. 38); (2) the correction for contraction of semidiameters at low altitudes (Inman's Tables, No. 39); (3) the correction of the auxiliary angle A for barometer and thermometer (Inman's Tables, No. 41); (4) the use of the *true* refractions in place of their *mean* values (Inman's Tables, No. 13); and (5) the application of the *true* parallaxes (*see* page 378).

It may be interesting to note the following among the various methods proposed at different times for obtaining longitude independently of a chronometer:

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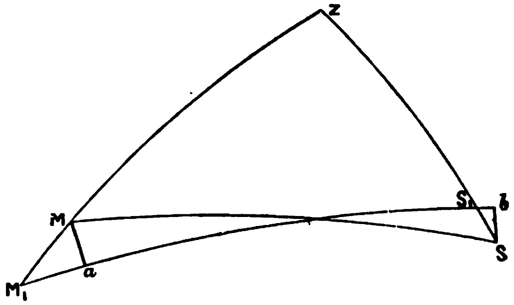
[1] The various approximations to clearing the lunar distance are based on the principle of dropping perpendiculars M*a*, S*b* (fig. 146) from M and S (the *true* places of

FIG. 146.



the moon and other heavenly body) on the observed distance $M_1 S_1$, and deriving the corrections to that observed distance from the small triangles $M M_1 a$ and $S S_1 b$.

This method of treatment appears due to Israel Lyons, afterwards astronomer with Captain Phipps in his Arctic voyage of 1773, and was published in the *Requisite Tables* of 1767. The reader will find much information on the subject of lunars in Shadwell's *Notes on Lunars*; in this book various refinements are considered which are not necessary for nautical purposes.

An accurate mode of clearing the lunar distance proposed by Sir G. Airy in 1883 is placed in the chart boxes supplied to H.M. ships.

In 1790 Margett published tables expressed graphically in curves, by which a lunar distance could be cleared by *measurement*. These were constructed on somewhat similar principles to those given by La Caille in 1759, and published in the *Conn. des Temps* for 1761, and by which a lunar distance could be *graphically* cleared within an error of 20″.

In 1678 Herne proposed to find longitude at sea by comparing the time of the moon's observed meridian passage with that given in the *Nautical Almanac* for Greenwich the interval from apparent noon being to the observer greater or less by the moon's proper motion due to difference of longitude (*see* page 217).

He proposed finding the time of the meridian passage by noting the times when it had equal altitudes on either side of the meridian. This method, though very inaccurate for sea observations, is in principle that of moon culminations used at present in astronomical observatories.

In 1771 Lemonnier proposed finding the longitude by the hour angle of the moon. This hour angle, obtained from an altitude, enabled an observer to find ship mean time, and hence the sidereal time of observation from which the right ascension of the moon could be found; this right ascension was then compared with that given in the *Nautical Almanac* for some previous time at Greenwich, and the longitude found similarly to the method by lunar distances. This method, which is not susceptible of great accuracy at sea, is used at present in astronomical observatories.

A method of finding the ship mean time (and thence the longitude) by observing the time of *true* sunrise or sunset (*see* page 333) was published in 1847 by Commander Weston, I.N.

To know this time, the altitudes of both the upper and lower limbs of the sun at the instant of the true centre being on the horizon are given in his tables for every half degree of latitude from the equator to 61° N. and S. and for every half degree of declination N. and S.

This method is, however, very unreliable in latitudes greater than 45° N. or S., and even in lower latitudes is liable to a probable error of half a minute in time, at least.

LONGITUDE BY EQUAL ALTITUDES.[1]

The principle of finding the apparent time of a heavenly body crossing the meridian by noting the times by a chronometer at which it had equal altitudes when near it on either side is applicable under certain circumstances at sea.

If the heavenly body observed is the sun, the mean of the times will (if the change of declination in the interval be neglected, and if the ship has been stationary) be the time by chronometer at ship apparent noon, from which the error of chronometer on ship

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[1] First used at sea by Picard, a French astronomer, in 1671.

mean noon can be found. If, then, the times of observation have been noted by a chronometer whose error on Greenwich mean time is known, a comparison can at once be made between ship and Greenwich mean times, and hence the longitude obtained.

This is equally true if the course in the interval has been East or West true, when the longitude found will be that at the middle time of observation, corresponding to the mean of the times.

By taking the observations shortly before and after noon the necessity for applying the correction due to change of declination is obviated, since the change in the interval is very small.

In latitude 10° N., the sun was observed to have equal altitudes when near the meridian at the following times by chronometer: **Example**

Before noon $6^h\ 10^m\ 22^s$; after noon $6^h\ 30^m\ 26^s$.

The run between the times of observation was West true 5 knots, the error of chronometer on Greenwich mean time was $2^h\ 2^m\ 10^s$ fast, and the equation of time 6^m additive to apparent time.

At the time of *apparent* noon on the meridian midway between those passing through the places of observation, the chronometer showed $6^h\ 20^m\ 24^s$, and at that instant the Greenwich mean time was $4^h\ 18^m\ 14^s$, while the ship mean time was $0^h\ 6^m\ 0^s$.

Hence the longitude of that meridian, or that of the ship at noon, was $4^h\ 12^m\ 14^s$, or 63° 3′ 30″ W.

When the course of the vessel is *not* East or West true between the times of observation, and it is desired to apply a correction for change of latitude, an approximate correction is applied to the middle time as follows:

Correction in seconds of time $=\frac{1}{15}$ change of lat. (expressed in seconds) × sec. lat. × cot. azimuth (*see* page 367), using as 'change of lat.' half the change in the interval between the observations.

This correction is subtracted from the mean of the times when the ship has approached the sun, and added when she has receded from it. The azimuth could be found from the compass bearing of the sun (corrected for total error), or it might be calculated from the formula sin az. = sin h . cos dec . sec alt., using for the value of h half the interval, or it may conveniently be taken from Weir's Azimuth diagram.

In low latitudes the correction for run may be omitted, if the latitude and declination are such that the observations are made under the most favourable circumstances—viz. with the sun on or near the prime vertical shortly before and after apparent noon.

NOTE.—The preceding problem is of restricted use as applied to the *sun*, since the movement in altitude will not be sufficiently rapid for accurate observation unless that altitude is at least 75°.

Suitable *stars* can, however, be found in *any* latitude, and in these cases, where the observations can be made in *twilight*, excellent results can be obtained, the difference between the ship mean time of meridian passage of the star and the mean of chronometer time corrected for error on G. M. T. (and also if necessary for run) being the longitude.

The reader may refer to Capt. Blackburne's *A* and *B Azimuth Tables* for illustrative examples of the utility of this method, and also of the facility with which by those tables the azimuth may be obtained in this case.

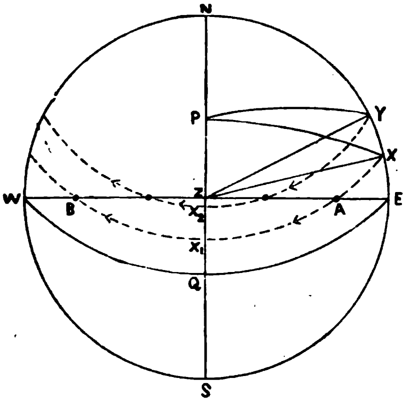
CHAPTER XI.

Astronomical observations for deviation of compass—Diurnal paths of heavenly bodies—Amplitudes—Altitude azimuths—Time azimuths—True bearing of terrestrial objects—Lengths of days—Time of twilight.

General observations

At sea, especially in iron vessels, throughout the day and night frequent suitable opportunities are taken of ascertaining the deviation of the compass; this is in all cases found by first computing to the nearest *minute* of arc the astronomical or *true* bearing of some heavenly body, and applying to that the variation (taken from the Variation chart), thus converting it into a *magnetic* bearing, which will differ from a *compass* bearing of the same body by an amount representing the deviation due to the direction of the ship's head at the time of observation.[1]

Fig. 147.



The deviations thus obtained from time to time are recorded in the Compass Journal, and compared with those obtained by swinging ship at some previous date; in all cases, however, the deviation *actually* deduced from observations at sea is that used for correcting courses.

Change of bearing

The changes in bearing of a heavenly body while above the horizon are illustrated by the accompanying figures, in which P represents the elevated pole, Z the zenith, and W Q E the equinoctial; in each case the daily path of the heavenly body is shown by a dotted line.

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[1] The *true* bearing is reckoned through 180° from North in N. lat. and from South in S. lat., being East or West according to the position of the heavenly body with regard to the meridian.

The *compass* bearing is necessarily, from the graduation of the card, read off from *either* North or South, increasing *only to* 90°.

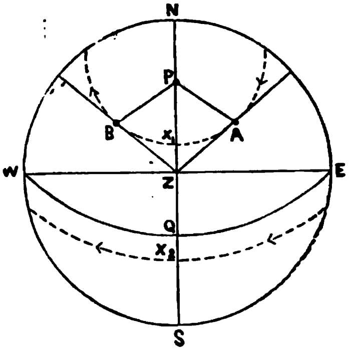
When the *deviation* has previously been carefully determined, the difference between the *observed* bearing (corrected for deviation) and the *true* bearing will give the variation; this, however, is seldom required to be determined in the practice of Navigation.

In fig. 147 the declination X_1 Q of the heavenly body X is supposed to be of the same name as the latitude, but not so great; in this case the azimuth P Z X, measured from the elevated pole P, increases from the time of rising until noon, and then decreases, the body crossing the prime vertical W Z E *twice*, once on the eastern side of the meridian at A, and again on the western side at B, at such times being equidistant from the meridian.[1] When the declination is nearly equal to the latitude as X_2 Q, the declination of a heavenly body Y, the times of crossing the prime vertical will be *near* that of crossing the meridian.

In fig. 148 the declination X_1 Q is supposed to be *greater* than the latitude and of the same name, in which case the body can never be on the prime vertical, and the azimuth or bearing from the elevated pole P will first *increase* until the body arrives at A (Z A being a tangent to the dotted curve, and the angle P A Z = 90°) and then *decrease* until the meridian passage, after which it will *increase* until the body arrives at B and then *decrease*.[2]

When the declination is of the *opposite* name to the latitude, as X_2 Q in fig. 148, the heavenly body can *never* be on the prime vertical.

FIG. 148.



When the declination *equals* the latitude, the body is in the zenith of the observer, and also on the prime vertical when crossing his meridian.

When the observer is on the equator and the body has no declination it is *always* on the prime vertical.

The reader should draw figures to illustrate these two latter cases.

The heavenly body will in all cases change its azimuth most rapidly when on the meridian, since it is then moving at right angles to the observer, and least rapidly when on the prime vertical or in the positions A and B (fig. 148).

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[1] This is *strictly* true only when the heavenly body considered is a fixed star whose declination does not vary.

[2] Thus in lat. 10° N., when the sun has 23° N. declination, it bears N. 66° 37′ E. on rising at 5^h 48^m A.M., and N. 69° 11′ E. at 7^h 40^m A.M., again bearing N. 66° 37′ E. at 9^h 12^m A.M., so that the shadow of an object at such a time would appear to 'go back' or retrograde 2° 34′.

True bearing

The true bearing of a heavenly body can be calculated by either of the three following methods:

First. By computing the true bearing from the East and West points at its rising or setting, termed its *amplitude.*

Second. By observing its altitude, and thence with its declination and the latitude of the observer determining the angle at the zenith or true bearing of the body: this angle, or the corresponding arc of the horizon which it subtends, is the *azimuth,* and this method of determining it is known as the Altitude Azimuth.

Third. By determining the hour angle; then, knowing the hour angle, declination, and latitude, the angle at the zenith or azimuth may be computed as before. This method is termed the Time Azimuth.

If the compass bearing is observed at the time of making either of the above observations, the *total error of the compass* —that is, the difference between the true and compass bearings— may be determined.

The total error of the compass is the combined effect of variation and deviation; the former is sufficiently well known for purposes of navigation over greater part of the navigable oceans, and is recorded on the Variation chart; the object in making the observation for error of compass is therefore almost always to ascertain the *Deviation,* which should only be expressed to the nearest minute of arc.

Note.—In those localities where the variation is not well known (which will be seen from the Variation chart) the navigator should find the variation by taking the difference between the true bearing of the sun and the mean of four or more equidistant compass bearings, which is the same as a magnetic bearing of the sun.

DEVIATION BY AMPLITUDE.

Definition

The amplitude of a heavenly body is the angle between the prime vertical and the circle of altitude passing through the body when on the horizon.

Theory

Proof of the rule for calculating the amplitude of a heavenly body.

In figs. 149, 150, let N W S E represent the horizon.
N Z S „ the celestial meridian.
W Z E „ the prime vertical.

Take Z Q = the latitude.
Q R = the declination.

Draw the parallel declination of R X, cutting the horizon at X.

Draw the circle of altitude Z X, and the declination circle P X.

In both figures P Z=90°−latitude.

When latitude and declination are of the same name

$$P X=90°-\text{decl.} \qquad \text{and } P Z X=90°-\text{ampl.}$$

When latitude and declination are of different names

$$P X=90°+\text{decl.} \qquad \text{and } P Z X=90°+\text{ampl.}$$

In the quadrantal triangle P Z X,

$$\text{Cos } P X=\sin P Z \cos P Z X.$$
$$\text{Cos } (90°\pm\text{decl.})=\sin (90°-\text{lat.}) \;.\; \cos (90°\pm\text{ampl.}).$$
$$\therefore \sin \text{decl.}=\cos \text{lat.} \;.\; \sin \;.\; \text{ampl.}$$
$$\therefore \sin \text{ampl.}=\sin \text{decl.} \;.\; \sec \text{lat.}$$

FIG. 149.

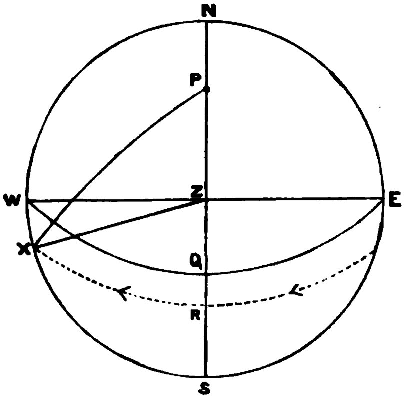
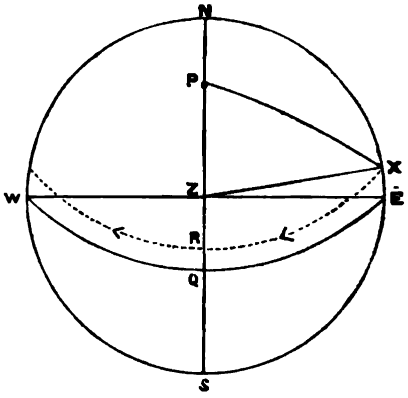


FIG. 150.



Also it is evident from the figure that the body will rise to the northward or southward of east and set to the northward or southward of west according as the declination is north or south.

Example Given the lat. of ship=47° 50′ N., long. 40° W., and the decl. of sun at rising=17° 54′ 44″ N.; to find the amplitude. If the compass bearing at the same time was N. 82° 50′ E., find the deviation of the compass, the ship's head being S.E. by S. by compass at time of observation.

(1) *To find the true bearing* (fig. 150).

Construction. Let X be the place of the sun at rising; then, in the quadrantal triangle Z P X are given the colat. P Z=42° 10′, the pol. dist. P X=72° 5′ 16″, and the zen. dist Z X=90°; hence the angle P Z X, and therefore its complement E Z X, may be found by the preceding formula.

sin ampl. = sin decl. . sec lat.
L sin decl.......... 9·487936
log sec lat........... 0·173090
∴ L sin ampl....... 9·661026
∴ amplitude = E. 27° 16′ N = E X.

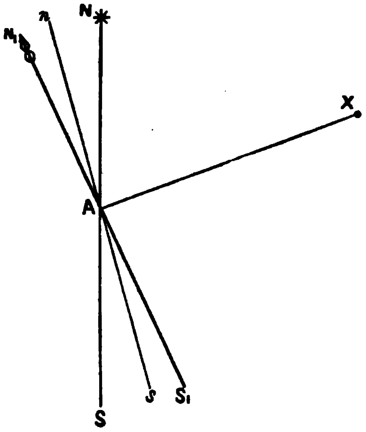
(2) *To find the deviation.*

Let N A S represent the true meridian; from the Variation chart the variation due to the ship's position is seen to be 33° W., hence N_1 A S_1 drawn at an angle of 33° with N A S will represent the magnetic meridian. Make N A X = 62° 44′ (90° − 27° 16′) = the *true* bearing of the sun; then X will be the true place of the sun (fig. 151).

The magnetic bearing of the sun will evidently be N. 95° 44′ E.,[1] which would also be the *compass* bearing but for the deviation.

The observed compass bearing being N. 82° 50′ E., if the angle X A *n* (= 82° 50′) is laid off from A, then *n* and *s* will represent the north and south points as shown by *compass*; hence the north point of the compass card *n* is drawn eastward of the magnetic north N_1 by an angle *n* A N_1, or, in other words, the deviation is 12° 54′ E.[2]

FIG. 151.



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[1] This may also be found by the fixed and movable compass cards on Plate 1.

[2] It is advisable at first to draw a figure for every case, but the reader will observe that when the true bearing, *however* determined, is to the *left* of the compass bearing, as seen from the centre of the compass card, the *total error* is *westerly*, and when to the *right* it is *easterly*.

Example September 19, at 5^{h} 51^{m} A.M. mean time nearly, in latitude 47° 25′ N., and longitude 72° 15′ W., the sun rose by compass N. 77° 50′ E. Required the deviation, the ship's head being E. by N.

Ship, Sept. 18........................ 17^{h} 51^{m}
longitude in time........................ 4 51 W.
Greenwich, Sept. 18 .... 22 42

or 22·7 hours—that is, 1·3 hours from noon, Sept. 19.

	Sun's Decl.	Var. in 1 Hour.		
Sept. 19	1° 37′ 13″ N.	58·2″	L sin decl.	8·457103
corr......	1 16	hours from noon, 1·3	log sec lat.	0·169628
	1 38 29	1746	L sin ampl.	8·626731
		582	amplitude	E. 2° 25′ N.
		75·66 = 1′ 16″	or true bearing...	N. 87° 35′ E.
			variation	13 30 W.
			magnetic bearing	S. 78 55 E.
			compass bearing	N. 77 50 E.
			deviation	23 15 E.

1. May 6, at 5^{h} 30^{m} A.M. mean time nearly, in lat. 50° 48′ N., and long. 47° 12′ E., the sun rose by compass S. 87° 50′ E. Required the deviation, the ship's head being S. by W. and the variation 24° 20′ W. *Ans.* 4° 44′ W.

2. Nov. 14, at 6^{h} 45^{m} P.M. mean time nearly, in lat. 32° 14′ S., and long. 100° E., the sun set by compass S. 74° 20′ W. Required the deviation, the ship's head being N.E. Var. 16° W.
Ans. 9° 44′ E.

3. Jan. 10, at 6^{h} 58^{m} A.M. mean time nearly, in lat. 31° 56′ N., and long. 75° 30′ W., the sun rose by compass S. 59° 50′ E. Required the total error, the ship's head being N.E. by E.
Ans. 3° 58′ W.

4. March 21, at 6^{h} 0^{m} A.M. mean time nearly, in lat. 42° 13′ N., and long. 90° E., the sun rose by compass S. 78° 20′ E. Required the deviation, the ship's head being W. by S. and the variation 3° 20′ W. *Ans.* 8° 46′ W.

5. March 31, at 6^{h} 0^{m} P.M. mean time nearly, in lat. 42° 13′ N., and long 124° W., the sun set by compass S. 78° 30′ W. Required the variation, the ship's head being N. (*see* Dev. Table, p. 21).
Ans. 20° 58′ E.

6. Dec. 4, at 7^{h} 50^{m} A.M. mean time nearly, in lat. 50° 40′ N., and long. 94° W., the sun rose by compass S. 79° 18′ E. Required the total error of the compass, the ship's head being N.E.
Ans. 26° 11′ E.

7. March 14, about 6^h 10^m A.M., in lat. 34° 50′ N., long. 14° 10′ E., the sun rose by compass E. by S.½S. Required the deviation of the compass, variation being 12° 10′ W. *Ans.* 1° 50′ W.

8. August 23, about 6^h 40^m A.M., in lat. 40° 30′ S., long. 80° E., the sun rose by compass N.E. by E.½E., the ship's head being S.S.W. Required the variation of the compass. *Ans.* 16° 5′ E.

9. October 3, in lat. 35° 54′ N., long. 15° 20′ E., the sun rose by compass E.S.E., the ship's head being S.S.W.½W. Required the variation of the compass from the azimuth tables.

Ans. 12° 37′ W.

10. May 3, in lat. 41° 20′ S., long. 102° 20′ E., the sun set by compass N.W. by W., the ship's head being S. by E. Required the variation of the compass from the azimuth tables.

Ans. 18° W.

11. May 25, about 8^h 40^m A.M. mean time, in lat. 14° 30′ S., long 9° 50′ W., when a chronometer showed 7^h 11^m 16^s, the sun bore by compass E by N.; the chronometer was slow on Greenwich mean time 2^h 7^m 40^s. Required the total error of the compass.

Ans. 10° 32′ W.

12. August 4, in lat. 49° 25′ N., long. 2° 23′ W., the sun rose by compass E.½N., the variation being 18° 40′ W. Find the deviation of the compass from the azimuth tables.

Ans. 2° 47′ W.

Elements from 'Nautical Almanac.'

Sun's Declination		Var. in 1 Hour.	Sun's Declination.		Var. in 1 Hour.
May 6	... 16° 43′ 42″ N.	... 41·62 incr.	Mar. 14	... 2° 15′ 0″ S.	... 59·09 decr.
Nov. 14	... 18 25 19 S.	... 38·57 incr.	Aug. 23	... 11 13 46 N.	... 51·21 decr.
Jan. 10	... 22 0 11 S.	... 22·13 decr.	Oct. 3	... 4 13 38 S.	... 57·96 incr.
Mar. 20	... 0 7 11 N.	... 59·22 incr.	May 3	.. 15 52 32 N.	... 42·65 incr.
Apr. 1	... 4 48 44 N.	... 57·71 incr.	May 25	... 21 4 25 N.	... 26·32 incr.
Dec. 4	... 22 20 44 S.	... 19·38 incr.	Aug. 4	... 17 4 47 N.	... 38·96 decr.

NOTE.—The observation for amplitude of the sun should be made when the centre of the *true* sun is on the horizon; this, from the effect of the great refraction on bodies on the horizon (34′), occurs in low latitudes (where the sun rises and sets almost perpendicularly to the horizon) at a time when the lower limb of the *apparent* sun has an altitude about equal to its semidiameter plus the dip due to the observer's height above the sea. In high latitudes, especially when the declination is of an *opposite* name to the latitude, the sun rises and sets very obliquely to the horizon, and the precise time for observation is more difficult to estimate, since the effect of the dip on such observations is also under these circumstances greater, and either the method of finding deviation by azimuth is to be preferred, or else make the observation when the *centre* of the body considered is *seen* on the horizon, and apply a correction from Inman's Tables, No. 26.[1] Observations of the moon and stars for amplitude are seldom available,[1] though occasionally the *setting* amplitude of a bright star or planet may be observed. In practice the amplitude of the sun is taken by inspection from the azimuth tables or that in Inman's Tables, No. 49, and the reader should compare the results of calculation in the examples with those obtained by inspection, employing the equation of time given in those tables for any leap year.

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[1] A table correcting amplitudes for this source of error was first published by Pierre Bouguer in 1731. This essay on the mode of finding variation at sea received

The true bearings of heavenly bodies whose declinations exceed 23° are conveniently taken from a graphic curve known as Weir's azimuth diagram.

On reference to fig. 152 it will be seen that to an observer on the equator the amplitude of the sun is equal to the declination, and that when the sun has *no* declination it rises and sets east and west *true* in *all* latitudes.

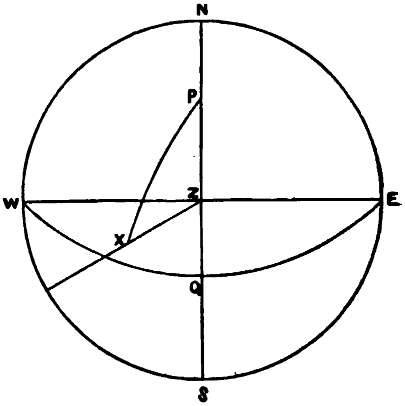
What is sometimes termed the *time* amplitude of a heavenly body is the interval between its rising and being on the six o'clock hour circle.

DEVIATION BY ALTITUDE AZIMUTH.

The altitude azimuth

Given the altitude and declination of a heavenly body, and latitude of the place; to find the azimuth.[2]

FIG. 152.



Let N S W E represent the horizon, N Z S the celestial meridian, P the pole, and Z the zenith. Then if X be the place of the heavenly body when its altitude was observed, we have given, in the spherical triangle P X Z, the three sides, to find an angle; namely, the zenith distance Z X, the polar distance P X, and the colatitude P Z, to find the angle P Z X = the azimuth or true bearing of X.

Example

Given the lat. = 51° 10′ S., long. 144° 20′ E., the true alt. of sun's centre = 15° 25′ 30″, and its decl. = 16° 41′ 42″ S.; required the true bearing, the sun being east of meridian. If the compass bearing was observed at the same time to be S. 70° 34′ E., required also the deviation of the compass. (Ship's head West.)

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a prize from the French Academy. The correction to be applied to an amplitude of the sun observed when the centre of the *apparent* sun is on the horizon is nearly 1° in lat. 50°, with declination 23° of opposite name to the latitude, and at a height of 16 feet above the sea.

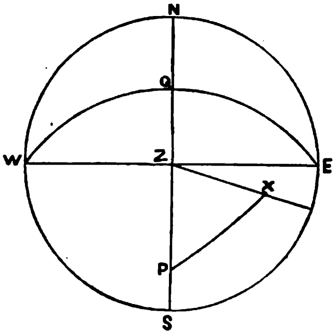
[1] On the horizon the sun is raised by refraction about 34′ and depressed by parallax about 8″; the moon is, however, more depressed by parallax (average 57′) than it is raised by refraction (34′), and consequently when the moon's centre is first *seen* it is *really* about 23′ *above* the horizon; the sun's centre, and also stars, when first seen, are nearly 34′ *below* the horizon (*rational* horizon in all cases).

[2] In 1790 Margetts published tables expressed graphically in curves, from which the azimuth could be taken by *measurement*.

(1) *To find the true bearing* (fig. 153).

In the spherical triangle P Z X are given the three sides, Z X the zenith dist. = 74° 34′ 30″, P X the pol. dist. = 73° 18′ 18″, and P Z the colat. = 38° 50′; to find the angle P Z X, the true bearing of the sun.

FIG. 153.



Z X......	74° 34′ 30″..............	log cosec	0·015932
P Z......	38 50 00	,, ...	0·202693
	35 44 30		
P X......	73 18 18		
	109 2 48	½ L hav	4·910810
	37 33 48	,, ...	4·507796
		L hav	9·637231
		∵ P Z X =	82° 23′
		or true bearing =	S. 82 23 E.

An alternative method would be to employ a formula similar to that on p. 203, from which we have

$$\sin \tfrac{1}{2}\ \text{azimuth} = \sqrt{\frac{\cos s \cos (s \sim p)}{\cos l \,.\, \cos a}}$$

where l = latitude; p = polar distance; a = altitude, and $s = \dfrac{a+l+p}{2}$, and the solution of the above example is as follows:

Polar dist.......	73° 18′ 18″		
Lat.......	51 10 00...	sec	·202693
Alt.......	15 25 30.........	,, ...	·015932
2)	139 53 48		
s =	69 56 54.........	cos	9·535091
P D − s =	3 21 24.........	,, ...	9·999255
		2)	19·752971
		sin ½ az. =	9·876485

or azimuth from North = N 97° 37′ E as before.

(2) *To find the deviation of the compass.*

On referring to the Variation chart it is seen that in the position of the ship the variation is 10° E., whence a similar figure may be drawn to fig. 151, or we may treat the true *bearing* as a true *course* and convert it into a *magnetic* bearing as on page 19.

True bearing of sun	S. 82° 23′ E.
Variation.................................	10 00 E.
Magnetic bearing of sun	S. 92 23 E.
Compass bearing of sun	S. 70 34 E.
Deviation	21 49 W.

Example June 7, at 5ʰ 50ᵐ A.M. mean time nearly, in lat. 50° 47′ N. and long. 99° 45′ W., when the sun bore by compass N. 44° 20′ E., the observed altitude of the sun's lower limb was 18° 35′ 20″, index correction + 3′ 10″, and the height of the eye above the level of the sea was 19 feet. Required the deviation, the ship's head being N.E. by compass.

Ship M. T., June 6.....................	17 50ᵐ
longitude in time	6 39 W.
Greenwich, June 7.....................	0 29 = 0·5 hour.

Sun's Declin.				Sun's Semi.		Obs. Alt.			
June 7 .	23°	43′	49″ N.	Var. in 1 hour 14·31″	15′ 46″		18°	35′	20″
corr. ...			7	hour from noon 0·5		Index corr....		3	10 +
	23	43	56	7·155			18	38	30
pol. dist.	66	16	4			dip		4	17 −
							18	34	13
						corr. in alt.		2	43 −
							18	31	30
						semid. ...		15	46 +
						true alt. ...	18	47	16

Latitude	50°	47′	0″	log sec.........	0·199108
altitude	18	47	16	„	0·023779
	31	59	44		
polar distance	67	16	4		
sum	99	15	48 (S.)	½ L hav (S.)	4·881893
difference....................	35	16	20 (D.)	½ „ hav (D.)	4·481384
				L hav......	9·586164

True bearing...............	N. 76° 46′ E.
Variation	15 25 E. (from Variation Chart)
Magnetic bearing.........	N. 61 21 E.
Compass „	N. 44 20 E.
Deviation	17 1 E.

13. On April 28, about 8h 40m A.M. mean time, in lat. 34° 45′ N., long. 48° 20′ W., the sun bore by compass S. 70° E., and the true altitude of its centre was 43° 56′. Required the total error of the compass. *Ans.* 5° 26′ W.

14. On May 4, about 7h 45m A.M. mean time, in lat. 51° 30′ N., long. 27° 5′ W., the sun bore by compass S. 80° E., and the observed altitude of the sun's L. L. at the same time was 29° 18′; the index correction was nil, and the height of the eye above the sea 19 feet, the ship's head being S.W. Required the deviation, the variation being 10° W. *Ans.* 11° 31′ E.

15. On December 3, about 4h 50m P.M. mean time, in lat. 42° S., long. 135° E., the sun bore West by compass, and the observed altitude of the sun's L. L. at the same time was 25° 22′, the index correction was −2′ 10″, and the height of the eye above the sea 18 feet, the deviation being 7° 45′ W. Required the variation. *Ans.* 0°.

16. On March 7, at 9h 10m A.M. mean time nearly, in lat. 51° 10′ N. and long. 89° 12′ E., the compass bearing of the sun was S. 54° 50′ E., and at the same time the observed altitude of the sun's lower limb was 21° 40′ 43″, the index correction was −2′ 18″, and the height of the eye above the sea was 14 feet. Required the total error of the compass. *Ans.* 5° 54′ E.

Elements from 'Nautical Almanac.'

	Sun's Declination.	Var. in 1 Hour.	Semidiameter.
April 28	14° 17′ 26″ N.	46·89″ incr.	15′ 56″
May 4	16 5 35 N.	43·14 ,,	15 53
Dec. 3	22 10 44 S.	20·73 ,,	16 16
Mar. 7	5 5 32 S.	58·38 ,,	16 9

DEVIATION BY TIME AZIMUTH.

Given the hour angle, declination of sun, and the latitude; to find the azimuth.

The time azimuth

(1) *To find true bearing of sun.*

Let N W S E (fig. 152) represent the horizon, N Z S the celestial meridian, P the pole, Z the zenith, and X the place of the sun when the time by chronometer was noted; then, if the error of the chronometer on apparent time at the place is known, the hour angle P may be computed; we have then, in the triangle P Z X, two sides and the included angle given to find an angle, namely.

the polar distance P X, the colat. P Z, and the included angle P, to find the angle P Z X, the true bearing or azimuth.

(2) *To find the deviation of the compass.*

Example

If the compass bearing of the sun was also observed when the time was noted, the deviation of the compass may then be determined as before in the cases of the amplitude and altitude azimuth.

Given the hour angle of the sun $=1^h\ 12^m\ 34^s$ (west of meridian), decl. $=13°\ 44'\ 21''$ N., latitude of the ship $=50°\ 48'$ N.; to find the true bearing. If the compass bearing at the time of observation was S. 51° 55′ W., required also the deviation of the compass; ship's head S.E. by S.

(1) *To find true bearing p z x* (fig. 152, page 322).

Let X be the place of the sun. Then, in the spherical triangle P Z X, are given P X $=76°\ 15'\ 39''$, P Z $=39°\ 12'$, and the angle P $=1^h\ 12^m\ 34^s$; to calculate the angle P Z X, the true bearing or azimuth.[1]

Calculation.

	39°	12′	0″		1.		2.
	76	15	39	L cot ½ P.	10·796826	L cot ½ P ...	10·796826
	115	27	39	„ cos ½ diff.	9·976882	„ sin ½ diff.	9·502137
	37	3	39	„ sec ½ sum	10·272522	„ cosec ½ sum	10·072869
½ sum	57	43	49	„ tan	11·046230	„ tan.........	10·371832
½ diff.	18	31	49		84° 51′ 45″		66° 59′ 15″
P =	1^h	12^m	34^s				84 51 45
½ P =	0	36	17			∴ true bearing = N.	151 51 0 W.
							= S. 28° 9′ W.

(2) *To find the deviation of the compass.*

The variation at the ship's position is taken from the Variation chart (36° W.), and on applying this as in the former examples to the true bearing of the sun the magnetic bearing is found to be S. 64° 9′ W.; the difference between this and the observed compass bearing (12° 14′) is the easterly deviation, due to the direction of the ship's head at time of observation.

The time azimuth problem may be conveniently solved by the application of the fundamental formula

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[1] In considering the case where the angle P Z Y is nearly 90′, and there is a doubt as to whether the angle found or its supplement is that required, it is well to remember that ½(P Z + P X) is of the same *affection* as ½(P Z X + P X Z); that is, they are both either greater or less than 90°.

The required angle may otherwise be obtained by first computing the third *side* of the triangle, and then applying the rule of sines, remembering that the greater angle is opposite to the greater side.

$$\cot A \sin B = \cot a \sin c - \cos B \cos c$$

which, applied to the data of the problem under discussion, becomes—

$$\cot A \sin h = \tan d \cos l - \cos h \sin l$$
$$= \cos h\left(\frac{\tan d}{\cos h} \cos l - \sin l\right)$$

Then, making $\frac{\tan d}{\cos h} = \tan \phi$, and dividing by $\sin h$

$$\cot A = \cot h\ (\tan \phi \cos l - \sin l)$$
$$= \frac{\cot h}{\cos \phi} (\sin \phi \cos l - \cos \phi \sin l)$$
$$= \frac{\cot h}{\cos \phi} \sin (\phi - l)$$

Or $$\tan A = \frac{\cos \phi \tan h}{\sin (\phi - l)}$$

which gives the required azimuth sought for, by aid of the auxiliary angle ϕ.

Example.—Latitude 10° 20′ N., declination 22° 14′ S., hour angle $1^h\ 44^m\ 17^s$ W. required the azimuth.

$\tan \phi = \frac{\tan d}{\cos h}$	$\tan A = \frac{\cos \phi \tan h}{\sin (\phi - l)}$
−9·611480	+9·959129
+9·953398	+9·689543
−9·658082	+9·648672
$\phi = -24°\ 28'\ 9''$	−9·756445
$l = +10\ \ 20\ \ 0$	−9·892227
$(\phi - l) = -34\ \ 48\ \ 9$	Azimuth = N. 142° 2′ 15″ W.

June 23, at $10^h\ 58^m$ A.M. mean time nearly, in lat. 50° 48′ N. and long. 1° 6′ W., when a chronometer showed $11^h\ 3^m\ 37^s$, the compass bearing of the sun was observed to be S. 6° 50′ E. (N. 173° 10′ E.), the error of the chronometer on Greenwich mean time being $0^m\ 54^s$ fast. Required the total error of compass. **Examples**

		Equation of Time.		Sun's Declination.
June 22, at	$22^h\ 58^m$	23d	1^m 42·68 sub.	23° 27′ 00″ N.
long. in time	4+	var. in 1 hr.	0·54	00
Gr., June 22	23 2		1 42·14	23 27 00
	(or 1 hour from			90
	noon, June 23).			pol. dist. 66 33 00

colat.	39°	12′	0″
pol. dist................	66	33	0
sum	105	45	0
difference	27	21	0
½ sum..................	52	52	30
½ difference	13	40	30

					(1)		(2)
Time by chron. ...	11^h	3^m	37^s + 12^h	L cot 31^m 41^s	10·856573	....	10·856573
error on Gr. M. T.		0	54 fast.	,, cos ½ diff. .	9·987511	L sin ½ diff....	9·373674
Greenwich, 22d. .	23	2	43	,, sec ½ sum .	10·219283	,, cosec ½ sum	10·098368
equation of time .		1	42 sub.	,, tan	11·063367	,, tan.........	10·328615
Greenwich apparent time	23	1	1				
longitude in time .		4	24 W.		85° 3′ 30″		64° 51′ 30″
ship app. time ...	22	56	37		64 51 30		
	24			True bearing N.	149 55 0 E.		
hour angle	1	3	23	comp. ,, N.	173 10 0 E.		
½ hour angle	0	31	41	total error	23 15 0 W.		

17. April 27, at 1^h 10^m P.M. mean time nearly, in lat. 50° 48′ N. and long. 1° 6′ W., when a chronometer showed 1^h 15^m 51^s, the bearing of the sun was observed to be S. 51° 55′ W., the error of the chronometer on Greenwich mean time being 1^m 18^s fast. Required the variation, the ship's head being S.S.W. (*see* page 21). *Ans.* 20° 46′ W.

18. December 14, at 10^h 22^m A.M. mean time nearly, in lat. 52° 10′ N. and long. 1° 30′ W., when a chronometer showed 10^h 30^m 48^s, the bearing of the sun was observed to be S. 0° 40′ E., the error of the chronometer on Greenwich mean time being 3^m 38^s fast. Required the total error of the compass. *Ans.* 21° 13′ W.

19. December 14, at 1^h 55^m P.M. mean time nearly, in lat. 48° 50′ N. and long. 1° 30′ W., when a chronometer showed 1^h 59^m 55^s, the bearing of the sun was observed to be S. 51° 40′ W., the error of the chronometer on Greenwich mean time being 0^m 5^s fast. Required the deviation, the variation being 12° 44′ W. *Ans.* 10° 44′ W.

20. December 14, at 11^h 11^m A.M. mean time nearly, in lat. 36° 40′ N. and long. 0° 40′ E., when a chronometer showed 11^h 19^m 43^s, the bearing of the sun was observed to be N. 167° 50′ E., the error of the chronometer on ship apparent time at 9 A.M. being 3^m 38^s fast, and the subsequent run S.E. (true) 20′. Required the variation, the ship's head being N.½E. (p. 21). *Ans.* 2° 50′ E.

21. March 7, at 9^h 59^m A.M. mean time nearly, in lat. 49° 48′ N. and long. 1° 10′ E., when a chronometer showed 10^h 24^m 8^s, the bearing of the sun was observed to be N. 164° 51′ E., the error of the chronometer on ship apparent time being fast 20^m 48^s. Required the total error of compass. *Ans.* 20° 28′ W.

22. May 26, at 9^h 48^m A.M. mean time nearly in lat. 50° 48′ N.

and long. 1° 6′ W., when a chronometer showed $9^h\ 47^m\ 37^s$, the bearing of the sun was observed to be S. 31° 7′ E., the error of the chronometer on Greenwich mean time being $3^m\ 17^s$ fast. Required the deviation, the variation being 33° 35′ W.

Ans. 10° E.

Elements from 'Nautical Almanac.'

	Sun's Declination.		Equation of Time.
April 27	13° 43′ 22″ N.	April 27	$2^m\ 24{\cdot}56^s$ add.
Var. in 1 hour,	47·47″ incr.	Var. in 1 hour,	$0{\cdot}397^s$ incr.
Dec. 14	23° 12′ 38″ S.	Dec. 14	$5^m\ 15{\cdot}08^s$ add.
Var. in 1 hour,	8·38″ incr.	Var. in 1 hour,	$1{\cdot}203^s$ decr.
March 7	5° 6′ 55″ S.	March 7	$11^m\ 10{\cdot}81^s$ sub.
Var. in 1 hour,	58·38″ decr.	Var. in 1 hour,	$0{\cdot}616^s$ decr.
May 26	21° 4′ 25″ N.	May 26	$3^m\ 19{\cdot}55^s$ add.
Var. in 1 hour,	28·62″ incr.	Var. in 1 hour,	$0{\cdot}265^s$ decr.

NOTE.—Tables from which the results of an altitude azimuth of the sun could be taken by inspection were calculated in 1825 by Commander Lynn, of the Indian Navy; but the more usual practice in the Royal Navy is to obtain the deviation by *time* azimuth, for which convenient tables, computed for each degree of lat. from 0° to 60° N. and S., each degree of declination from 23° N. to 23° S., and every four minutes of apparent time are placed in the chart boxes; the reader should carefully read the introductory remarks in these, and compare the results of calculation in each of the examples here given with those obtained from these tables.

The errors in observed bearings resulting from the compass card not being truly horizontal increase rapidly with the altitude of the body observed; and therefore, these tables do not extend to greater altitudes than 60°.

It will be also seen from these tables that a heavenly body moves least rapidly in azimuth when on the prime vertical, which is therefore the most favourable time for observation; in this case the *true* bearing is East or West, and a comparison with a *compass* bearing at such a time gives the total error of the compass, and hence the deviation.

The tables above referred to are also applicable to all those heavenly bodies whose declinations lie between the same limits as the declination of the sun (*i.e.* 23° N. to 23° S.); this includes the moon during greater part of each month, the principal planets, and several conspicuous stars.

A list of available stars is given in the introduction to these tables.[1]

Towson's great circle tables and lunar index also give the true bearings of a heavenly body, whose hour angle is known, since the true bearing is the great circle course to the place where the body is in the zenith.

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[1] The earliest and long the only method used to obtain the error of a compass was by bearings of the Pole-star.

A table giving the true bearing of Polaris for each hour of sidereal time is given in Inman's Nautical Tables. It is seen on referring to fig. 131, page 251, that if the first corr. is taken out of the *Nautical Almanac* for sid. time + 6 hours it will give the rectangular deviation (d) of the Pole-star from the meridian, the corresponding angle on the horizon, or bearing, being found from this formula:—$\sin \text{bearing} = \sin d \,.\, \sec \text{alt}$.

When observing on the prime vertical for this purpose, an error of several minutes in the apparent time will produce but small effect; the altitude should not, however, be great.

In finding the deviation by star observations preference should be given to those near the pole, their movement in azimuth being necessarily slow.

The principle of Captain Blackburne's valuable A and B Tables (which are useful among other purposes for determining azimuths) is as follows:

In fig. 152, on page 322, by the ordinary spherical formula,

$$\cot \text{PX} \sin \text{PZ} = \cot \text{PZX} \sin \text{ZPX} + \cos \text{PZ} \cos \text{ZPX},$$

whence by transposition and using the usual abbreviations

$$\cot \text{azimuth} = -(\tan l \cot h) + (\tan d \cos l \cos h).$$

The quantities on the right-hand side of this equation are tabulated in natural numbers for various values of l, d, and h, and termed A and B, by the addition or subtraction of which the azimuth is readily determined.

The deviation of a compass can also be found from the bearing of a heavenly body on the *meridian*, since it then must be North or South true, but this is an unfavourable position for observation unless the lat. and dec. are of opposite names and the body far from the zenith.

TO FIND THE TRUE BEARING OF A TERRESTRIAL OBJECT.

The true bearing of a terrestrial object from a given position may be found by observing with a sextant its angular distance from the sun or any other heavenly body whose true bearing is known or can be calculated.[1]

If the object is on the horizon of the observer, the apparent zenith distance of its place is evidently 90°; if elevated above the horizon, as the summit of a mountain, its altitude must be observed, and thus its zenith distance determined.

Theory Given the angular distance of a terrestrial object from the sun, and the true bearing of the sun; to find the true bearing of the object (fig. 154).

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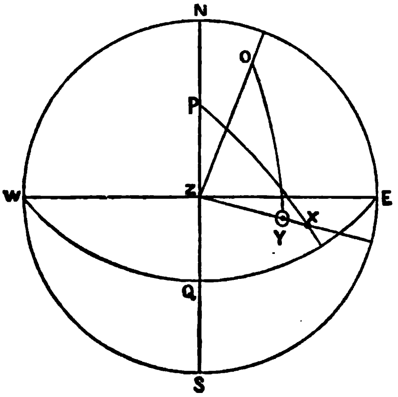
[1] This problem is chiefly of use in nautical surveying, but is sometimes employed in navigation when the deviation of the compass is to be ascertained from the bearing of a distant object.

The more usual practice is, however, to measure the *horizontal* angle, or difference of bearing required, by means of the azimuth circle of the standard compass, or by taking the difference between the two *compass* bearings of the sun and object.

Let O be the place of the terrestrial object (that is, its projection on the celestial concave), X the true place of the sun's centre; and let N S represent the celestial meridian, P the pole, and Z the zenith of the observer. Through O and X draw the circles of altitude O Z, Y Z, and through X the circle of declination P X. Then the true bearing P Z O of the object O is found by computing (1) the true bearing of the heavenly body X, namely the angle P Z X; and (2) the angular distance of the object O from X, namely the angle O Z X: the difference (in this case) will be the bearing of O, namely the angle P Z O.

FIG. 154.



(1) *To find the angle p z x (by altitude azimuth).*

In the spherical triangle P Z X are given P Z the colatitude, P X the polar distance, and Z X the true zenith distance of the sun; to calculate the angle P Z X, the bearing of sun.

(2) *To find the angle p z x (by time azimuth).*

In the spherical triangle P Z X are given P Z the colatitude, P X the polar distance, and hour angle Z P X (determined from ship mean time and the equation of time); to calculate the azimuth P Z X.

(3) *To find the angle o z y.*

Let Y be the apparent place of the sun's centre; then O Y is the observed angular distance of the object O from the sun's centre (= distance measured with a sextant from sun's nearest limb, corrected for instrumental errors, + sun's semi.). In the spherical triangle O Z Y are given O Z = apparent zenith distance of point O, Z Y = apparent zenith distance of sun's centre, and O Y = observed angular distance of object from sun; to calculate the angle O Z Y. The triangle O Z Y is quadrantal when the object is on the horizon, since, then, Z O = 90°.[1]

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[1] In this case, which is the more frequent in practice, it will be seen that, from the properties of quadrantal triangles,

$$\text{cos diff. bearing of sun and object} = \frac{\text{cos apparent distance}}{\text{cos apparent alt. of sun}}.$$

[over

The most favourable circumstances for observation are when the sun is low or on the prime vertical, and the object having the same altitude as the sun is 90° from it. Thus when the sun is rising or setting the circumstances are favourable and the calculation simple.

The preceding illustration and remarks are applicable to *any* heavenly body.

Note.—The true bearing of a distant object thus determined will differ from the bearing taken from a Mercator's chart by an equal amount, and for the same reason that the *rhumb* course from the ship to the distant object differs from the *great circle* course; it follows from this that three terrestrial objects which to the eye appear to be in line will not in general be thus represented on a Mercator's chart (*see* also note on page 39).

It will also be remarked that since the meridians passing through two places A and B are nowhere parallel to one another except at the equator, the bearing of A from B will not be the reverse of that of B from A unless A and B are true north and south of one another.

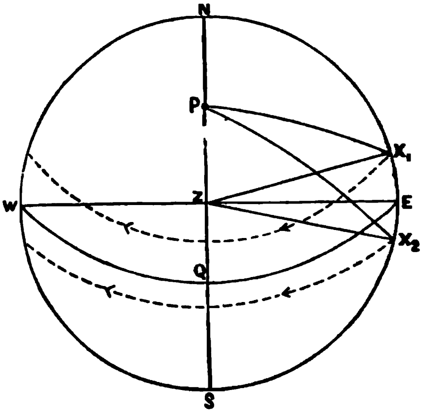
LENGTH OF DAY.

The interval of time between the rising and setting of a heavenly body is (if change of declination be neglected) twice its hour angle, or angular distance from the meridian, when on the horizon.

This hour angle is termed the semi-diurnal arc.

Thus in fig. 155 if P and Z represent the pole and zenith, N Z S being the meridian and W Q E the equinoctial, a heavenly body X_1 or X_2 will be above the horizon for a period measured by twice the angle ZPX_1 or twice ZPX_2.

Fig. 155.



The value of this angle is readily calculated by spherical trigo-

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The angle O Z X will be less or greater than the arc measured by the sextant, according as that arc is less or greater than 90°.

If the difference of bearing is measured with the azimuth circle of the standard compass, or by taking the difference of the *compass* bearings of the two objects (the ship's head in both cases remaining perfectly steady), no correction or reduction is required.

nometry, since in either of the cases of the declination being of the same or opposite names to the latitudes the triangles ZPZ_1 and ZPX_2 are quadrantal, the zenith distance ZX_1 or ZX_2 being 90°; knowing, then, the polar distance PX_1 or PX_2 and the colatitude PZ, the required angles are easily found from the formula $\cos h = -\tan l \,.\, \tan d$ or its equivalent $\tan \frac{h}{2} = \sqrt{\frac{\cos (l \sim d)}{\cos (l + d)}}$.[1]

If the heavenly body considered be the sun, then the angular distance from meridian, when on the horizon, is the apparent time from noon of rising or setting, and twice this angle will be (approximately) the length of the apparent solar day.

If the body considered be other than the sun, this angle applied to the time of meridian passage will give the time of rising and setting.

It is also seen from the figure that if PE and PW be joined by arcs of circles of declination, every heavenly body on arriving on these arcs will be at an angular distance of 6 hours or 90° from the meridian; such circles (as previously stated) are termed *six o'clock hour circles.*

From fig. 155 it will be seen that if we neglect the *change* of declination—

(1) When the declination is 0 the sun rises at 6 A.M. and sets at 6 P.M. in *all* latitudes (since WQ and QE each = 90°).

(2) When the latitude is 0 the sun rises at 6 A.M. and sets at 6 P.M., whatever the declination may be, the angle at the pole ZPX_1 being in this case always 90° or 6 hours.

(3) When the declination is of the *same* name as the latitude the sun rises before 6 A.M. and sets after 6 P.M., the apparent solar day being *more* than 12 hours in length.

If the polar distance is less than the latitude the sun neither rises nor sets, but is always above the horizon.

(4) When the declination is of the *opposite* name to the latitude the sun rises after 6 A.M. and sets before 6 P.M., the apparent solar day being *less* than 12 hours in length.

(5) The longest day occurs when the declination of the sun is greatest and of the same name as the latitude, and the shortest day occurs when the sun has a maximum declination of the opposite name to the latitude.

Also in every place the sum of the longest and shortest days is 24 hours.

In London the number of hours the sun is above the horizon varies from 18 to 6, and the number of hours the moon is above the horizon varies from 17½ to 7.

Times of rising and setting

The times of rising and setting of a star can readily be found by calculating the time of meridian passage and applying to this the calculated semi-diurnal arc.

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[1] For the sun and those heavenly bodies whose declinations do not exceed that of the sun these angles or times of rising and setting have been calculated, and form No. 49 of Inman's Tables; they are also given in the Time Azimuth tables previously mentioned. The local or ship times of rising and setting are of course the same for all places on the same parallel of latitude, but it must be remembered that these calculations can only be made for a position at the centre of the earth and referred to the rational horizon, in other words no allowance is made for dip, refraction, or parallax.

In the cases of the sun, moon, and planets the result obtained by working with an *approximate* declination would require a further correction for the *change* of declination.

The preceding remarks on rising and setting and lengths of days apply to the *centres* of the heavenly bodies considered, and to the *true* times and rational horizon; those of *apparent* rising and setting will depend upon the dip and refraction, and in the case of the moon on parallax.

Dip and refraction will accelerate the time of rising and retard that of setting, while parallax will produce contrary effects.

The actual time of *visible* sunrise or sunset may be found by calculating the hour angle for a zenith distance of 90° + dip + 33′ (or Refr. on horizon) and the difference between this hour angle and that found by working with zenith distance 90° (or taken out of the azimuth tables) will be the time by which the sun will rise *earlier* and set *later* than the predicted time.

In the case of the moon a similar calculation must be made, using zenith distance $= 90° - \text{hor. par.} + \text{dip} + \text{Refr.}$ but since the hor. par. exceeds on board ship the dip + refr. he rising will be *retarded* and the setting *accelerated*.

Twilight

Owing to the refraction of light and its reflection from the upper clouds in the tmosphere a certain amount of light is received from the sun when *below* the horizon.

Theoretically this twilight begins or ceases when the sun is 18° below the horizon, and the time when it does so may be calculated by finding the value of the angle ZPX_1 (fig. 155), on the supposition that $ZX_1 = 90° + 18°$, or 108°.

When the sum of the latitude and declination is not less than 72° (the complement of 18°) twilight lasts all night; thus in the latitude of London there is twilight throughout the night from May 22 to July 21, since during that time the sun is never more than 18° below the horizon. Twilight is necessarily short within the tropics, since the apparent path of the sun is more perpendicular to the horizon than in higher latitudes, and hence the 18° of additional zenith distance is more rapidly traversed.

CHAPTER XII.

Errors of chronometers—The artificial horizon—Absolute altitudes—Equal altitudes —Advantages of equal altitudes—Rates of chronometers—Secondary meridians.

Error by time ball

AFTER the chronometers have been received on board from the Royal Observatory their errors on Greenwich mean time and rates are usually daily ascertained before commencing a voyage by noting the time of falling of a time ball dropped from some conspicuous place, generally at 1 P.M. Greenwich mean time.[1] This time is noted by the hack watch, which is compared with all the box chronometers immediately before or afterwards.

Examples

At Portsmouth on May 1 the time ball dropped at $4^h\ 15^m\ 8{\cdot}4^s$, as shown by hack watch, and a few minutes afterwards the following comparisons were made with chronometers A and B. Required their errors on Greenwich mean time.

	(1)	(2)
	A $5^h\ 10^m\ 20^s$	B $3^h\ 12^m\ 40^s$
Hack watch	4 18 32·8	4 19 5·2

On reference to the Time Signal list it is seen that the ball dropped at 1 P.M. Greenwich mean time; hence the hack watch is $3^h\ 15^m\ 8{\cdot}4^s$ fast. On taking the difference between the times under (1) it is seen that chronometer A is $0^h\ 51^m\ 47{\cdot}2^s$ fast of the hack watch; hence A must be $4^h\ 6^m\ 55{\cdot}6^s$ fast of Greenwich. On similarly taking the difference of the times under (2) it will be found that chronometer B is $1^h\ 6^m\ 25{\cdot}2^s$ slow on the hack watch, and therefore $2^h\ 8^m\ 43{\cdot}2^s$ fast on Greenwich.

1. On April 2, when in the Downs, the time ball at Deal dropped at $2^h\ 12^m\ 8{\cdot}4^s$ by hack watch, and shortly afterwards comparisons were made as follows with chronometers A, B, and C:

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[1] A complete list of places in the world where time balls are dropped or other time signals made is placed in the chart box; this list gives both local and Greenwich mean times at which the signals are made.

	(1)	(2)	(3)
	A $10^h\ 50^m\ 10^s$	B $7^h\ 20^m\ 0^s$	C $1^h\ 11^m\ 54^s$
Hack watch	2 15 4·2	2 15 56·8	2 16 20

2. On April 5 at Portsmouth similar observations were made, the time ball there dropping at $2^h\ 12^m\ 21{\cdot}2^s$ by hack watch, and the following being the subsequent comparisons with A, B, and C:

	(1)	(2)	(3)
	A $10^h\ 48^m\ 41^s$	B $7^h\ 18^m\ 30^s$	C $1^h\ 10^m\ 26^s$
Hack watch	2 14 1·5	2 14 30	2 15 20·1

Required the errors of A, B, and C on Greenwich mean time on April 2 and April 5, together with their daily rates (*see* page 354).

NOTE.—From the list of time signals it is seen that at each place the time ball dropped at 1 P.M. G. M. T.

Ans. Errors on April 2: A $2^h\ 12^m\ 45{\cdot}8^s$ slow; B $5^h\ 43^m\ 48{\cdot}4^s$ slow; C $0^h\ 7^m\ 42{\cdot}4^s$ fast.

Errors on April 5: A $2^h\ 12^m\ 59{\cdot}3^s$ slow; B $5^h\ 43^m\ 38{\cdot}8^s$ slow; C $0^h\ 7^m\ 27{\cdot}1^s$ fast.

Daily rates: A $4{\cdot}5^s$ losing; B $3{\cdot}2^s$ gaining; C $5{\cdot}1^s$ losing.

From numerous observations of this character an opinion is formed of the merits of each chronometer, excellence depending upon the *regularity* with which a rate, however large, is maintained.

Where facilities do not exist for obtaining errors and rates as in the above examples, these must be found from astronomical observations, a description of which will be preceded by that of the artificial horizon.

Artificial horizon

The artificial horizon is an instrument which usually consists of a shallow rectangular basin B C filled with pure mercury, the surface of which (except near the edges) forms a perfectly horizontal plane (fig. 156).[1]

Theory

A ray of light from a body S will be reflected from such a plane in a direction O A, making an angle A O B with the plane of reflection B C equal to the angle S O C. An observer at A, near B C, will see, on looking at the mercury, an image of S proceeding apparently from a point S_1 in a straight line with A O, as far *below* the horizontal plane as S is really *above* it; since the angle A O B is equal to the angle S_1 O C, the angle S_1 O C equals the angle S O C, and an observer at A measuring an angle between S and its image in the mercury will obtain twice its altitude above the real horizon B C.[1]

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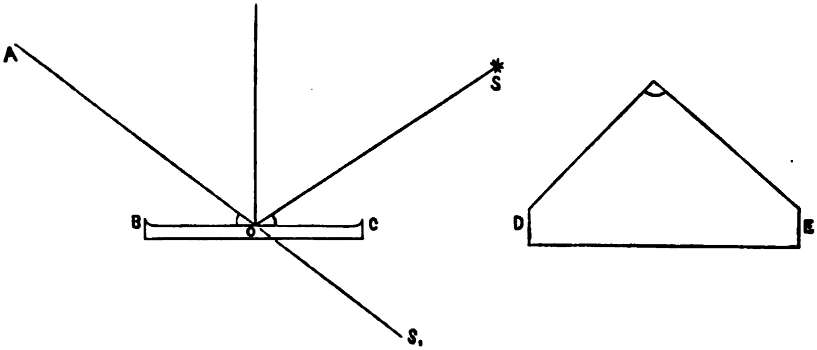
[1] Experience shows that the level of the surface of the mercury is sensibly affected by the *near* vicinity of masses of land, such as mountains.

The artificial horizon does not as a rule admit of measuring altitudes of less than 20°; at lower altitudes the rays of light from the object observed strike the mercury very obliquely, and the observer has to sit so far away that the object is only seen through a very narrow aperture of the roof.

To obviate the disturbing effect of wind on the surface of the mercury it is usual to cover the horizon with a glass roof D E, the sides of which meet at the top at an angle of 90°.

FIG. 156.



Any fluid will answer the purpose of forming an artificial horizon, but mercury is usually preferred, on account of its high reflecting power and great specific gravity.[2]

Observations

If observing *without* the roof the altitude measured, on being corrected for instrumental errors, gives twice the apparent altitude, but if using the roof an additional source of error may exist from the inner and outer surfaces of the glass sides not being exactly parallel to one another; this is partially obviated by turning the roof round while observing, so that half the altitudes may be measured each way, and then taking their mean.[1]

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[1] This is only strictly true for an observer *very close* to O, but for all celestial bodies the distance of the observer from the horizon may be disregarded.

[2] Oil (except in cold weather) forms an excellent substitute, and is much employed in the French navy; its reflecting power is not so great as that of mercury, but it is not so readily affected by motion of the ground in the vicinity.

Excellent results are obtained by using a mercurial horizon with a shallow *copper* trough.

Some observers amalgamate the mercury with tinfoil: on pouring it out the amalgam which floats can be drawn on one side with a folded piece of paper, leaving a perfectly bright surface, free from dust or impurities.

The form of artificial horizon used by travellers is a glass plate with accurately parallel faces blackened on one side and capable of being levelled by screws and small spirit levels.

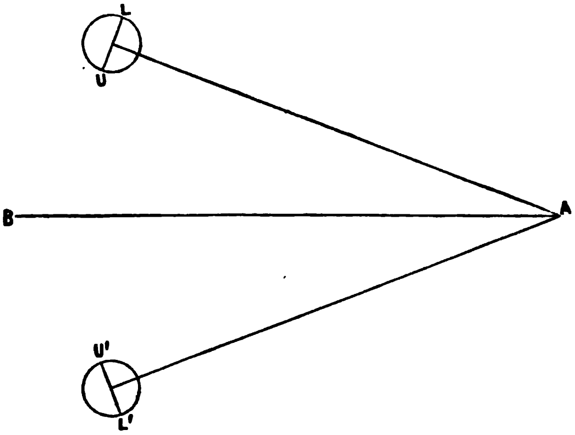
The observations should invariably be made in or near the *centre* of the horizon, since near the edges the surface of the mercury is not horizontal but convex.

Altitudes of sun

In observing the altitude of the lower limb of the sun in an artificial horizon the apparent upper limb as seen in the horizon glass of the sextant (the movable sun as seen through the *inverting* tube) is brought into contact with the lower limb of the image in the mercurial horizon or fixed sun, so that the reflected image in the mercury (or *fixed* sun) will appear to be the uppermost.

If the upper limb is being observed the apparent lower limb as seen in the horizon glass (the movable sun) is brought into contact with the upper limb of the image in the artificial horizon (the fixed sun); in this case the reflected image in the mercury (or *fixed* sun) will appear to be the undermost.

FIG. 157.



From this it follows that in the forenoon *upper* limbs are *closing* suns and in the afternoon *opening* suns, while *lower* limbs are *vice versâ*.

In both forenoon and afternoon observations of the sun it is well to remember that the greater altitude must be that of the upper limb.

If A B (fig. 157) represent the plane of the artificial horizon, and U L the upper and lower limbs of the sun as seen through an *inverting* telescope by an observer at A, then U′ L′ will represent the reflected images of U L seen through the same telescope.

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[1] Except in observing *equal* altitudes (*see* page 343), when, the object being at the second observation to measure the *same* altitude as at the first, the horizon is retained in the same position.

The glass sides are carefully ground parallel by hand.

The double altitude of L is measured by bringing it into contact with L′, and hence if the sun is *rising* these will be separating from one another, and if falling overlapping one another, while the reverse occurs when observing the upper limb.

For nautical purposes the artificial horizon is chiefly used for observations of the sun (on shore) to determine the error of chronometer, and even when the natural horizon *can* be seen the mercurial horizon is employed for this purpose in preference, since by its use all uncertainty is removed as to the true place of the horizon at the time[1] (*see* page 360).

Example

The observed altitude of the sun's lower limb in an artificial horizon was 98° 14′ 10″, index correction – 4′ 10″. Required apparent altitude of sun's lower limb.

Observed alt. $\underline{\odot}$	98°	14′	10″
index corr.		4	10 –
2)	98	10	0
∴ app. alt. sun's L. L.	49	5	0

This apparent altitude is then corrected for refraction, parallax, and semidiameter, as on page 197.

Error of chronometer

The error of a chronometer on Greenwich mean time, being the difference at any instant between its indication and the Greenwich mean time, is usually found in harbour at a place whose longitude is known, by calculating from an altitude of a heavenly body the ship mean time, and thence finding the Greenwich mean time, which is compared with the time by chronometer when the altitude was observed.[2]

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[1] In observations of altitudes made with the artificial horizon as compared with those made with the visible horizon, it is to be remarked that the effect of any error in determination of index error is reduced by half; that due to error of collimation is increased, while that owing to error of centering may be either increased or decreased.

Coloured glasses over the eyepiece of the telescope should always be used when possible instead of the shades, and the brightness of the two suns carefully equalised by the up and down piece.

The use of a sextant-stand greatly facilitates observations with the artificial horizon.

[2] The box chronometers are not taken on shore, the times being noted by the hack watch, which is compared with the chronometers on board both before and after; the latitude and longitude of the place of observation is taken from the plan of the harbour, as explained on page 42. If inconvenient to land and the sea horizon is

Absolute altitudes Shore observations for error of chronometer are usually of precisely the same *character* as those for determining the longitude by sun chronometer at sea, the apparent and mean times of place being calculated as in that observation.

The observations themselves are most conveniently made by carefully setting the sextant at readings 10′ apart and noting the instants of contact.

This method of determining error is known as that by *single or absolute altitudes*, and is liable to the same sources of error as are involved in the similar observation for longitude at sea. The most favourable time for its use is when the sun is on the prime vertical (*see* page 368).

Equal altitudes The second and more accurate method of obtaining error of chronometer is that known as *equal altitudes*, which involves carefully noting the times when the sun has equal altitudes on both sides of the meridian.

Such observations as single and equal altitudes can be made of *any* heavenly body, but, as in practice the error of chronometer is almost always obtained from observations of the sun, our illustrations will be limited to examples of that character.

Error of Chronometer by Absolute Altitudes

The calculation of the apparent time being effected in working absolute altitudes in precisely the same way as in the problem of finding longitude by chronometer, it is not necessary to repeat the remarks made on pp. 365, 368, but we illustrate the method by the following example:

Examples May 10, at $8^h\ 45^m$ A.M. mean time nearly, in latitude 50° 48′ N. and long. 1° 6′ W., when a chronometer showed $8^h\ 26^m\ 59{\cdot}7^s$, the observed altitude of the sun's lower limb in the artificial horizon was 78° 24′ 55″,[1] index correction + 4′ 24″. Required the error of the chronometer on Greenwich mean time.

N.B.—The approximate error of chronometer was 21 minutes slow on Greenwich mean time.[2]

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visible from the anchorage, fair results may sometimes be obtained by using that instead of the artificial horizon.

For illustrations of the occasional use of artificial horizons on board large ships the reader may refer to the remarks in Mr. Blackburne's *A* and *B Azimuth Tables*.

[1] Usually (but not always) the mean of a set of five or more observations taken by setting the sextant to altitudes an equal number of minutes apart.

[2] In observations for error of chronometer the approximate error is always known beforehand.

May 9, M. T. of pl.	20^h 45^m	Time by chron. ...	8^h 27^m	Sun's semidiameter 15′ 51″
Long. in time	4 W.	approx. error slow	0 21	
Greenwich date	20 49	approx. mean time at Greenwich	8	$48^h + 12 = 20^h$ 48^m May 9, or 3·2 from noon May 10.

Sun's Declination.		**Var. in 1 hour.**	Obs. alt. ...	78° 24′ 55″
May 10th	17° 35′ 18″ N.	39·7″ incr.	index corr....	4 24+
change...	2 7	3·2		2)78 29 19
	17 33 11	79·4		39 14 39
		1191	corr. in alt...	1 12−
		127·04		39 13 27
Equation of Time.		**Var. in 1 hour.**	semidiam ...	15 51+
10th...	3^m $48·3^s$ sub. from	0·12 decr.	true alt.	39 29 18
change	0·4 app. time	3·2		90
	3 48·7	·384	zen. dist. ...	50 30 42

Lat.	50° 48′ 0″ N.	log sec lat.	0·199263
decl.	17 33 10 N.	„ sec decl.	0·020707
diff.	33 14 50	½ L hav S.	4·824496
zen. dist.	50 30 42	½ „ hav D.	4·176356
sum.	83 45 32(S.)	L hav	9·220822
diff.	17 15 52(D.)		

∴ apparent time	20^h	47^m	29^s	
equation of time..............		3	48·7−	
∴ ship mean time May 9	20	43	40·3	
long. in time[1]		4	24·0	W.
Greenwich mean time May 9	20	48	04·3	
chronometer showed (adding 12 hours) ...	20	26	59·7	
∴ error on Greenwich mean time		21	4·6	slow.[2]

NOTE.—The error is generally required on *Greenwich* mean time, but can of course be similarly obtained on either *ship* apparent or mean time.

3. Find the error of a chronometer on ship mean time, having given the latitude of the observer = 53° N., the sun's true altitude = 23° 17′ 20″ (west of meridian), the declination = 2° 0′ 30″ N.,

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[1] This should be taken out to the nearest second of time and the logarithms employed in finding the hour angle taken out *accurately*, the error being found to at least one place of decimals.

[2] It is customary to consider the chronometers in a ship as either *all* slow or *all* fast on G. M. T.

and the equation of time = $6^m\ 58^s$, to be added to apparent time; the chronometer showing at the instant of observation $3^h\ 24^m\ 46^s$.

Ans. $11^m\ 42^s$ slow.

4. Find the error of a chronometer on mean time at the place of observation, having given the latitude of the observer = 47° 30′ N., the sun's true altitude = 20° 11′ 45″ (west of meridian), the declination = 20° 1′ 30″ N., and the equation of time = $3^m\ 45^s$, to be subtracted from apparent time; the chronometer showing at the instant of observation $5^h\ 10^m\ 32^s$.

Ans. $0^h\ 12^m\ 6{\cdot}1^s$ slow.

5. Find the error of a chronometer on mean time at the place of observation, having given the latitude of the observer = 50° 48′ S., the sun's true altitude = 33° 4′ 15″ (east of meridian), the declination = 14° 35′ 20″ S., and the equation of time = $14^m\ 5^s$, to be added to apparent time; the chronometer showing at the instant of observation $8^h\ 51^m\ 20^s$.

Ans. $11^h\ 42^m\ 34{\cdot}8^s$ slow.

6. May 20, at $5^h\ 20^m$ P.M. mean time nearly, in lat. 47° 20′ N. and long. 94° 30′ E., when a chronometer showed $11^h\ 5^m\ 20^s$, the observed altitude of the sun's lower limb in an artificial horizon was 39° 56′ 20″, the index correction − 4′ 10″. Required the error of chronometer on Greenwich mean time.

Ans. $1^m\ 12{\cdot}62^s$ fast.

7. Feb. 3, at $10^h\ 30^m$ A.M. mean time nearly, in lat. 49° 30′ N. and long. 22° W., when a chronometer showed $0^h\ 2^m\ 30^s$, the observed altitude of the sun's lower limb in an artificial horizon was 38° 46′ 20″, the index correction + 3′ 20″. Required the error of chronometer on Greenwich mean time.

Ans. $9^m\ 13{\cdot}2^s$ fast.

8. March 25, at $3^h\ 20^m$ P.M. mean time nearly, in lat. 52° 10′ N. and long. 36° 58′ 15″ W., when a chronometer showed $5^h\ 40^m\ 58^s$, the observed altitude of the sun's lower limb in an artificial horizon was 50° 14′ 30″, the index correction − 6′ 10″. Required the error of chronometer on Greenwich mean time.

Ans. $8^m\ 42{\cdot}8^s$ slow.

9. May 19, at $3^h\ 0^m$ P.M. mean time nearly, in lat. 49° 50′ N., and long. 21° 4′ 45″ E., when a chronometer showed $1^h\ 23^m\ 20^s$, the observed altitude of the sun's lower limb in an artificial horizon was 85° 45′ 10″, the index correction + 4′ 10″. Required the error of chronometer on Greenwich mean time.

Ans. $10^m\ 17{\cdot}0^s$ slow.

Elements from 'Nautical Almanac.'

	Sun's Declination.	Equation of Time.	Semi.
May 20	19° 54′ 26″ N.	3^m 46·2^s sub.	15′ 49″
	Var. in 1 hour, 30·94″ incr.	0·141^s decr.	
Feb. 3	16° 36′ 41″ S.	14^m 5·6^s add.	16 14
	Var. in 1 hour, 44·51″ decr.	0·243^s incr.	
Mar. 25	1° 40′ 56″ N.	6^m 13·8^s add.	16 3
	Var. in 1 hour, 58·85″ incr.	0·769^s decr.	
May 19	19° 41′ 37″ N.	3^m 49·5^s sub.	15 49
	Var. in 1 hour, 31·79″ incr.	0·120^s decr.	

ERROR OF CHRONOMETER BY EQUAL ALTITUDES.

Equal altitudes

When the sun's centre is on the meridian of any place, the local or ship apparent time is then either 0^h or 24^h. To obtain mean time at the same instant, we have only to apply the equation of time with its proper sign. We thus find mean time at the instant the sun is on the meridian, and if we can also ascertain what a chronometer showed at the same instant, it is manifest that the error of the chronometer on mean time at the place is known, since it will be the difference between the two times.

To find the time shown by the chronometer at apparent noon, we have recourse to the method of equal altitudes, which consists in noting the time shown by the chronometer when the sun has the same altitude on both sides of the meridian; half the interval between the observations being added to what the chronometer showed at the first observation will be the time shown by the chronometer when the sun was on the meridian, if *the declination is supposed to be invariable* in the interval between the observations.

But the sun's declination is not invariable, but increases or decreases by a small amount during the interval; so that the declination at the second equal altitude is not the same as at the first; and therefore half the interval between the observations being added to the time shown by chronometer at the first observation will *not* be the time the chronometer shows when the sun is on the meridian, but will differ by a *few seconds.* This difference expressed in apparent time is called *the equation of equal altitudes.*[1]

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[1] The equation is nothing at the time of the sun being in the solstices (June 21 and December 21), since the declination then does not change, and is a maximum when the declination is changing most rapidly (March and September).

The method of finding the time of meridian passage of a heavenly body by equal

Theory

Given the elapsed time, and change of declination in the interval; to investigate an expression for the equation of equal altitudes.

Let P be the pole, Z the zenith, and X and Y the places of the sun at the times of the observations when the zenith distances Z X and Z Y are equal.

First. Suppose the polar distance to be increasing (fig. 158); then the polar distance P Y will be greater than P X, and therefore the angular distance Z P X is greater than the hour angle Z P Y, or greater than half the interval X P Y, since the sun will, owing to the change of declination, arrive *earlier* on the circle of equal altitude X T Y. Make $\text{ZPT} = \text{ZPY}$, and let the angle $\text{XPT} = 2x$, and the elapsed time, namely the angle $\text{XPY} = \varepsilon$.

Then $\varepsilon = \text{YPT} + \text{XPT} = 2\,\text{ZPT} + 2x$,

$\therefore \frac{1}{2}\varepsilon = \text{ZPT} + x$: add x to both sides,

$\therefore \frac{1}{2}\varepsilon + x = \text{ZPT} + 2x = \text{ZPX}$ = angular distance from the meridian, or the time that must elapse (by chronometer) before the sun is on the meridian P Z.

Second. Suppose the polar distance to be decreasing (fig. 159), or P Y less than P X; then in a similar manner it may be shown that the time to noon or the angle $\text{ZPX} = \frac{1}{2}\varepsilon - x$, since in this case, owing to the change in declination, the sun will arrive on the circle of equal altitude, passing through X *later* than it otherwise would.

FIG. 158. Polar distance increasing.

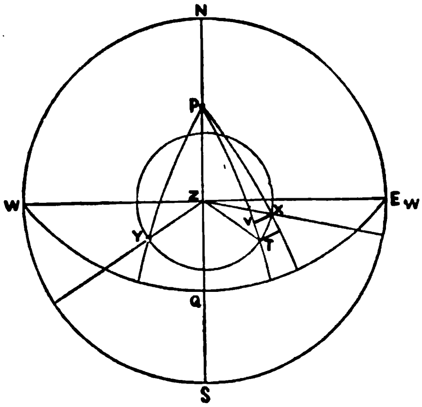
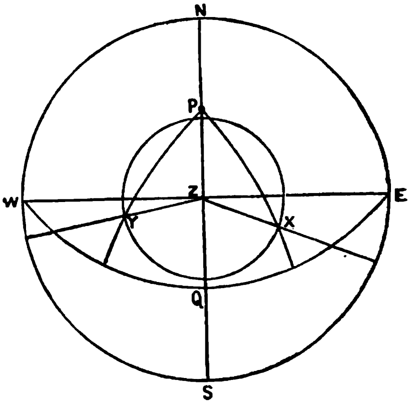


FIG. 159. Polar distance decreasing.



The value of x, or half the difference of the two hour angles in seconds of time, is called the EQUATION OF EQUAL ALTITUDES,

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altitudes was employed in astronomical observations in 1666, and used at sea (see footnote, page 312) by Picard, a French astronomer, in 1671.

and is generally (except in low latitudes) *additive* to the mean of the two times by chronometer of observing the same altitude when the sun's distance from the elevated pole (north or south) is increasing, and *subtractive* when it is decreasing.

Investigation of Equation of Equal Altitudes.

About Z as a centre (fig. 158) with radius ZX describe the circle XTY, and join TZ. Then PT = PY (for YZ = TZ, and ZPT was made equal to ZPY, and PZ is common to the two triangles). Again, about P as a centre describe the arc of a parallel of decl. XV: then TV is the difference between PX and PT or PY; that is, it is the change of declination in the interval of elapsed time ϵ.

Let the declination at apparent (local) noon = D.

Let the change of declination from the meridian to the time of either observation = d''.

Then in fig. 158, $D + d''$ will be the declination at time of A.M. observation, and $D - d''$ that at time of P.M. observation.

Let the latitude of place = l.

In the triangle ZPX

(1) $$\cos ZX = \sin l \sin (D + d'') + \cos l \cos (D + d'') \cos ZPX.$$

In the triangle ZPY

(2) $$\cos ZY = \sin l \sin (D - d'') + \cos l \cos (D - d'') \cos ZPY.$$

But $\cos ZX = \cos ZY$, and by subtracting the second equation from the first we have

$$0 = \sin l \,[\sin (D + d'') - \sin (D - d'')] + \cos l \,[\cos (D + d'') \cos ZPX - \cos (D - d'') \cos ZPY].$$

But $ZPX = (\tfrac{1}{2}\epsilon + x)$ and $ZPY = (\tfrac{1}{2}\epsilon - x)$.

On substituting these values, and then expanding and multiplying out, we find

$$0 = 2 \sin l \cos D \sin d'' - 2 \cos l \sin D \sin d'' \cos \tfrac{1}{2}\epsilon \cos x - 2 \cos l \cos D \cos d'' \sin \tfrac{1}{2}\epsilon \sin x.$$

By transposing and dividing by the coefficient of $\sin x$

$$\sin x = \frac{\tan d'' \tan l}{\sin \frac{1}{2}\epsilon} - \frac{\tan d'' \tan D}{\tan \frac{1}{2}\epsilon} \cos x.$$

But since x and d'' are always small quantities, we may substitute x for its sine, d'' for its tangent, and unity for $\cos x$, when

$$x = \frac{d'' \tan l}{\sin \frac{1}{2}\epsilon} - \frac{d'' \tan D}{\tan \frac{1}{2}\epsilon} \text{ (in arc).}$$

$$\therefore x \text{ (in time)} = \tfrac{1}{15} d'' \,.\, \tan l \,.\, \operatorname{cosec} \tfrac{1}{2} \varepsilon - \tfrac{1}{15} d'' \,.\, \tan \mathrm{D} \,.\, \cot \tfrac{1}{2}\varepsilon.$$

$$\text{Let } \mathrm{M} = \tfrac{1}{80} d'' \,.\, \tan l \,.\, \operatorname{cosec} \tfrac{1}{2}\varepsilon,$$

$$\text{,, } \mathrm{N} = \tfrac{1}{80} d'' \,.\, \tan \mathrm{D} \,.\, \cot \tfrac{1}{2}\varepsilon\,;$$

then the equat. of equal alt. $x = \mathrm{M} - \mathrm{N}$.

It will be seen that in the case illustrated by fig. 159 the equation would equal $-(\mathrm{M} - \mathrm{N})$ or $\mathrm{N} - \mathrm{M}$.

Hence generally the equation of equal altitudes $= \mathrm{M} \underset{\wedge}{+} \mathrm{N}$.

To determine the signs of M and N in any particular case it will be remarked that in the preceding investigation d'' has been considered *positive* when the sun is moving in declination from north to south, and therefore it must be considered to be *negative* during the opposite season of the year.

Tan l will also change signs with change of latitude from north to south, and tan D will similarly change signs as the declination changes from north to south.[1]

From these considerations the following general rules are derived:

(1) M is *positive* when declination is decreasing and of same name as latitude, or increasing and of a different name.

(2) M is *negative* when declination is decreasing and of a different name from latitude, or increasing and of the same name.

(3) N is positive when declination is increasing. (4) N is negative when declination is decreasing.	When elapsed time is *greater* than 12 hours cot $\frac{1}{2}\varepsilon$ changes sign, and therefore in this case the sign of N in each case is reversed.

In other words, M is + from the summer to the winter solstice of the hemisphere in which the observer is, and — from the winter to the summer solstice.

N is + between the equinoxes and soltices and — between the solstices and equinoxes except when the elapsed time is greater than 12 hours, when the sign of N is reversed.

It will be remarked that in connection with the values of M and N,

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[1] The work of calculation may be shortened without any appreciable loss of accuracy by using Inman's Tables, No. 43, where the factors depending on ϵ and $\frac{1}{2}\epsilon$ are tabulated.

(1) When $l=0$, $\text{M}=0$
(2) „ $\text{D}=0$, $\text{N}=0$
(3) „ $d''=0$, both M and $\text{N}=0$
(4) „ l equals or exceeds D, M exceeds N; but when D exceeds l, N will first equal and then exceed M for certain values of ϵ; thus with lat. 5°, declination 9° 55′ 30″ (of same name), and elapsed time 8 hours, N and M are *equal*, and with the same lat. and *greater* declination N will exceed M.

NOTE.—The preceding rules for the signs of the two parts of the equation of equal altitudes apply to the usual case of observing equal altitudes of the sun in the morning and following afternoon; where circumstances of weather necessitate employing an afternoon altitude, and the corresponding equal altitude the next morning, the error thus found is that at midnight. In this case the sign of M must be changed, but not that of N.

The sign of M changes since equal zenith distances observed in latitude $+l$ are equivalent to the supplements of such equal distances observed in latitude $-l$ at the antipodes.

Hence, since tan l, and therefore M, changes sign, the usual formula, with that alteration, will give the equation of equal alts. for *noon* in lat. $-l$ corresponding to midnight in lat. $+l$.

This method is theoretically as accurate as that previously given, but during the longer elapsed time the rate of the chronometer may probably not be so uniformly maintained. The mode of observation is exactly the same, except that they are taken in reverse order, the last of the afternoon being the first in the following forenoon.

The method of calculation will now be illustrated by the following examples, observing that the observations will stand recorded in the following form:

Altitude.	Times by watch, A.M.	Times by watch, P.M.	Sums.
65° 0′	3h 25m 29s	8h 36m 13·5s	12h 1m 42·5s
10	25 50	35 52	42
20	26 11·5	35 81·5	43
30	26 32·7	35 10	42·7
40	26 53	34 49·5	42·5
50	27 13·5	34 28·3	41·8
66 0	27 35·3	34 7·5	42·8
10	27 56·5	Lost.	
20	Lost.	33 25	
30	28 38	33 3·5	41·5
		Mean	12 1 42·38
		Approximate time by watch at apparent noon	6 0 51·19

The close agreement of the seconds in the last column being an excellent criterion of the quality of the sights.

Examples

On Aug. 7, in lat. 50° 48′ N. and long 1° 6′ W., the sun had equal altitudes at the following times by chronometer:[1]

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[1] These times correspond to the means of sets of observations, usually five or seven in number, separated by equal differences of double altitude of 10′ or less.

The *rate* of the chronometer, however large, does not (if regular) affect the result, since the mean of A.M. and P.M. times truly shows the chronometer time at mid-day.

A.M.	P.M.
$9^h\ 25^m\ 42{\cdot}5^s$	$2^h\ 59^m\ 55{\cdot}6^s$

Required the error of chronometer on mean time at the place, and also on mean time at Greenwich.

August 7, local apparent time......	0^h	0^m
long. in time..............................		4
Greenwich app. time, August 7....	0	4

Note.—The equation of time and declination are to be taken out of the *Nautical Almanac* for apparent noon and corrected for Greenwich apparent time.

Equation of Time.		**Variation in 1 Hour.**	**Chronometer Times.**			
Aug. 7 ... 5^m $33{\cdot}00^s$	4^m is $\frac{1}{15}$	$0{\cdot}3^s$ sub.	P.M....	14^h	59^m	$55{\cdot}6^s$
0·02		0.02	A.M....	9	25	42·5
5 32·98 +			elapsed T.	5	34	13·1
			½ elap. T.	2	47	6·55

Sun's decl. 7th, 16° 34′ 28″ N.	Var in 1 hr. 41·41″ decr.		41·41″
corr. 2	hrs. from noon	·06	½ el. time 2·78
16 34 26		2·4846	33128
			28987
			8282
			d'' = 115·1198

(1)		(2)	
L tan lat.	10·088533	L tan decl.	9·473680
,, cosec ½ el. time........	10·176373	,, cot ½ el. time.....	10·048960
log change of decl. (d'')	2·061075	log change of decl. (d'')	2·061075
	2·325981		1·583715
log 15	1·176091	log 15	1·176091
	1·149890		0·407624
	M = + 14·12s		N = − 2·56s
	N = − 2·56		
eq. of equal alt.	+ 11·56		

A. M. chronometer time	9^h	25^m	$42{\cdot}5^s$
½ elapsed time	2	47	6·55
	0	12	49·05
equation of equal altitude			11·56 +
time by chronometer at apparent noon......		13	0·61
apparent time at apparent noon...............	0	0	0
equation of time		5	32·98 +
mean time at apparent noon	0	5	32·98
time by chronometer at apparent noon	0	13	0·61
error of chronometer on mean time at place		7	27·63 fast.

To find error on Greenwich mean time.

Mean time at apparent noon	0^h	5^m	$32{\cdot}98^s$
longitude in time		4	$24{\cdot}00+$
mean time at Greenwich............................	0	9	$56{\cdot}98$
time by chronometer..................................	0	13	$0{\cdot}61$
ERROR OF CHRONOMETER on Gr. mean time		3	$3{\cdot}63$ fast.

NOTE.—If using Inman's Tables, No. 48, the two parts of the equation of equal altitudes in preceding example would be obtained as follows:

log A	9·4451	log B	9·3178
log tan l	10·0885	log tan d	9·4737
log 41·41″	1·6171	log 41″·41	1·6171
	1·1507		0·4086
$\therefore$ M =	+14·13	$\therefore$ N =	−2·56

Practically agreeing with the former result.

10. Aug. 7, in lat. 50° 48′ N. and long. 1° 6′ W., the sun had equal altitudes at the following times by chronometer:

A.M.	P.M.
$9^h\ 3^m\ 42{\cdot}3^s$	$3^h\ 21^m\ 54{\cdot}2^s$

Required the error of the chronometer on Greenwich mean time.

For elements from *Nautical Almanac* see preceding example.

Ans. $3^m\ 2{\cdot}83^s$ fast.

11. Aug. 20, in lat. 50° 48′ N. and long. 1° 6′ W., the sun had equal altitudes at the following times by chronometer:

A.M.	P.M.
$10^h\ 49^m\ 15{\cdot}4^s$	$1^h\ 27^m\ 27{\cdot}6^s$

Required the error of the chronometer on Greenwich mean time.

Ans. $1^m\ 9{\cdot}02^s$ fast.

Elements from 'Nautical Almanac' at App. Noon.

Equation of time $3^m\ 1{\cdot}94^s+$, var. in $1^h = 0{\cdot}606^s-$

Declination, 20th, 12° 34′ 59′ N. Var. in 1^h, 49·58″ decr.

12. Sept. 9, in lat. 50° 48′ N. and long. 1° 6′ W., the sun had equal altitudes at the following times by chronometer:

A.M.	PM.
$9^h\ 45^m\ 55{\cdot}2^s$	$2^h\ 20^m\ 39{\cdot}9^s$

Required the error of the chronometer on Greenwich mean time.

Ans. $0^h\ 2^m\ 9{\cdot}66^s$ fast.

Elements from 'Nautical Almanac' at App. Noon.

Equation of time, $2^m\ 58{\cdot}43^s-$, var in $1^h = 0{\cdot}866^s +$
Declination, 9th, 5° 27′ 27″ N. Var. in 1^h, 56·75″ decr.

13. May 13, in lat. 50° 48′ N. and long. 15° 0′ W., the sun had equal altitudes at the following times by chronometer:

A.M.	P.M.
$10^h\ 46^m\ 57{\cdot}0^s$	$1^h\ 39^m\ 42{\cdot}0^s$

Required the error of the chronometer on the mean time at the place, and also on Greenwich mean time.

Ans. Fast on mean time at place, $17^m\ 4{\cdot}7^s$.
Slow on Greenwich mean time, $42^m\ 55{\cdot}3^s$.

Elements from 'Nautical Almanac' at App. Noon.

Equation of time, $3^m\ 53{\cdot}8^s-$, var. in $1^h = 0{\cdot}01^s-$
Declination, 13th, 18° 28′ 49″ N. Var. in 1^h, 36·33″ incr.

To find the approximate time by chronometer when the P.M. *altitudes should be observed.*

After taking the observations in the morning, it will always be convenient to estimate nearly at what *time by the chronometer* the observer should prepare to take the P.M. sights. To do this the error of the chronometer on mean time *at the place* must be supposed to be known within a few minutes. Thus suppose (as in the last example) a chronometer is known to be about 17 minutes *fast* of mean time at the place, the time of the A.M. observation was by chronometer $10^h\ 46^m\ 57^s$, equation of time 4 minutes additive to mean time. It is required to find the time the chronometer will show in the afternoon when the sun has the same altitude.

Let a = estimated error of chronometer on mean time at place (supposed fast),
t = time shown by chronometer at A.M. observation;
then $t-a$ = mean time at A.M. observation nearly.
Let E = equation of time (supposed additive to mean time).
$\therefore\ t-a+E$ = apparent time at A.M. observation.

$\therefore\ 12-(t-a+\text{E})=$ apparent time from noon of A.M. observation.
$=$ apparent time of P.M. observation.
$\therefore\ 12-(t-a+\text{E})-\text{E}=$ mean time of P.M. observation.
And $12-(t-a+\text{E})-\text{E}+a=$ time of P.M. observation *by chronometer.*

$\therefore$ time of P.M. observation as shown by the chronometer

$$=12-(t-a+\text{E})-\text{E}+a$$
$$=12-t+2(a-\text{E}).$$

Thus in the supposed case $t=10^{\text{h}}\ 47^{\text{m}}$, $a=17^{\text{m}}$, $\text{E}=4^{\text{m}}$;
$\therefore$ time by chronometer $=1^{\text{h}}\ 13^{\text{m}}+26^{\text{m}}=1^{\text{h}}\ 39^{\text{m}}$.

It appears from this that the observer need not prepare to take his P.M. sights until $1^{\text{h}}\ 30^{\text{m}}$ by chronometer.

A similar formula may be made to suit any other case.

From another point of view let $\text{T}=$ *mean* time of transit of sun $=0^{\text{h}}\pm\text{E}$, the equation of time,

$a=$ estimated error of chronometer on *mean* time,
$a_1=$ estimated error of chronometer on *apparent* time $=a\pm\text{E}$,
$t=$ time shown by chronometer at *last* observation A.M.

Then $(\text{T}\pm a)-t=$ angular distance from meridian nearly, and, therefore, approximate *apparent time* of P.M. observation.

Hence on applying a_1 to the apparent time we find what the chronometer will show at that instant. In the example given $\text{T}=11^{\text{h}}\ 56^{\text{m}}$, and $(\text{T}+a)-t=1^{\text{h}}\ 26^{\text{m}}$. This is the *apparent* time for P.M observation, and hence the chronometer at that instant will show $1^{\text{h}}\ 26^{\text{m}}+13^{\text{m}}=1^{\text{h}}\ 39^{\text{m}}$.

Second method for equal altitudes

Equal altitudes of the sun may also be computed as in the following example:[1]

At Cape Passaro Castle, Sicily, on August 1, 1866, the *true* double altitude of the sun's centre was observed to be 64° at $6^{\text{h}}\ 2^{\text{m}}\ 23{\cdot}2^{\text{s}}$ by chronometer, and again at $2^{\text{h}}\ 27^{\text{m}}\ 17{\cdot}2^{\text{s}}$.

Latitude of castle 36° 41′ N., declination of sun at time of first observation 18° 2′ N., variation of declination in 1 hour 37·75″.

The half-interval between the observations is $4^{\text{h}}\ 12^{\text{m}}\ 27^{\text{s}}$, and the change of declination in the time 159″.

On calculating the sun's hour angle at the time of the first observation by the usual method (page 285) it is seen that a change of $+120''$ in declination (increasing) will alter the value of L sec declination by $+0{\cdot}000082$, the half L haversine of the sum (76° 39′) by $-0{\cdot}000159$, and the half L haversine of the difference by $+0{\cdot}000353$.

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[1] Taken from a paper in *Naval Science* for 1874 by the late Admiral Shortland.

Hence a change of $120''$ in declination will give a change of $+0{\cdot}000276$ in the L haversine of the hour angle.

But with hour angle $4^h\ 12^m\ 27^s$ a change of 0·000051 in the L haversine gives 1 sec. change in the hour angle, and therefore $120''$ change of declination will give a change of 5·4 sec. in the hour angle, and ∴ $159''$ will give 7·1 sec., which is the equation of equal altitudes to be *added* to the half sum of the times in this case, because the sun arrives sooner at this altitude in the afternoon by twice the above interval than it would have done if it had not moved in declination.

Thus 7·1 seconds being added to $10^h\ 14^m\ 50{\cdot}2^s$ (the mean of the times) gives $10^h\ 14^m\ 57{\cdot}3^s$ as the chronometer time of meridian passage of the sun.

Remarks When the time is obtained from single altitudes, the result is directly dependent on the correctness of the three elements of the calculation employed, the latitude, true altitude, and declination; the former may not be accurately known, error may exist in the altitude from defects of observation, instrumental or personal, or erroneous estimation of the refraction, and the declination may be vitiated by errors in the assumed time and longitude. From any of these causes, taken singly or in combination, errors may arise in the time deduced from the observations. Equal altitude observations, on the other hand, are in a great measure independent of these sources of error. If the object observed is a star, the time can be deduced at once from the observation without any calculation, the mean of the times observed being the time corresponding to the star's meridian transit; and if the object observed be the sun, as is more generally the case, 'the equation of equal altitudes' required on the occasion is but slightly affected by small errors in the observer's latitude or in the sun's declination. The value of the method of equal altitudes consists in the same altitude being noted on the opposite side of the meridian without regard to its precise measure.

A small error in the latitude is not of importance,[1] and hence in a harbour whose latitude is only approximately known equal altitudes should be observed in preference to single altitudes.

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[1] An error of 5′ in latitude will only affect the value of x by the hundredth part of its value even under the most unfavourable circumstances.

In low latitudes, where the sun moves rapidly, three observations may be obtained at one altitude by observing both limbs and the centre.

In high latitudes the value of M (depending upon tan *l*) becomes very large, and equal altitudes are not so accurate as in lower latitudes. The most suitable times for observation are when the sun is on the prime vertical, and it is recommended when practicable not to observe nearer to the meridian than when the double altitude changes 10′ in 30 seconds. The mean result of errors by single altitudes, observed A.M. and P.M. at *nearly* the same altitude, will be almost the same as that obtained from observations of *equal* altitudes, which circumstances of weather may prevent; this method is also *preferable* to equal altitudes in high latitudes, where tan *l* (in the value of M), as already remarked, would be a large quantity. If equal altitudes of a star were observed, the mean of the chronometer times of observation would, without any correction, at once give the chronometer time of meridian passage, and thence by subtracting this from the local time of meridian passage accurately calculated as on page 213 the error on *sidereal* time of place, from which the error on sidereal and therefore on mean time of Greenwich could be found.

In such an observation errors in latitude and declination produce no effect, since these elements do not enter into the calculation.

EXAMPLE.[1]—On March 15, at the Cape of Good Hope, lat. 33° 56′ S., long. $1^h\,13^m\,56^s$ E. equal altitudes of Spica were observed as follows:

	East.			Double alt. Spica.	West.		
	10^h	20^m	$0{\cdot}5^s$	104° 0′	2^h	40^m	38^s
		20	28	10		40	10·5
		20	55	20		39	42
Means	10	20	27·83		2	40	10·17
					10	20	27·83
				Sum of times by chron.	25	0	38·00
				Mean of times, or time by chron. at star's transit	12	30	19·00

To find the error of the chronometer on Greenwich mean time, we have—

L. M. T. of star's transit at the Cape	13^h	42^m	$29{\cdot}55^s$	(*see* page 213).
Longitude	1	13	56·00	E.
G. M. T. of star's transit at the Cape	12	28	33·55	
Time by chronometer	12	30	19·00	
Chronometer fast on G. M. T.		1	45·45	

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[1] Taken from Chauvenet's *Astronomy*.

The following ingenious method of obtaining the error and rate of a chronometer by equal altitudes of a star on the *same* side of the meridian was proposed by Riddle in 1820.

The method of equal altitudes as applied to the stars, although both accurate and simple, is, however, affected by the material inconvenience that it is necessary to wait a long time at unsuitable hours during the night for the duplicate observation west or east of the meridian, according as the star observed passės above or below pole.

If, however, the observations are restricted to altitude on the *same* side of the meridian this difficulty vanishes, and both the error and rate can be determined from them by repeating the operations on subsequent nights.

A star attains the same altitude on the same side of the meridian at intervals of a sidereal day (23^h 56^m $4{\cdot}09^s$), or in 3^m $55{\cdot}91^s$ less than a mean solar day; therefore, if the difference of the two consecutive times at which a star attains the same altitude, whatever it may be, on the same side of the meridian be less than 3^m $55{\cdot}91^s$ the chronometer will have gained, if more it will have lost so much in a sidereal day; and if the observations are made at an interval of n days, the nth part of the difference compared with 3^m $55{\cdot}91^s$ will in like manner give the rate in a *sidereal day*, and this rate multiplied by 1·00274 will be the rate in a mean solar day.

The *error* at the mean epoch between the two observations for rate (separated by an interval of not less than five nor more than ten days) being obtained by calculating two star chronometers from which the errors on local mean times are known, and hence, in a place whose longitude is known, the errors on Greenwich mean time.

It is desirable to select for observation stars on or near the prime vertical, preference being given to those situated near the equator.

RATE OF CHRONOMETER.

The *mean daily rate* of a chronometer is found by dividing the increase or decrease in its error by the number of days (usually, if circumstances permit, not less than five nor more than ten) elapsed between the times when the observations were taken to determine its error; thus, suppose on April 20, when in harbour, at 9^h 30^m A.M., the error of a chronometer was found to be fast 10^m $10{\cdot}5^s$ on Greenwich mean time, and that on April 30 about the same hour its error was found to be $10^m\ 40{\cdot}5^s$ fast: then it appears that in the ten days elapsed between the observations the chronometer has gained 30^s, hence its mean daily rate in harbour is 3^s gaining.

Strictly speaking, the rate thus found is that which corresponds to the date mid-way between the times when the observations for error were taken, and the error should also be referred to that epoch; thus in the case considered the error on April 25, at 9.30 A.M., was $10^m\ 25{\cdot}5^s$ fast, and rate at that date 3 seconds daily gaining.

It is customary to consider a *gaining* rate as positive or +, and a *losing* rate as negative or −, and refer both error and rate to the instant of the nearest Greenwich noon. If the performance of the chronometer were perfectly uniform, the best results for rate would be found by employing observations separated by as long an interval as possible, since errors of observation would then produce the least effect. The advantage of such a long interval is, however, more than counterbalanced by the irregularities in performance of the chronometers due to changes of temperature, &c.

The rate (if determined, as it usually is, from absolute observations) should be found by comparing A.M. sights with A.M. sights, or P.M. observations with similar P.M observations, *not* when avoidable by a comparison of A.M. sights at one time with P.M. sights at another.

This is a requisite precaution to observe, since the error of centering will affect the errors on different sides of the meridian in opposite directions, and it is essential to accuracy that the observations upon which the rate depend should be similarly affected by the possible errors in the data on which they depend.

When the error and rate of a chronometer are given, we may determine what its error will be at some future time, provided the rate of the chronometer continues uniform in the interval.

This is illustrated by the following example.

Example

On June 13, at $10^h\ 52^m$ P.M. mean time nearly, in long. 60° W., an observation was taken when a chronometer showed $2^h\ 50^m\ 42^s$. On June 1, at G. M. noon, its error was known to be $3^m\ 10{\cdot}3^s$ fast on Greenwich mean time, and its mean daily rate was $3{\cdot}5^s$ gaining. Required mean time at Greenwich when the observation was taken.

Ship, June 13	10^h	52^m
long. in time .	4	0 W.
Gr., June 13 .	14	52

Interval from June 1 noon to June 13 at 14^h 52^m, is 12^d 14^h $52^m = 12^d$ 15^h nearly or 12·6 days

Daily rate .	$3{\cdot}5^s$
	12·6
	630
	378

Accumulated rate			44·10 gaining
original error		3	10·3 fast
present error		3	54·4 fast
chronometer showed ...	2	50	42·0
Greenwich, June 14 ...	2	46	47·6 A.M.
∴ Greenwich, June 13 ...	14	46	47·6

On August 10, at 3^h 42^m A.M. mean time nearly, in long. 100° 30′ W., an observation was taken when a chronometer showed 10^h 30^m $45{\cdot}5^s$. On August 1 at G. M. noon its error was known to be 12^m $10{\cdot}5^s$ slow on Greenwich mean time, and its rate $11{\cdot}2^s$ gaining. Required mean time at Greenwich when the observation was taken.

Ship, Aug. 9...	15^h	42^m
long. in time...	6	42 W.
Gr., Aug. 9 ...	22	24

Interval from Aug. 1 noon to Aug. 9, at 22^h 24^m, is 8^d 22^h 24^m or 8·97 days

Daily rate ...	$11{\cdot}2^s$
	8·97
	784
	1008
	896
	$100{\cdot}664 = 1^m$ $40{\cdot}4^s$

Accumulated error		1^m	40·4 gaining
original error		12	10·5 slow
present error		10	30·1 slow
chronometer showed ...	10	13	45·5
Greenwich, Aug. 10	10	24	15·6 A.M.
∴ Greenwich, Aug. 9	22	24	15·6

14. On Nov. 20, at 6^h 42^m P.M. mean time nearly, in long. 32° 0′ E., an observation was taken when a chronometer showed 4^h 30^m 6^s. On Oct. 9 at G. M. noon its error was known to be 5^m $52{\cdot}4^s$ slow on Greenwich mean time, and its rate $2{\cdot}7^s$ losing. Required mean time at Greenwich when the observation was taken. *Ans.* 4^h 37^m $52{\cdot}3^s$.

15. On Dec. 31, at 10^h 10^m A.M. mean time nearly, in long. 150° E., an observation was taken when a chronometer showed 0^h 0^m $22{\cdot}3^s$. On Nov. 20 at G. M. noon its error was known to be 3^m $52{\cdot}4^s$ slow on Greenwich mean time, and its rate $2{\cdot}7^s$ losing. Required mean time at Greenwich when the observation was taken. *Ans.* 12^h 6^m 4^s.

16. On April 11, at 3^h 14^m P.M. mean time nearly, in long. 56° 42′ W., an observation was taken when a chronometer showed 7^h 2^m $10{\cdot}5^s$. On March 15 its error at G. M. noon was known to

be 1^m 32·7^s fast on Greenwich mean time, and its daily rate 6·3^s losing. Required mean time at Greenwich when the observation was taken. *Ans.* 7^h 3^m 29·7^s.

Error at sea

At sea every opportunity is taken when passing within sight of an island or point of land whose position is well known to find the *error* of chronometer, and also, by comparing the longitude ascertained by chronometer with that taken from the ship's position on the chart, to find if the *rate* of the chronometer has differed from that allowed when commencing the voyage.

Examples

On leaving Monte Video, Jan. 1, to proceed to the Cape of Good Hope the chronometer at noon S. M. T. was considered to be 2^h 17^m 12^s slow on Greenwich mean time, rate 5·5^s losing; on Jan. 11 the island of Tristan d'Acunha was sighted, and when its summit, lat. 37° 2′ S., long. 12° 17′ W., bore S.E. true, distant 8 miles,[1] observations at 5 P.M. were taken, from which the error of chronometer on ship mean time was found to be 1^h 28^m 40^s slow. Required the error on Greenwich mean time, and also the actual rate while at sea.

The true longitude at time of observation (found either by calculation of course and distance from the summit or by measurement on the chart) was 12° 24′ W., or, expressed in time, 0^h 49^m 36^s.

Chron. slow on S. M. T. ...	1^h	28^m	40^s
Longitude in time	0	49	36
Chron. slow on G. M. T. ...	2	18	16 on Jan. 11, at 5 P.M. ship M. T.
" " " ...	2	17	12 on Jan. 1, noon Monte Video M. T.
Change in 10$\frac{1}{12}$ days		1	4 or 64 seconds.

∴ daily rate 6·35 sec. losing.

If the longitude by chronometer is *west* of the *true* longitude, and the vessel has come from the westward, then the rate if gaining must be *less*, and if losing *more* than was allowed for; if, on the contrary, the position by chronometer is *east* of the true position, then the rate if gaining must be *more*, and if losing *less* than that supposed; opposite conclusions would be drawn in each case if the vessel had been steering to the westward.

When there is a considerable difference of longitude between the two places it is important, especially when the rate is large, to take into consideration the fraction of a day due to this difference of longitude. Thus the interval of time between noon on January 1 and noon on January 11 at a place 30° eastward of the first position is 10 days minus 2 hours, or the difference of longitude; and similarly the interval is 10 days plus 2 hours if the second place is 30° westward of the first.

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[1] Known either by estimation or by one of the methods described in Chap. IV.

Harbour rate and sea rate

As indicated in the preceding examples, the rate of a chronometer at sea is seldom exactly the same as in harbour even at the same temperature. This is probably an effect of the motion of the ship on the mechanism.

In ascertaining the longitude at sea the known error of chronometer on Greenwich mean time at some former date is corrected (usually) for accumulated rate as determined in *harbour*.

Under certain circumstances the actual rate at sea may be correctly found in the following way:

Secondary meridians

A large number of the most frequented seaports in the world have been telegraphically connected together and with Greenwich, so that their longitudes may be considered to be accurately known.

An observation for error of chronometer made at such a place will, on applying the longitude in time, give the error on Greenwich mean time at that instant. Hence if observations for error in the course of a voyage have been made at *two* such places at an interval of a certain number of days apart, since each observation gives the error on Greenwich mean time, the difference of such errors divided by the interval in days will give the actual mean daily rate during the voyage between those places, and on the supposition of the chronometer being approximately compensated for temperature will also give the probable sea rate during the next voyage.

Examples

On a voyage from the Cape of Good Hope (long. 18° 26′ 00″ E.) to Bombay, calling at Mauritius (long. 57° 31′ 42″ E.), the error of chronometer on ship mean time on March 1 (noon) at the Cape was 2^h 10^m $14{\cdot}18^s$ slow, and on March 12 (noon) the error on ship mean time at Mauritius was 4^h 47^m $20{\cdot}24^s$ slow. Find the sea rate.

Mar. 1, error slow on Cape	2^h	10^m	$14{\cdot}18^s$	Mar. 19, error slow on Mauritius	4^h	47^m	$20{\cdot}24^s$
long. in time E.	1	13	44·00	long. in time	3	50	6·8
Mar. 1, error slow on G.M.T.	0	56	30·18	Mar. 12, error slow on G. M. T.	0	57	13·44
Mar. 12 „ „	0	57	13·44				
change in $10\frac{7}{8}$ sea days ...		00	43·26				
∴ daily sea rate			3·98 seconds losing.				

The rate thus determined should be used during the remainder of the voyage from Mauritius to Bombay in preference to a harbour rate at Mauritius, the determination of which would involve a detention of several days at that place.

17. On Jan. 4 at noon a chronometer at Plymouth (long. 4° 10′ 12″ W.) was $1^h\ 10^m\ 12{\cdot}26^s$ fast on ship mean time, and on Jan. 14 at noon the same chronometer at Madeira (long. 16° 54′ 45″ W.) was $2^h\ 1^m\ 30{\cdot}68^s$ fast on ship mean time. Find the sea rate. *Ans.* 1·87 sec. gaining.

18. On June 2 at noon a chronometer at Monte Video was $3^h\ 2^m\ 21{\cdot}4^s$ slow on ship mean time, and on June 16 at noon the same chronometer was $8^h\ 2^m\ 24{\cdot}6^s$ slow on ship mean time at the Cape of Good Hope. Required the sea rate.

Ans. 5·23 sec. losing.

Long. of observing spot at Monte Video ...... 56° 14′ 00″ W.
„ „ „ Cape of Good Hope 18 28 45 E.

In the case of a vessel after a short absence returning to the port sailed from, the sea rate may be found by dividing the difference between the *errors* on departure and on arrival by the number of days elapsed.

CHAPTER XIII.

Errors of observation—Errors affecting latitude—Errors affecting hour angles—Rates of changes of altitudes—Errors affecting azimuths—Errors caused by spheroidal form of earth—Precession of the equinoxes.

In this chapter we shall first explain the sources of various errors which generally affect the results of some of the more important practical problems of Nautical Astronomy, and then investigate the circumstances of observation under which such errors will be the least possible.

Change in place of horizon

The solution of almost all practical problems depends, as we have already seen, upon the measurement of altitudes of the heavenly bodies above the visible horizon. This horizon, even when apparently well defined, is often raised above its true position or depressed below it by irregular refraction to an uncertain and unknown extent.

The actual state of the horizon at any time might be ascertained either by an instrument termed a dip sector, or by observing the altitude of a heavenly body and its supplement at the same instant[1]; the sum of these, but for refraction (which may or may not be the same for opposite points of the horizon), should be 180° plus twice the *true* dip due to the height of the observer.

Observations of this kind are much required, as well authenticated cases exist of great deviations in the position of the sea horizon at different hours of the day.

Experiments made in the early part of this century show that, when the surface temperature of the sea is *higher* than that of the air, within a few feet above it, the tabular correction for apparent dip is too small, and when the sea surface is *colder* than the air the apparent dip taken from the table is too great; errors of 3′ to 4′ were found in observed altitudes with a difference of 9° F. in these temperatures.

This uncertainty with regard to the position of the horizon renders it unnecessary at sea to work to *seconds* of arc, the nearest

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[1] This can only be done with ordinary sextants when the altitude exceeds 60°; the supplement instead of the altitude is sometimes observed in night or twilight observations when the horizon is more clearly defined at a point opposite to that under the heavenly body.

minute in all cases where altitudes are involved being sufficiently accurate and logarithms taken out to the first four places only; this is especially the case in all star observations.

Night observations

This source of error is augmented in night observations by the difficulty usually experienced in *seeing* the horizon, but there exists a great difference of opinion on the subject of the reliability of star observations at night.

Some observers by taking the means of observations at, if possible, about the same altitude north and south of the zenith for latitude, and east and west of the meridian on the prime vertical for time, consider the results *perfectly* reliable; while others consider a latitude thus determined, even under favourable circumstances, liable to an error of at least 5 miles, and the longitude thus obtained to be still more doubtful.

All observers, however, agree that most excellent results for both latitude and time are obtained from *twilight* observations of the stars and planets.[1]

Errors in latitude by mer. alt.

It is evident on referring to figs. 118, 119, 120 that latitudes determined from meridian altitudes of heavenly bodies, either above or below pole, are affected by the *whole error* in such observed altitudes.

The altitude of a heavenly body may increase after crossing the meridian from two causes: (1) when the declination is rapidly changing so that the body is approaching the observer, and (2) when the observer is rapidly approaching the body, a circumstance which will produce the same effect as an equal change of its declination in the contrary direction. The time (T) from that of meridian passage when the maximum altitude is attained is given by the following equation:

$$\text{T} = \frac{d_1}{810000 \sin 1''} \, \frac{\sin (l \pm d)}{\cos l \, . \cos d}$$ (*see* Chauvenet's 'Astronomy').

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[1] The great speed of many modern vessels of war enables them to run distances of fully 300 miles in the winter season during the time when the *sun* is either invisible or unfavourably situated for observations; this alone must of necessity lead to efforts to improve the present character of star observations, and may also lead to the invention of a form of artificial horizon adapted to use at sea.

On this important subject the reader is referred to the opinions expressed by Captain Moriarty, R.N., in the *Encyclop. Brit.*, art. 'Navigation'; by Captain Wharton, R.N., F.R.S., the Hydrographer of the Admiralty, in *Hydrographical Surveying*; and by Captain Lecky, R.N.R., in *Wrinkles in Navigation.*

Among various improvements in the construction of sextants, by which they have been rendered more efficient for night observations, may be mentioned (1) an increased size given to the object glass of the telescope; (2) a cylindrical lens placed between the mirrors, by which the reflected image of a star is seen as a horizontal *line* of light; and (3) a prism of double-refracting material, placed in the same position, by which *two* images of a star are seen and the horizon brought between them.

Where T is expressed in seconds of time, d_1 is the hourly change of declination expressed in seconds of arc, and l and d represent the latitude and declination respectively.

This is arrived at from the following considerations:

From the observation made of the *maximum* altitude we can, from the formula used in the ex meridian problem, find the reduction to apply to obtain the *meridian* altitude. From the formula on page 249 neglecting the second term, which is inappreciable,

$$x = \frac{\cos l \,.\, \cos d}{\sin (l \underset{\sim}{+} d)} \; \frac{2 \sin^2 \frac{1}{2} h}{\sin 1''}$$

In this case x will be the change of altitude due to the apparent change of declination (d_1) during the interval h, or $\frac{d_1 \, h}{3600}$ in seconds of arc, and on substituting and reducing we arrive at the equation given.

The reduction (y) of the maximum to the meridian altitude is given by—

$$y = \frac{(15\text{T})^2 \sin 1''}{2} \; \frac{\cos l \,.\, \cos d}{\sin (l \underset{\sim}{+} d)}$$

where y is expressed in seconds of arc.

Suppose a vessel steaming 20 knots an hour south (true) in lat. 50° N. to observe the maximum altitude of the moon when it has no declination, but the hourly *change* of decl. is 17′ to the northward; the result of the combined movements of the ship and the moon will produce the same effect as if the ship were stationary, and the declination changed 37′ an hour.

If the observer and the heavenly body are *approaching* one another the maximum altitude will be attained *after* the time of crossing the meridian, and if receding from one another *before* that time for bodies above pole, and the reverse for the minimum altitude for bodies below pole.

On substitution in the above equations it will be found that the maximum altitude will exceed the meridian altitude by 3′ 28″, and will occur $11^{\text{m}}\ 14^{\text{s}}$ after the time of meridian passage.

In the case of the sun under similar circumstances, with a maximum hourly change of declination of 60″, the maximum altitude would exceed the meridian altitude by 1′ 7″, and occur at $6^{\text{m}}\ 22^{\text{s}}$ after noon.

If the value of T be substituted in the second formula, it gives

$$y = \frac{225 \text{ T} \sin 1''}{2} \,.\, \frac{\cos l \,.\, \cos d}{\sin (l \underset{\sim}{+} d)} \,.\, \frac{d_1}{810000 \sin 1''} \,.\, \frac{\sin (l \underset{\sim}{+} d)}{\cos l \,.\, \cos d}$$

$$\therefore \; y = \frac{d_1}{2} \,.\, \frac{\text{T}}{3600}.$$

Now d_1 is the apparent hourly change in the declination, and $\frac{T}{3600}$ is the number of hours from mer. passage; therefore we get the following:

1. The reduction of the maximum to the meridian altitude is equal to one-half the change in the apparent declination which occurs in the interval.

Also we know that

2. For altitudes near the meridian their differences from the meridian altitude vary directly as the squares of the elapsed times from the time of meridian passage.

The given example may therefore be worked without reference to the formulæ, thus:

With lat. 50°, dec. 0°, and $h=10^m$, the difference of altitude from the meridian altitude is calculated to be $2'\ 44{\cdot}5''=164{\cdot}5''$

$\therefore$ for 1 min. the difference is $1{\cdot}645''$.

The change of declination in 1 min. $=\frac{37'}{60}=37''$

$\therefore$ half „ „ „ $=18{\cdot}5''$.

Then if x denotes the number of minutes of time elapsed between meridian altitude and maximum altitude.

$$\begin{aligned}1{\cdot}645x^2 &= 18{\cdot}5x\\ 1645x &= 18500\\ x &= \underline{11^m\ 14^s}\end{aligned}$$

And the reduction $=18{\cdot}5''\times 11\frac{7}{30}=3'\ 28''$.

Error in latitude by alt. near meridian

In the case of finding the latitude by an altitude *near* the meridian, the practical rule was deduced from the formula

(1) $\text{vers Z}=\text{vers z}-\cos l\ .\ \cos d\ .\ \text{vers}\ h$ (*see* page 243).

The resulting meridian zenith distance Z is affected by errors in (1) z or the observed zenith distance, (2) in h, the supposed hour angle,[1] and (3) in l or the estimated latitude.

The effect of (1) is almost identical with the effect on a *meridian* altitude.

The effect of (2) may be thus investigated:

Suppose that the true value of the hour angle is h_1, then, remembering that an error in h will occasion an error of *the same sign* in Z when east of meridian (that is to say, too large an hour angle will give too great a zenith distance and *vice versâ*), and of the *opposite sign* when west, the true value of Z may be taken as Z_1.

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[1] In all the following investigations it should be remembered that hour angles are estimated from 0^h to 24^h.

$$(2) \qquad \therefore \text{vers}\,\mathrm{Z}_1 = \text{vers}\,z - \cos l \,.\, \cos d \,.\, \text{vers}\,h_1.$$

By subtracting (1) from (2)

$$\text{vers}\,\mathrm{Z}_1 - \text{vers}\,\mathrm{Z} = \cos l \,.\, \cos d\,(\text{vers}\,h - \text{vers}\,h_1),$$
$$\cos \mathrm{Z} - \cos \mathrm{Z}_1 = \cos l \,.\, \cos d\,(\cos h_1 - \cos h).$$

$$(3) \qquad -2 \sin \tfrac{1}{2}(\mathrm{Z} + \mathrm{Z}_1) \sin \tfrac{1}{2}(\mathrm{Z} - \mathrm{Z}_1) = -2 \cos l \,.\, \cos d.\ \sin \tfrac{1}{2}\,(h_1 + h) \sin \tfrac{1}{2}(h_1 - h).$$

Now $\frac{1}{2}(\mathrm{Z} + \mathrm{Z}_1)$ is approximately equal to Z, and $\frac{1}{2}(h_1 + h)$ to h, while $\frac{1}{2}(\mathrm{Z} - \mathrm{Z}_1)$ and $\frac{1}{2}(h_1 - h)$ being small, their sines are approximately equal to the arcs in seconds multiplied by $\sin 1''$, $\therefore$ equation (3) may be written

$$\sin \mathrm{Z}\ (\mathrm{Z} - \mathrm{Z}_1) \sin 1'' = \cos l \,.\, \cos d \,.\, \sin h\ (h_1 - h) \sin 1''.$$

$$(4) \qquad \therefore \mathrm{Z} - \mathrm{Z}_1 = \frac{\cos l \,.\, \cos d \,.\, \sin h}{\sin \mathrm{Z}}\ (h_1 - h).$$

But near the meridian Z is nearly equal to z, and from fig. 126, p. 242, we see $\frac{\sin \mathrm{PX}}{\sin \mathrm{XZ}} = \frac{\cos d}{\sin z} = \frac{\sin \text{azimuth}}{\sin h}$.

On substituting this value in equation (4) we find—

$$\left.\begin{array}{l}\text{Error in meridian zen. dist.}\\ \quad\text{and} \therefore \text{error in latitude}\end{array}\right\} = \cos l \,.\, \sin \text{azimuth} \,.\, \text{error in hour angle.}$$

Hence we see that the *lower* the latitude and the nearer the azimuth is to 90°, the greater the error in meridian zenith distance (and therefore of latitude) for a given error in h.

Thus on the equator with the azimuth one point from the meridian the error in latitude arising from an error of one minute in estimated time is about 3 miles, and under similar circumstances in lat. 60° only 1½ miles.

Hence we also see that under *all* circumstances the nearer the body is to the meridian the more accurate will be the result; but that if the object can be observed at equal, or at nearly equal distances on each side of the meridian the mean result will be free from the effect of an error in h, since such error will be + in one case and − in the other.

On similarly investigating the effect of an error $(l_1 - l)$ in the assumed *latitude* we find

$$\mathrm{Z} - \mathrm{Z}_1, \text{or error in mer. zen. dist.} = \frac{\sin l \,.\, \sin \text{azimuth} \,.\, 2 \sin^2 \frac{h}{2}}{\sin h} \,.\, (l_1 - l)$$

But $\sin h = 2 \sin \frac{h}{2} \,.\, \cos \frac{h}{2} = 2 \sin \frac{h}{2}$, for $\cos \frac{h}{2}$ approximately = 1, since h is very small.

On substituting this value of sin h and reducing

$$(5) \quad z - z_1 = \sin l \,.\, \sin \text{azimuth} \,.\, \sin \frac{h}{2} \,.\, (l_1 - l).$$

Hence it is seen that the error in meridian zenith distance (and therefore in latitude), arising from an error in estimated *latitude*, increases with the azimuth and also increases as the latitude increases.

In practice this error is eliminated by reworking the observation (*see* page 243).

Remarks

Latitudes obtained from altitudes near meridian may be considered as reliable as those from meridian altitudes (if the limits indicated in the preceding investigations are attended to), with the advantage that *several* can often be observed.

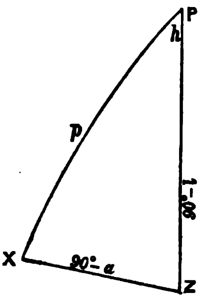
If observations cannot be obtained on *both* sides of the meridian, and if the position of the ship has not been determined astronomically for some time, the effect of any estimated error in time should be found by substitution in equation (4).

As extreme limits it may be shown that when the time is well known the second term may be neglected in lat. 60° with declination 23° of the *same* name for observations taken 35 minutes before *or* after noon, that interval being extended to 80 minutes before *or* after noon in the same latitude and with declination 23° of an *opposite* name. In lat. 30° under similar circumstances the limits may be considered to be 10 minutes and 50 minutes. The usual practical rule being that the hour angle in minutes of time should not be greater than the meridian zenith distance in degrees.

Errors in hour angles

The errors in the calculation of hour angles (upon which our knowledge of the longitude depends) are of these characters:

Fig. 160.



The hour angle (fig. 160) is calculated from the *assumed* values of P Z (90° − lat.), Z X (90° − alt.), and P X (90° − declin.); of these sides P Z at the time of making an observation is frequently only roughly known, since it generally depends upon the dead reckoning carried on from some former astronomical determination; and Z X is also liable to error, chiefly from uncertainty with regard to the horizon above which the altitude is measured.

P X may for purposes of navigation be always considered as accurately known, since the slow change occurring since the time when it is taken from the *Nautical Almanac* for Greenwich mean noon can be readily and correctly allowed for.

We have, then, to consider (1) the effect of an error in altitude on the hour angle, and (2) that of an error in latitude.[1]

In fig. 160, let a represent the altitude, p the polar distance, l the latitude, and h the hour angle.

$$\text{Cos } \mathrm{ZX} = \cos \mathrm{PZ} \,.\, \cos \mathrm{PX} + \sin \mathrm{PZ} \,.\, \sin \mathrm{PX} \,.\, \cos \mathrm{ZPX},$$

$$(1) \quad \text{or } \sin a = \sin l \,.\, \cos p + \cos l \,.\, \sin p \,.\, \cos h.$$

If a supposed error $+a_1$ be made in the altitude there will result an error of $+h_1$ or $-h_1$ in the hour angle, according to the position of the heavenly body observed with regard to the meridian; hence

$\sin(a+a_1) = \sin l \,.\, \cos p + \cos l \,.\, \cos p \,.\, \cos(h \pm h_1)$ where a_1 and h_1 are both expressed in arc.

On expanding this, and considering $\cos a_1$ and $\cos h_1$ each equal to unity, and $\sin a_1$ and $\sin h_1$ equal to $a_1 \sin 1''$ and $h_1 \sin 1''$ respectively (since a_1 and h_1 are both supposed to be small quantities), we find

$$(2) \quad \sin a + \cos a \,.\, a_1 \sin 1'' = \sin l \,.\, \cos p + \cos l \,.\, \sin p \,.\, \cos h$$
$$\pm \cos l \,.\, \sin p \,.\, \sin h \,.\, h_1 \sin 1''.$$

From equations (1) and (2) we have by subtraction

$$\cos a \,.\, a_1 \sin 1'' = \pm \cos l \,.\, \sin p \,.\, \sin h \,.\, h_1 \sin 1'',$$

$$(3) \quad \text{or } h_1 = \pm a_1 \frac{\cos a}{\cos l \,.\, \sin p \,.\, \sin h}.$$

From the original triangle PZX (fig. 160, p. 365) we find

$$\frac{\cos a}{\sin h} = \frac{\sin \mathrm{ZX}}{\sin \mathrm{ZPX}} = \frac{\cos l}{\sin \mathrm{PXZ}} = \frac{\sin p}{\sin \mathrm{PZX}}.$$

These values introduced into equation (3) give (expressed in time)

$$(4) \quad \begin{cases} h_1 = \pm a_1 \dfrac{1}{\sin \mathrm{PZX} \,.\, \cos l} \,.\, \dfrac{1}{15} \\ \text{or } h_1 = \pm a_1 \dfrac{1}{\sin \mathrm{PXZ} \,.\, \sin p} \,.\, \dfrac{1}{15} \end{cases}$$

(h_1 being expressed in seconds of time, and a_1 in seconds of arc).

Hence it follows that for any particular values of l and p, h_1, or the error in hour angle occasioned by an error a_1 in altitude, is least when $\sin \mathrm{PZX} = 1$; that is, when PZX or the azimuth is 90°, and therefore the heavenly body is on the prime vertical, either east or west of meridian: or when $\sin \text{PXZ}=1$; that is, when $\text{PXZ}=90°$, which, as seen on page 316, can only occur when the declination of the heavenly body observed exceeds the latitude of the observer.

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[1] Tables showing the effect of errors in both latitude and altitude on the hour angle will be found in *Wrinkles in Navigation*. *See* also Lecky's A, B, C and D tables.

The investigations given in the text do not apply when the body observed is near the meridian.

It has already been shown that for the *same* heavenly body the angle P Z X and P X Z cannot *both* be 90°.

The supposed error will also evidently produce the least effect when $l=0$, or when the observer is on the equator, and will consequently be less in a low than in a high latitude; the errors in h and a will have the *same* sign when west of meridian, and *opposite* signs when east.

In considering the effect on the hour angle of an error l_1 made in the assumed latitude, we have, as in equation (1) of preceding article,

$$(1) \qquad \sin a=\sin l \,.\, \cos p+\cos l \,.\, \sin p \,.\, \cos h.$$

Hence with the erroneous latitude $l+l_1$ we find

$\sin a=\sin (l+l_1) \cos p+\cos (l+l_1) \sin p \,.\, \cos (h+h_1)$ where l_1 and h_1 are both expressed in arc, an error of $+l_1$ producing different effects on the hour angles depending upon the true bearing of the body at time of observation.

On expanding this and considering $\cos l_1$ and $\cos h_1$ each $=$ unity, and $\sin l_1$ and $\sin h_1$ equal to $l_1 \sin 1''$ and $h_1 \sin 1''$ respectively (since l_1 and h_1 are both supposed to be small quantities), and also neglecting the term involving $\sin^2 1''$,

$$(2) \quad \sin a=\sin l \,.\, \cos p+\cos l \,.\, \cos p \,.\, l_1 \sin 1''+\cos l \,.\, \sin p \,.\, \cos h-\cos h \,.\, \sin l \,.\, \sin p \,.\, l_1 \sin 1''-\cos l \,.\, \sin h \,.\, \sin p \,.\, h_1 \sin 1''.$$

From equations (1) and (2) we have by subtraction and transposition

$$h_1 \sin 1'' (\cos l \,.\, \sin h \,.\, \sin p)=l_1 \sin 1'' (\cos l \,.\, \cos p-\cos h \,.\, \sin l \,.\, \sin p),$$

$$\text{or } h_1=l_1 \left(\frac{\cos l \,.\, \cos p-\cos h \,.\, \sin l \,.\, \sin p}{\cos l \,.\, \sin p \,.\, \sin h}\right),$$

$$(3) \qquad \text{or } h_1=l_1 \frac{\cos l \,.\, \cot p-\cos h \,.\, \sin l}{\cos l \,.\, \sin h}.$$

But from the original triangle P Z X by spherical trigonometry

$$\cos l \,.\, \cot p-\cos h \,.\, \sin l=\sin h \,.\, \cot \text{PZX}.$$

Hence h_1 in arc $=l_1 \dfrac{\cot \text{PZX}}{\cos l}$ or $l_1 \dfrac{\cot \text{PZX}}{\cos l} \dfrac{1}{15}$ in time,[1]

(h_1 being expressed in seconds of time and l_1 in seconds of arc), and the error in hour angle h_1 occasioned by an error l_1 in the latitude is least when $\cot PZX$ is least—that is, when PZX or the azimuth is 90°, and the heavenly body is therefore on the prime vertical.

---

[1] From this expression we see that when *east* of meridian the error of hour angle has the *opposite* sign to the error of latitude (that is, too much latitude gives too small an hour angle, and *vice versâ*) when the true bearing of the body observed lies between

This error is also less in a low than in a high latitude.

Hence the error in hour angle (and therefore in longitude) occasioned by an error in either altitude or latitude is least when the body observed is on the prime vertical, and at that time varies directly as such errors.

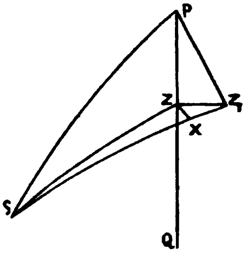
the elevated pole and prime vertical; and they have the *same* signs when the body lies on the equatorial side of the prime vertical.

When the body is *west* of meridian the signs are the *same* when the true bearing lies between the elevated pole and prime vertical, and *opposite* when the true bearing lies on the equatorial side of the prime vertical, the hour angle being supposed to increase from 0 hr. to 24 hrs., and the bearing to be estimated from the elevated pole: this follows from the change of sign of $\cot PZX$ as it alters in value.

Capt. H. S. Blackburne, late of the P. & O. S. N. Co., has published 'A and B Tables' for obtaining the angle PZX or azimuth by inverting this formula. Practically, the effect of an error in lat. is usually found as in the example on page 285.

The error of hour angle arising from (1) an error in altitude (only), and (2) an error in latitude (only), may be also graphically shown as follows:

FIG. 161.



(1) In fig. 161 let P represent the pole, S the sun, and Z and Z_1 the true and erroneous positions of the zenith, both lying on the same parallel. Through P and S draw arcs of circles of declination and altitude passing through Z and Z_1; with S as pole draw an arc of a small circle ZX, passing through Z; and with P as pole draw an arc of a small circle ZZ_1; then ZPZ_1 will represent the error in hour angle corresponding to an error Z_1X in altitude.

From the right-angled spherical triangle ZPZ_1 (the right angle being at Z) we have $\sin ZZ_1 = \sin ZPZ_1 \sin PZ_1$, or $ZZ_1 = ZPZ_1 \sin PZ$; since ZZ_1 and ZPZ_1 are both small and $PZ = PZ_1$.

$$\therefore ZPZ_1 = \frac{ZZ_1}{\sin PZ} = \frac{Z_1X}{\sin Z_1ZX \sin PZ}.$$

Since the triangle ZXZ_1 being small may be considered as plane, and has ZXZ_1 a right angle; also, since Z_1ZX and QZS are both approximately complements of QZX, this last expression becomes

$$\frac{Z_1X}{\sin QZS \sin PZ};$$

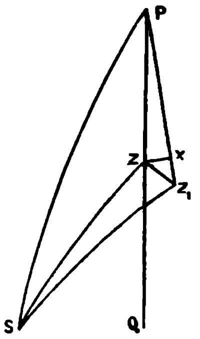
or error in hour angle = error in alt . cosec azimuth . sec lat.

(2) To similarly investigate the effect of an error in latitude, let Z and Z_1 (fig. 162) represent the positions of the true and erroneous zenith, both lying at equal zenith distances from S; then, on making a similar construction to fig. 161 and drawing ZX a parallel round P, ZPZ_1 will represent the error of hour angle corresponding to an error of latitude Z_1X.

Then, as before,

$$ZPZ_1 = \frac{ZX}{\sin PZ} = \frac{Z_1X}{\tan Z_1ZX \sin PZ} = \frac{Z_1X}{\tan QZS \sin PZ};$$

or error in hour angle = error in lat . cot azimuth . sec lat.



Remarks

In the case of the sun or any heavenly body whose declination does not exceed 23° N. or S. the time when it is on the prime vertical may be taken from either the azimuth tables or from Inman's Tables (No. 37); in low latitudes this may occur shortly before and after noon, enabling the observations for lat. and long. to be taken within a short interval of one another. In those cases where the declination exceeds the latitude the best time for observation will be also shown by the azimuth tables, since the azimuth does not then change for some time.

When the latitude and declination are of opposite names the observation should be made when (if possible) the altitude is not less than 10°, in order to obviate the effect of irregular refraction at low altitudes. For heavenly bodies whose declinations exceed 23° N. or S. the hour angle when on the prime vertical is found from the formula

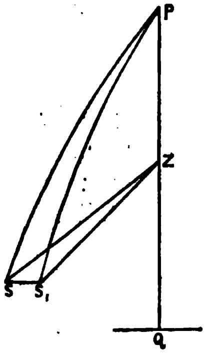
$$\cos \text{hour angle} = \cot \text{lat} \ . \ \tan \text{decl.},$$

which hour angle applied to the time of meridian passage gives the time of crossing the prime vertical.

Rate of change of altitude

Before proceeding to investigate the effects of errors on azimuths we shall show the rates at which the altitudes of heavenly bodies change when in different positions with regard to the observer; and first prove that the altitude of a heavenly body changes most rapidly when it is on the prime vertical :—

FIG. 163.



Let S and S_1 (fig. 163) be the places of any heavenly body when two altitudes are taken at a small interval of time apart.

Let h = hour angle S P Z, z = zenith distance Z S,
h_1 = hour angle S_1 P Z, z_1 = zenith distance Z S_1,
l = latitude and d = declination for the middle time between the observations.

Then $h - h_1$ = difference of observed times,
$z - z_1$ = corresponding difference of altitudes.

$$\text{In triangle PSZ, } \cos h = \frac{\cos z - \sin d \ . \ \sin l}{\cos d \ . \ \cos l},$$

$$\text{,, } \quad \text{P}S_1\text{Z, } \cos h_1 = \frac{\cos z_1 - \sin d \ . \ \sin l}{\cos d \ . \ \cos l},$$

$$\therefore \cos h - \cos h_1 = \frac{\cos z - \cos z_1}{\cos d \ . \ \cos l};$$

or $2 \sin \frac{1}{2}(h + h_1) \,.\, \sin \frac{1}{2}(h - h_1) = \dfrac{2 \sin \frac{1}{2}(z + z_1) \,.\, \sin \frac{1}{2}(z - z_1)}{\cos d \,.\, \cos l}$

(1) or $\sin h \,.\, (h - h_1) \sin 1'' = \dfrac{\sin z}{\cos d \,.\, \cos l} \,.\, (z - z_1) \sin 1''$.

Since $\frac{1}{2}(h + h_1)$ is nearly equal to h, and $\frac{1}{2}(z + z_1)$ is nearly equal to z; also since $\frac{1}{2}(h - h_1)$ and $\frac{1}{2}(z - z_1)$ are small quantities, their arcs, multiplied by sin 1″, may be substituted for their sines.

From (1) we have by transposition and division

$$z - z_1 = \frac{\cos d \,.\, \cos l \,.\, \sin h}{\sin z} (h - h_1)$$

But

$$\frac{\cos d}{\sin z} = \frac{\sin \text{PS}}{\sin \text{ZS}} = \frac{\sin \text{PZS}}{\sin h}.$$

Making this substitution on the right-hand side of the equation we have

$$\text{(A)} \qquad z - z_1 = \cos l \,.\, \sin \text{PZS}\, (h - h_1).^1$$

Hence when PZS is 90°, and therefore the heavenly body is on the prime vertical, the value of $z - z_1$, corresponding to a small change in the hour angle $(h - h_1)$, is a maximum; or, in other words, the body will in that position rise or fall most rapidly in any one latitude.

When PZS never attains the value of 90°, which, we have seen (page 316), occurs (1) either when the declination is greater than the latitude (both of the *same* name), or (2) when the latitude and declination are of *opposite* names, then the rate of change of altitude is greatest when the angle PZS attains its value *nearest* to 90°.

In the first case this occurs when the angle of position PSZ is 90°, and in the second case when the heavenly body is rising or setting. On the equator a heavenly body on the prime vertical changes its altitude 1′ in 4 secs. of time, and in any other latitude the change on prime vertical is to that on the equator as departure is to diff. long.

Altitude varies as hour angle on prime vertical

The same equation shows that the *rate* of motion for small intervals of time varies as the cosine of the latitude when the body considered is on the prime vertical, and approximately so when near it; hence at the equator such motion will be most rapid. Moreover, since in the position considered (on or near the prime vertical) differences of altitude *vary* as differences of hour

---

[1] From this equation it is seen that all bodies on the same or opposite azimuths increase or decrease their altitudes at the same rate, irrespective of what their declinations may be.

angles, the mean altitude of a set of sights truly corresponds with the mean time of observation, and *differences* of altitude should bear the same proportion to one another as the corresponding *differences* of times of observation, thereby affording a test of their accuracy.

Thus the following observations, taken near the prime vertical, are known to be good, since they fulfil this condition :

Time.	Altitude.
$7^h\ 10^m\ 20^s$	$54^\circ\ 3'\ 20''$
$11\ \ 10$	$6\ \ 30$
$12\ \ 30$	$11\ \ 30$

Change of altitude near meridian

In the expression for the value (x) of the reduction to the meridian, on page 249,

$$x = \frac{\cos l\,.\,\cos d}{\sin (l \pm d)}\,.\,\frac{2\sin^2 \frac{1}{2}h}{\sin 1''}$$

since h is small $\dfrac{2\sin^2\frac{1}{2}h}{\sin 1''} = \dfrac{2(\frac{1}{2}h\sin 1'')^2}{\sin 1''} = \dfrac{h^2}{2}\sin\ 1''$

$$\therefore\ x = \frac{\cos l\,.\,\cos d}{\sin (l \pm d)}\,.\,\frac{\sin 1''}{2}\,.\,h^2.$$

Hence, near the meridian the *change* of altitude varies as the *square* of the hour angle or of the angular distance from the meridian, and therefore the *mean* altitude of a series does *not* correspond to the mean of the times of observation.

The expression showing at *any instant* the change of altitude ($z - z_1$) corresponding to a change of hour angle ($h - h_1$) is

$$z - z_1 = \cos l \sin \mathrm{PZX}\,(h - h_1) + \frac{\cos l \sin \mathrm{PZX} \cos \mathrm{PZX} \cos \mathrm{PXZ}}{\sin h}\,.\,\frac{(h - h_1)^2}{2}.$$

When either PZX or $\mathrm{PXZ}_1 = 90^\circ$ the second term on the right side vanishes and we obtain equation (A), page 370.

Errors in altitude azimuth

In determining the deviation of the compass from an altitude azimuth, any error in the observed altitude will produce the least effect when the motion in altitude bears the greatest possible ratio to the motion in azimuth.[1]

The *relative* changes of altitude and azimuth of a heavenly body may be illustrated by the accompanying figure drawn on the plane of the horizon, fig. 164.

---

[1] It may be shown that if x is the altitude of the sun when it is changing most rapidly $\sin x = \dfrac{\text{sin declination}}{\text{sin latitude}}$ or $\dfrac{\text{sin latitude}}{\text{sin declination}}$ according as the latitude is greater or less than the declination, and if y is the altitude when its bearing is changing *slowest* $\sin y = \tan \dfrac{x}{2}$, therefore an error of altitude produces the least effect at some altitude between x and y (see *Naval Science* for 1874).

If X *a b* Y represent the diurnal path of a heavenly body X, equal portions (*a b*) of this arc will be described in equal times owing to the earth's uniform motion of rotation.

If the motion in *a b* be resolved in the direction of a circle of altitude and also at right angles to it, then *a c* will represent the change in altitude, and the arc of the horizon corresponding to *b c* will represent the corresponding change in azimuth.

Similar figures should be drawn by the reader to illustrate these relative changes for other values of the latitude and declination.

The effects of errors in altitude and latitude on the azimuth may be investigated in the same way as on pp. 366, 367, but the following will afford an illustration of another and more graphic method of arriving at the same result:

FIG. 164.

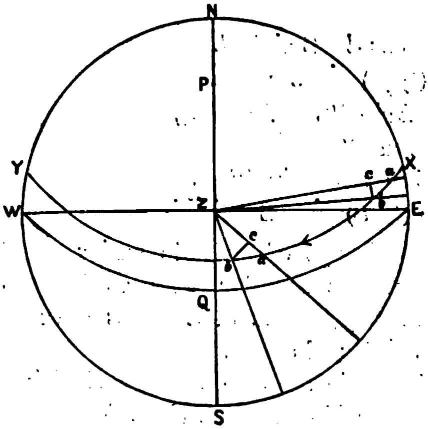
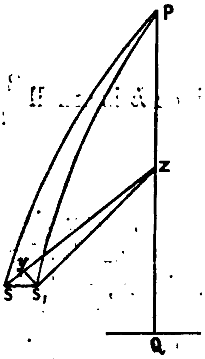


FIG. 165.



In fig. 165, if $S S_1$ and $S_1 Y$ are arcs of circles drawn round P and Z as poles, it is evident that $S Z S_1$ represents the error in azimuth arising from an error S Y in the altitude, and that each of the angles $P S S_1$, $P S_1 S$, $Z Y S_1$, and $Z S_1 Y$ is approximately a right angle.

From the triangle $S_1 Y Z$ we have

$$S_1 Y = S_1 Z S \sin S_1 Z$$

$$(1) \quad \therefore S_1 Z S = S_1 Y \operatorname{cosec} S_1 Z$$

But from the triangle $S Y S_1$, which, being small, may be considered to be a plane triangle, we have

$$S_1 Y = S Y \tan S_1 S Y$$

$$(2) \qquad = S Y \cot P S Z.$$

Substituting in (1) this value of $S_1 Y$,

$$S_1 Z S = S Y \cot P S Z \operatorname{cosec} S_1 Z,$$

or error in azimuth = error in alt. sec alt. cot P S Z.

Hence the error is least when the altitude is low and P S Z or the angle of position is 90°.

On similarly investigating the effect of an error in *latitude* on the azimuth we find:

Error in azimuth = error in lat. sec lat. cot hour angle. Hence an error in latitude produces its least effect when cot h vanishes—that is, when $h=90°$ and the heavenly body observed is on the six o'clock hour circle; or when cos l is a maximum or the observer is on the equator.

Error in time azimuth

In the case of a time azimuth the effect of an error in the apparent time on the resulting azimuth may be found from equation (1) of the preceding article (fig. 165, page 372).

$$\text{S}_1\,\text{Z}\,\text{S} = \text{S}_1\,\text{Y}\ \text{cosec}\ \text{S}_1\,\text{Z}.$$

$$\text{But}\ \text{S}_1\,\text{Y} = \text{S}\,\text{S}_1 \cos \text{P}\,\text{S}\,\text{Z}$$

$$\text{and}\ \text{S}\,\text{S}_1 = \text{S}\,\text{P}\,\text{S}_1 \sin \text{P}\,\text{S}_1,$$

$$\therefore \text{S}_1\,\text{Z}\,\text{S} = \text{S}\,\text{P}\,\text{S}_1 \cos \text{P}\,\text{S}\,\text{Z}\,\frac{\sin \text{P}\,\text{S}_1}{\sin \text{Z}\,\text{S}_1},$$

$$\text{or error in azimuth} = \text{error in time}\ \frac{\sin \text{azimuth} \,.\, \cos \text{P}\,\text{S}\,\text{Z}}{\sin \text{hour angle}},$$

which will be a minimum when either the hour angle is 90° or 6 hours, or when the angle of position P S Z is 90°, at which time the azimuth does not change.

Remarks

Although the preceding investigations indicate the *best* times for observing azimuths, the deviation can in practice be obtained at almost *any* time, the chief limitation being that the altitude of the heavenly body observed should not be too great, since the errors arising from the compass card not being quite horizontal at the time of observation rapidly increase with the altitude.

The deviation is often conveniently obtained by calculating the time of a heavenly body crossing the prime vertical (*see* page 369) and observing its compass bearing at that instant, or by observing the compass bearing when on the meridian, provided the altitude be not too great.

Double altitudes. Limits of observation

The following investigation showing the limits of observation for the double altitude problem (direct method) was given by Dr. Inman.

Let P Z Q be the celestial meridian, P the pole, Z the true zenith, X the true place of the body observed at the *greater* bearing X Z Q, Y the true place of the body observed at the *less* bearing Y Z Q = (suppose) b; and let A Z B, C Z D be parallels round poles X and Y. Instead of the true zenith distance Z X, let an incorrect one equal to Z′ X be used in the computation by the rule for finding the latitude. Then, if the zenith distance ZY be supposed to remain correct, it is evident, since Z′Y is equal to ZY, that by the said computation the point Z′ will be determined for the zenith instead of Z, the real zenith; so that, if Z′T, Z′S be parallels round the poles X and P, ZT will be the error in zenith distance of X, and ZS the corresponding error in the colatitude; that is, $PS = PZ'$ will be determined for the colatitude instead of PZ.

Now $ZS = ZZ' \cos Z'ZS$, and $ZZ' = \dfrac{ZT}{\cos Z'ZT}$; consequently $ZS = ZT \, . \, \dfrac{\cos Z'ZS}{\cos Z'ZT}$, where $Z'ZS$ or CZQ is the *complement* of YZQ, the less bearing, that is, of b; and $Z'ZT$ is the *complement* of XZY, the difference of the bearings, that is, of d, if d denote that difference. It appears therefore that—

FIG. 166.

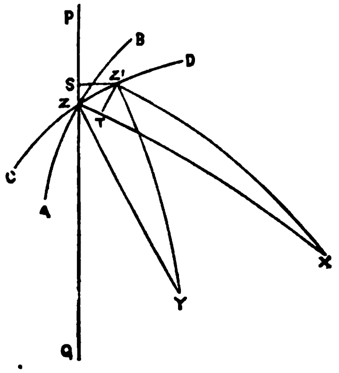
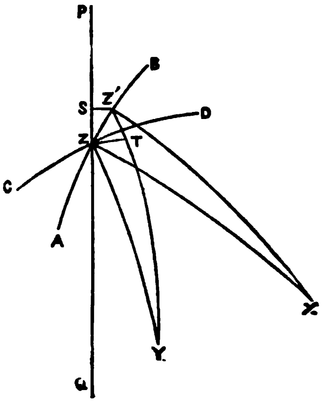


FIG. 167.



$$\text{Error in lat.} = \text{error in alt.} \frac{\sin b}{\sin d} \, .$$

Since, according to the rules given on page 256, b should be always less than d and also less than $180° - d$, it follows that $\sin b$ is always less than $\sin d$. Consequently $\dfrac{\sin b}{\sin d}$ is always less than 1. If, therefore, the error in altitude at the greater bearing be 2′, which is the utmost that should be made at sea, the consequent error in the computed latitude will be less than 2′, supposing the observation at the less bearing to be correct.

If $\sin b$ be less than one-half, one-third, &c., of $\sin d$, then the error in the computed latitude arising from an error in the altitude at the greater bearing considered singly will be less than one-half, one-third, &c., of 2′, or than 1′, 40″, 30″, &c.

Let the error be now supposed to be in the zenith distance at the *less bearing* (consequent on an error in the observed altitude), and let the zenith distance at the *greater bearing* be supposed correct. Thus, instead of Z Y let Z′ Y be found from observation. Then since $Z'X = ZX$, it is evident that Z′ will be the zenith determined by the computation, and that Z′ T and Z S will be corresponding errors in the zenith distance at the less bearing and in the colatitude, Y and P being poles of Z T and Z′ S.

Now, $ZS = ZZ' \cos Z'ZS$, and $ZZ' = \dfrac{Z'T}{\sin Z'ZT}$; consequently,

$ZS = Z'T \cdot \dfrac{\cos Z'ZS}{\sin Z'ZT}$, where Z′ Z S or A Z Q is the *complement* of the greater bearing X Z Q, that is, of $b + d$; and $Z'ZT = XZY = d$, each having the same complement, X Z T. Hence it appears that—

$$\text{Error in lat.} = \text{error in alt.} \frac{\sin (b+d)}{\sin d}$$

Now, since $\sin (b+d)$ is less than $\sin b + \sin d$, it follows that $\dfrac{\sin (b+d)}{\sin d}$ is less than $\dfrac{\sin b + \sin d}{\sin d}$ or $\dfrac{\sin b}{\sin d} + 1$. And since $\dfrac{\sin b}{\sin d}$ has been shown before to be less than 1, it follows that $\dfrac{\sin (b+d)}{\sin d}$ is less than 2. So that supposing the error in the zenith distance (or in the observed altitude) at the less bearing to be 2′, the other altitude being correct, the consequent error in the computed latitude will be less than 2′ × 2, or 4′.

If $\sin b$ be much less than $\sin d$, as one-half, one-third, &c., thereof, this error will be considerably less than 4′.

If errors be made in both altitudes, as of 2′ in each, and these errors concur in their effects on the computed latitude, then the whole error in the said latitude will be less than 2′ + 4′, or 6′.

General Relations of Right Ascension and Declination as compared with Longitude and Latitude.

The very remarkable relations which exist between the right ascension (a), declination (δ), latitude (λ), longitude (L), obliquity (ω), and angle (θ) at a star between the circles of declination and latitude passing through it, can all be exhibited by the aid of a single triangle.

In fig. 168 on next page, let A Q be the equator, A L the ecliptic, P and P′ their respective poles. Let P A be the equinoctial, and

P′PLQ the solstitial colure. Let S be the place of a star; PSD and P′SI circles of declination and latitude passing through it. Draw the arc P′A.

Then the arcs AQ, AL, PA, PD, P′A, and P′I are all quadrants, or 90°, and the angles APP′ and AP′P are both right angles.

Hence, therefore—

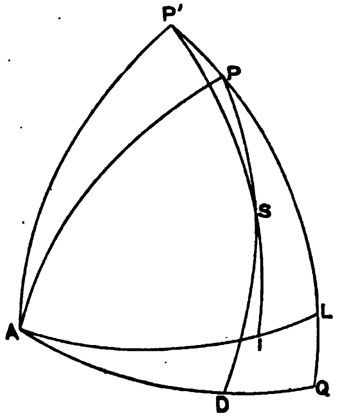
$$\text{SPP}' = 90° + a \;;\quad \text{SP}'\text{P} = 90° - \text{L} \;;\quad \text{P}'\text{SP} = \theta$$
$$\text{PS} = 90° - \delta \;;\quad \text{P}'\text{S} = 90° - \lambda \;;\quad \text{P}'\text{P} = \omega$$

$$\text{Also} \quad \left.\begin{array}{l} \sin \text{SPP}' = \cos a \\ \cos \text{SPP}' = -\sin a \end{array}\right\} . \quad \text{And} \quad \left.\begin{array}{l} \sin \text{SP}'\text{P} = \cos \text{L} \\ \cos \text{SP}'\text{P} = \sin \text{L} \end{array}\right\}$$

And applying these values to the three fundamental formulæ of spherical trigonometry, viz.—

$$\cos a = \cos b \cos c + \sin b \sin c \cos \text{A} \qquad \text{(I.)}$$
$$\sin a \sin \text{B} = \sin b \sin \text{A} \qquad \text{(II.)}$$
$$\cot \text{A} \sin \text{B} = \cot a \sin c - \cos \text{B} \cos c \qquad \text{(III.)}$$

Fig. 168.



in the triangle P′PS, the following equations can easily be obtained:

$$\tan a = \cos \omega \tan \text{L} - \sin \omega \tan \lambda \sec \text{L} \qquad (1)$$
$$\sin \delta = \cos \omega \sin \lambda + \sin \omega \cos \lambda \sin \text{L} \qquad (2)$$
$$\tan \text{L} = \cos \omega \tan a + \sin \omega \tan \delta \sec a \qquad (3)$$
$$\sin \lambda = \cos \omega \sin \delta - \sin \omega \cos \delta \sin a \qquad (4)$$
$$\cos \delta \cos a = \cos \lambda \cos \text{L} \qquad (5)$$
$$\cot \theta = \cot \omega \cos \delta \sec a + \tan a \sin \delta \qquad (6)$$
$$= \cot \omega \cos \lambda \sec \text{L} - \tan \text{L} \sin \lambda \qquad (7)$$

Note.—In the above expressions (1) is obtained from formula (III.) by making S P P correspond to A and S P′P to B. Similarly, (3) is arrived at from (III.) by making S P′P to be represented by A and S P P′ by B.

Equations (2) and (4) are obvious applications of (I.).

So, also, (5) is directly deduced from (II.).

Finally, the expressions (6) and (7) are derived from (III.) by successively making the angles A and B correspond, first to θ and P, and then to θ and P′.

These equations are very remarkable: (1) and (2) give expressions for the right ascension and declination in terms of the obliquity, and the latitude and longitude; (3) and (4) in a similar manner express the longitude and latitude in terms of the right ascension and declination.

Equation (5) gives the general relation between the right ascension and declination as compared with the longitude and latitude.

Finally, (6) and (7) give expressions for the angle of situation, θ, the first in terms of the right ascension and declination, and the second in terms of the longitude and latitude.

The figure represents the first quadrant of the celestial sphere; in the second quadrant S P′P would be L − 90°, and in the same way that angle and S P P′ would alter in the other quadrants: but the algebraical results above deduced are of course general, and independent of the particular position of the geometrical figure whence they were derived.

In applying the formulæ, however, to a numerical solution in any particular case, attention must be paid to the algebraic signs of the functions of the various quantities.

SPHEROIDAL FORM OF EARTH.

Although the circumstances under which observations are taken at sea render extreme precision unattainable, and therefore the application of small corrections due to the form of the earth not being that of a sphere unnecessary, it is advisable that the reader should understand the nature and extent of the errors thus introduced.

Direct measurement of many arcs of both latitude and longitude in various localities combined with astronomical observations show the form of the earth to be that of an oblate spheroid whose shorter axis passes through the poles, the proportion of its length to that of the longer axis being nearly 299 : 300.[1]

The section of such a solid figure taken through the poles will be an ellipse.

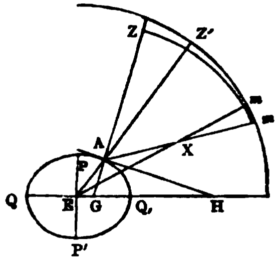
In fig. 169 let $P\ Q\ P_1\ Q_1$ be such a section passing through a place A on the surface. Let E be centre of the ellipse; join

---

[1] The quantity $\frac{300-299}{300} = \frac{1}{300}$ is termed the compression; the actual lengths of the polar and equatorial diameters are 7,899 and 7,925 miles.

E A; then if A G be drawn at right angles to the tangent at A, meeting $Q\,Q_1$ in G, the angle $A\,G\,Q_1$ is the latitude as determined by observation or the *geographical latitude*, the angle A E G is the reduced or *geocentric latitude*, and the angle E A G ($=A\,G\,Q_1 - A\,E\,Q_1$) is the angle of the vertical. This angle of the vertical, or difference between the geographical and geocentric latitudes, attains its maximum value (11′ 30″) in lat. 45° and vanishes at the poles and at the equator. The *geocentric* is always less than the *geographical* latitude except at the poles and equator, where they coincide.

Fig. 169.



The positions of heavenly bodies given in the *Nautical Almanac* are those they would occupy as seen from E, but, since the positions of places on the earth are laid down on charts according to their *geographical* latitudes, the determination of this is the object in Nautical Astronomy.

If G A and E A be produced to Z and Z′ on the celestial concave, then Z is termed the astronomical zenith and Z′ the reduced or geocentric zenith, the angular distance between them being equal to the angle of the vertical.

Let X be the position of a heavenly body, and join A X, E X; then

the angle Z A X is the apparent zenith distance of X (corrected for refraction)
„ Z′ A X „ apparent reduced zenith distance of X „ „
„ Z′ E X „ true „ „ „ „ „ „

The parallax of X in the given position is really the angle A X E, which is sometimes termed the *diurnal* parallax, to distinguish it from the parallax in altitude when parallax is supposed to take place in a circle of altitude passing through Z instead of in a circle passing through Z′[1]; the error involved in this assumption even in the case of the moon is very trifling, and still less appreciable in the case of the more distant heavenly bodies.

Refraction, being a phenomenon affecting the position of a heavenly body as seen from the *surface* of the earth, is unaffected by its spheroidal form, or, in other words, refraction acts in a vertical circle passing through the *astronomical* zenith, while parallax acts in a circle passing through the *geocentric* zenith.

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[1] Practically this means that the apparent altitude of a heavenly body, from which its parallax in altitude is calculated (*see* page 153), should be increased by the angle Z A Z′ when the body is on the meridian, and by the amount of this angle multiplied by the cosine of its azimuth when not on the meridian.

PRECESSION OF EQUINOXES.

The slow annual movement of the first point of Aries alluded to in the chapter on Time (page 124) is termed the *Precession of the Equinoxes,* since it causes the positions of successive equinoctial points to *precede* one another and the sun to arrive at the first point of Aries about 20 minutes earlier every year.[1]

Precession is occasioned by the attraction of the sun and moon[2] on the spheroidal protuberance of the earth round the equator.

The effect of this attraction if the earth were not rotating on its axis would be to make the planes of the equator and ecliptic coincide, but combined with the rapid diurnal rotation it causes a slow conical motion of the axis of the earth (and therefore of the heavens) round an axis passing through the poles of the ecliptic.

The direction of this revolution is *opposite* to the direction in which the earth travels in her orbit, and therefore as seen from the northern side would be *with* the hands of a watch (*see* fig. 56, page 121).

This revolution of the axis of the earth involves a similar revolution of the points where the planes of the equinoctial and ecliptic intersect, since the plane of the equinoctial varies in position with that of the terrestrial equator, and that of course changes with the movement of the earth's axis. The result is that in the interval of 25,870 years the pole of the earth, and therefore that of the heavens, will have described a circle round the pole of the ecliptic at a mean distance of 23° 27′, during which time every star on such a circle will in turn become our Pole-star, and the first points of Aries and Libra will have described a complete circle round the ecliptic.

The tendency of the sun and moon to bring the planes of the equator and ecliptic into coincidence is greatest when they have extreme declination, and ceases when they have none.

As all the elements of the *Nautical Almanac* are calculated from the position of the *true* equinox—that is, from the *actual place* of the first point of Aries—on every day of the year, no correction for precession is applied in any of the problems of Nautical Astronomy.

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[1] **This was first discovered by Hipparchus in B.C. 160, and explained by Newton in 1687. In this elementary description the effects of *planetary* precession and *nutation* are disregarded.**

[2] **Of the total precession about 14½″ is owing to the effect of the sun, and 35½″ to that of the moon.**

It will be remarked that precession does not affect the obliquity of the ecliptic nor the *latitudes* of heavenly bodies, but it causes a constant increase in their *longitudes*, and will affect them in declination and right ascension in different ways according to their positions in the heavens. This may be seen by referring to the catalogue of mean places of stars given in the *Nautical Almanac* with the annual variations in right ascension and declination.

The *astronomical* seasons of spring and autumn commence when the sun's centre passes respectively through the first points of Aries and Libra; summer and winter commence when the sun attains longitude 90° and 270° respectively, measured from the *actual* first point of Aries.

Hence the commencement of each season occurs earlier each year by the time the sun takes to apparently describe an arc of 50·22″, or by about 20 minutes.

It is to be remarked that precession does not alter the *relative* positions of the stars, since it only shifts the circles of reference to which these positions are referred.

Independently of this, however, slow motions in various directions of the stars themselves exist, termed their 'proper motions,' by which in the course of very many years the familiar appearance of the constellations will be greatly changed.

CHAPTER XIV.[1]

Latitude and longitude from observations at same time—Sumner's method—Marcq Saint-Hilaire's method—Double chronometer rule.

THE problem of making observations for determining the longitude of a ship available at the same time for ascertaining the latitude appears to have been originally investigated by the celebrated French astronomer Lalande in 1764.[2]

The principle involved is that if two observations for longitude of the same or different heavenly bodies are made at the same place,[3] and the results on being calculated with an assumed latitude exactly agree, that assumed latitude is correct; it will not follow that the resulting *longitude* is correct, though the hour angles will be so, for in problems of this character, as in all others for determination of longitude at sea except the lunar, it must be remembered that we are *entirely dependent* upon the chronometer for a knowledge of Greenwich mean time, and therefore of longitude.

Among several methods of solution of this valuable problem that known as 'Sumner's' method first claims our attention.

SUMNER'S METHOD.[4]

In Chapter IV., the student has been shown how the position of a ship can be fixed by the intersection of two lines determined by compass bearings of known objects, and we shall now show how the principle of fixing positions by the intersection of two lines on a chart can be extended by means of celestial observations.

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[1] In compiling this chapter a partial translation has been made of portions o the excellent chapter on the 'Rectification du Point Estimé' in Dubois's *Cours de Navigation*, and the works of Hilleret, Ledieu, and Fasci on this subject have been consulted.

[2] It was first published in England by Samuel Dunn, in 1786, from what appears to be an independent investigation, and was much used in the Royal Navy in the early part of this century, being known as the method by trial and error.

[3] The correction for run enables us to consider both observations to have been made at the same place.

[4] The reader should also refer to the chapters on Sumner's method in *Hydrographical Surveying*, by Rear-Admiral Wharton, R.N., F.R.S., the Hydrographer of the Admiralty, and in *Wrinkles on Navigation*, by Captain Lecky, R.N.R.

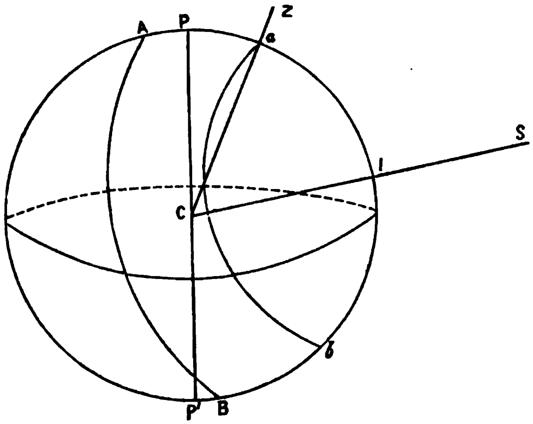
The first *published* work on the subject was by Captain Sumner, of Boston, U.S., in 1843, which book contains a very clear description of the chord method (page 385).

Lines of position

A line, in whatever way obtained, which passes through the point on a chart representing the position of a ship, is called a *line of position.* Such a line is not necessarily *straight*; for instance, a danger angle gives us a *circle* of position, and consecutive soundings an *irregular* line, but all these are lines of position, and the intersection of any two will give the latitude and longitude of the observer.

We are in fact about to use the bearings of two heavenly bodies, or of the same body in two different positions, in very much the same way as when fixing a position by cross bearings of terrestrial objects, hence the original expressive name given to this method, viz. *cross bearings of the sun.*

FIG. 170.



Geographical position of sun

In the following investigation we shall see how lines of position can be obtained from observed altitudes of the sun and other heavenly bodies. We shall first show how to trace on the globe a series of circles dependent upon the sun's altitude, and then turn our attention to the projections of these circles on a Mercator's chart and then of similar circles derived from other heavenly bodies.

At any given moment the sun is in the zenith of *some* particular place and is shining over half the world. This hemisphere is bounded by a great circle called the *terminator* and sometimes the *circle of illumination*. The spot vertically below the sun, being the pole of this great circle, is termed the geographical position of the sun, and may be defined as the point where the straight line joining the centres of the sun and earth (considered spherical) cuts the surface of the latter, and its position can readily be determined by taking the sun's declination as its latitude and the Greenwich apparent time (expressed in arc) as its longitude. This will be evident when we consider that at any given Greenwich apparent time, as say 4^h, it must be apparent noon to all places on the 60th meridian of west longitude, and at the *one* place on that meridian whose latitude equals the sun's declination the sun must be in the zenith. This pole is not a fixed point, but, owing to the rotation of the earth, travels round it in a direction and at a rate corresponding to the apparent diurnal motion of the sun.

Let O be the centre of the earth (fig. 170), A B a part of the circle of illumination, *a b* a small circle parallel to A B, and S the sun; I represents the geographical position of the sun, and P P′ the poles of the earth.

FIG. 171.

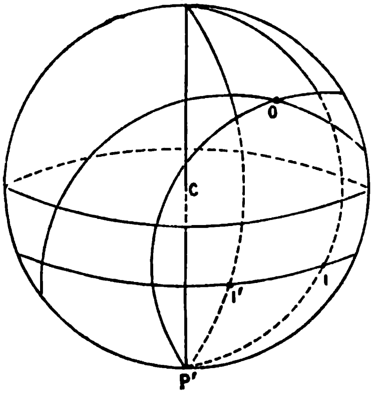
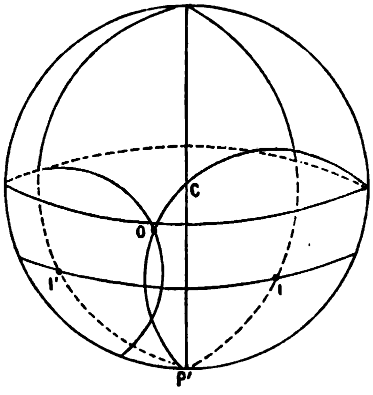


FIG. 172.



Parallels of equal altitude

It is evident that the angle *a* C S is constant for all positions of *a* on the small circle *a b*; but *a* C S is the sun's zenith distance (true) at *a*; therefore the sun's zenith distance (and altitude) must be constant for all points on *a b*. Small circles round I are therefore called *parallels of equal altitude.*

If, then an altitude of the sun be known at any given moment, and a small circle described round I as pole at a distance equal to the sun's zenith distance, the observer must be situated on the circumference of this circle; and if without changing place he obtains a second altitude when I has moved to some other position I′ (figs. 171, 172), then a second parallel of equal altitudes can be found, and it is certain that the observer must be situated at one or the other of the points of intersection of these two parallels. As these two points will in general be very far apart, there will be no difficulty in deciding upon the position required.

Curve of equal altitude

The projection of a parallel of equal altitudes upon a Mercator's chart is called a *curve of equal altitude.*

These curves vary very much in form, changing not only with the altitude but also with the declination. In the accompanying figure (173, page 386) the sun's declination is assumed to be 10° N. and the Greenwich apparent time $4^h\,0^m$ P.M., corresponding to longitude 60° W.

There will be no great difficulty in tracing any particular curve—say that of 30° altitude or Z.D. 60°. First obtain the limits of latitude, where such an altitude is possible, which will be the points where the curve cuts the meridian of I.

Zenith distance	60° N.	Zenith distance	60° S.
declination	. 10 N.	declination	. 10 N.
N. latitude	. . 70	S. latitude	. 50

That is to say, in lat. 70° N. the meridian altitude of the sun will be 30° bearing south, and in lat. 50° S. its meridian altitude will be 30° bearing north.

Nearer the poles than 70° N. and 50° S. the sun will not rise to as great an altitude as 30°.

Now take any intermediate latitudes, *e.g.* 30° N. and 10° S., and work out with altitude 30° the corresponding longitudes.

lat.	30° N.	....0·062469	lat.	10° S.	....0·006649
decl.	10 N.	....0·006649	decl.	10 N.	....0·006649
	20	4·808067		20	4·808067
Z.D.	60	4·534052	Z.D.	60	4·534052
	80	9·411237		80	9·355417
	40	61° 1′ E. or W. of I.		40	56° 52′ E. or W. of I.
long. of I.		60 W.			60 W.
		1 1 E.			3 8 W.
		and 121 1 W.			116 52 W.

Each latitude gives us two longitudes, the easterly longitude corresponding to a P.M. and the westerly to an A.M. altitude of 30°.

Any required number of points can thus be found, and the curve sketched in by hand.

In actual practice, since the position of the ship is always approximately known by the dead reckoning, it would be sufficient to determine *two* points on the curve of equal altitude by calculating with the observed altitude the longitudes corresponding to two latitudes lying one a few minutes north and the other a few minutes south of the D. R. latitude, and assuming that the *straight line* joining the two positions so found is identical with the *arc* of the curve between them. This method of dealing with such curves (known as the *chord* method) was that originally proposed; but since each curve requires for its delineation at least two calculations of longitude, and we must have two curves drawn, it involves a considerable amount of work, and is now (since the publication of azimuth tables and diagrams) generally superseded by what is known as the *tangent* method, where a tangent drawn through the point on the curve corresponding to the dead reckoning latitude and the longitude calculated with that latitude is assumed as the direction of the curve; such a tangent is termed a 'Sumner' line.

Sumner line

By careful consideration of the curve, when obtained, the student will readily grasp the following important conclusions.

(1) When the observer is nearly E. or W. of I (or, in other words, when the sun is near the prime vertical), two points on the curve differing considerably in latitude have a proportionally insignificant difference of longitude, a fact we have already recognised as indicating the most suitable position for making observations for determining ship apparent time when the latitude is not accurately known (*see* page 368).

(2) When the sun is near the meridian, points differing slightly in latitude have large differences in longitude; while, on the other hand, points differing considerably in longitude differ but slightly in latitude.

(3) Any great circle in the heavens passing through the sun's centre cuts at right angles all the parallels of equal altitude round the sun.

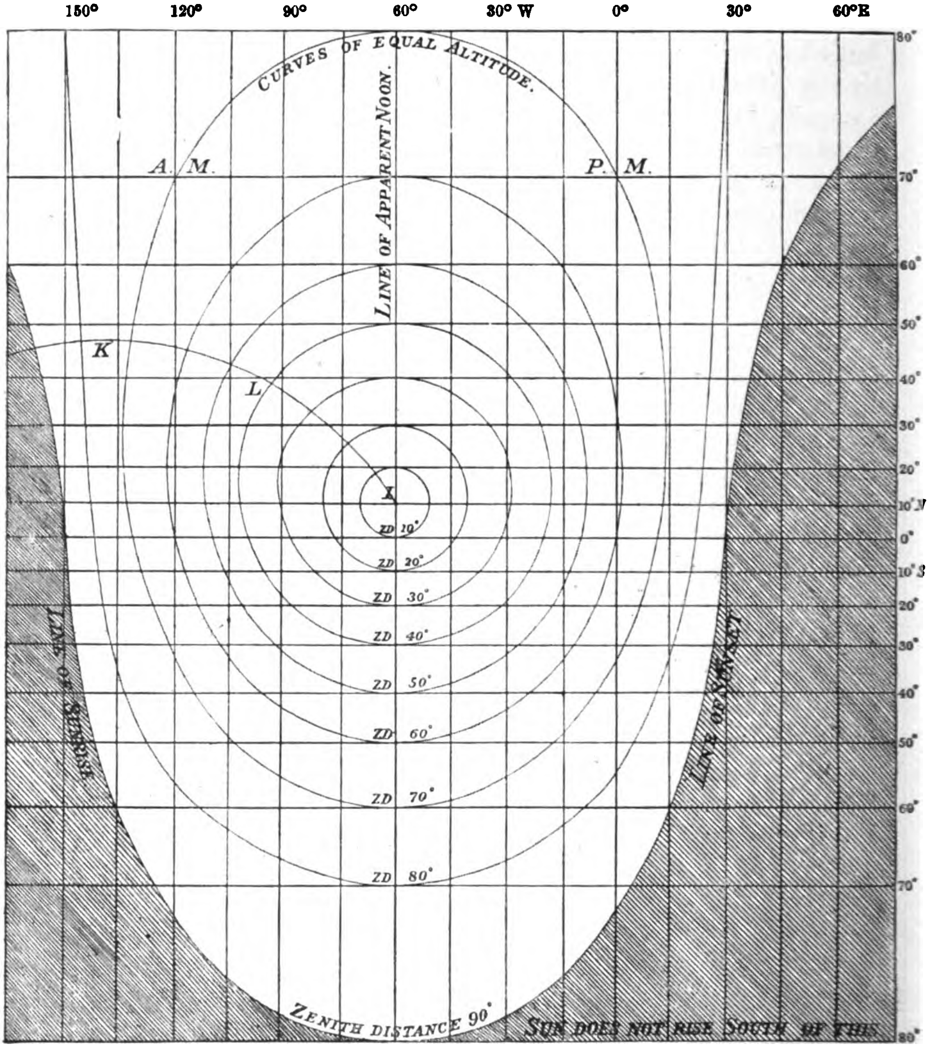
(4) The projection of such a great circle on a Mercator's chart (fig. 173) cuts at right angles all the curves of equal altitude. Thus the curved line I L K on the Mercator's chart is the line of the sun's true bearing, because it is the projection of the great circle passing through the zenith and the sun, and it cuts at right angles every curve of equal altitude.

(5) The tangent to a curve of equal altitude is consequently at right angles to the sun's true bearing, and represents the *direction* of the curve at that point, so that a line drawn at right angles to the true bearing of the sun through the dead reckoning position of the ship may be considered as representing the curve itself for a considerable length; for even with a zenith distance of

only 10° or altitude 80° we are dealing with an arc of a circle of radius 600 miles.

The sun's true bearing at any moment can be taken out by

FIG. 173.



inspection from the azimuth tables, or from Weir's azimuth diagram, or by compass bearing with the dev. and var. applied.

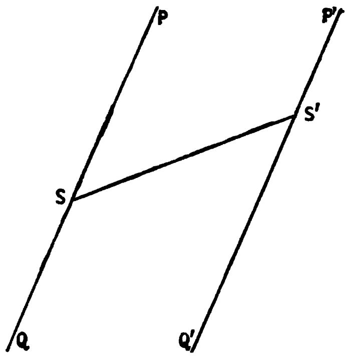
If, as is usually the case, two observations to enable us to draw the two Sumner lines are both taken sufficiently far from the meridian for them to be worked as 'sun chronometers,' the two positions obtained—viz. dead reckoning latitudes and longitudes by observation—are laid down on the chart, and the intersection of the Sumner lines through them gives the position at time of the last observation.

To obtain a good intersection the angle between these lines, which is the same as the difference of the two bearings of the sun, should not be less than 25° or more than 155°.

If the first observation is taken far from the meridian and the second near the meridian, use estimated latitude and work out a 'longitude by chronometer' with the first observation; and work out the second observation as an 'ex-meridian,' taking care to utilise the hour angle obtained by the earlier observation.

The second line will in general be as dependable as the first, but the azimuth tables show very readily when a difference of a degree in latitude or longitude makes any marked difference in the direction of the tangent.

FIG. 174.



Up to the present we have supposed that the ship has not changed her position between the times of observation, but as in practice she does, we proceed as follows:

The correction for run between the times of observation may be applied as a correction to the first *altitude* (as on page 221), and thus both observations reduced to the same instant, but by proceeding *graphically* on the chart, the calculation can be commenced while waiting to observe the second altitude.

Let P Q (fig. 174) be a Sumner line determined by observation—that is, a line drawn on the chart through S, the dead reckoning position of the ship at right angles to the sun's true bearing.

From S draw S S' in the direction of and equal to the run between times of observation; through S' draw P' Q' parallel to P Q.

Then if the ship was in P Q at the commencement of the run it must be in P' Q' at the termination, as all lines drawn from P Q parallel and equal to S S' terminate in P' Q' and the latitude of S' will be that to use in working the second observation.

Example 1 On February 22, at 8^h 30^m A.M., in lat. 50° 12′ N., long. 8° 35′ W. (by D. R.), the true alt. of the sun was 14° 31′; G. M. T. 21^h 17^m 10^s; decl. 10° 5′ S.; eq. of time 13^m 30^s add. to App. time.

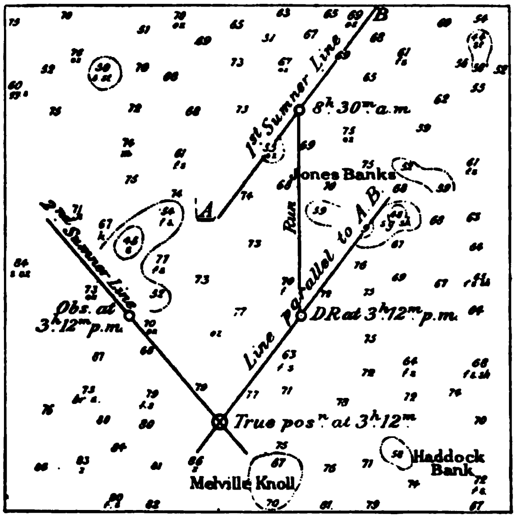
The vessel then ran S. (true) 32 miles, and at 3h 12m P.M., when the G. M. T. was 4h 0m 4s, the true alt. of the sun was 17° 13′; decl. 10° 1′ S.; eq. of time 13m 28s.

Find lat. and long. of second position of ship.

On working the first observation as a sun chronometer, using the lat. by D. R., we find a position lat. 50° 12′ N., long. 8° 14½′ W., which is laid down on the chart (see fig. 175) and a Sumner line A B drawn through this at right angles to the sun's true bearing, N. 126° E., taken from the azimuth tables.

The run is laid off from this first position and through its extremity, which will be the D. R. position at 3h 12m P.M.; a line is drawn parallel to A B, or, in other words, the first Sumner line is transferred to the second D. R. position. With the latitude of this second D. R. position (49° 40′ N.), the second observations are worked, also as a 'chronometer,' giving lat. 49° 40′ N., long. 8° 54¼′ W., and plotted on the chart as the position by observation at 3h 12m P.M. Through this last position the second Sumner line is drawn at right angles to the sun's true bearing at that time, N. 130° W., and the intersection of this with the transferred first Sumner line gives the true position of the ship, lat. 49° 24′ N., long. 8° 33′ W., at 3h 12m P.M.

Fig. 175.



Example 2 On April 21, in lat. 49° 22′ N., long. 8° 42′ W. (by D. R.), at 8h 32m A.M. the true alt. of the sun was 33° 32′; G. M. T. 21h 6m 8s; decl. 12° 2′ N.; eq. of time 1m 28s subtr. from App. time.

The ship then ran S. 73° E. (true) 36 miles, and at about

11^h 35^m A.M. (the elapsed time by chronometer being 3^h 1^m 10^s) the following observation was made near the meridian :

True alt. of sun 52° 31′ ; decl. 12° 4′ N. ; eq. of time 1^m 29^s.

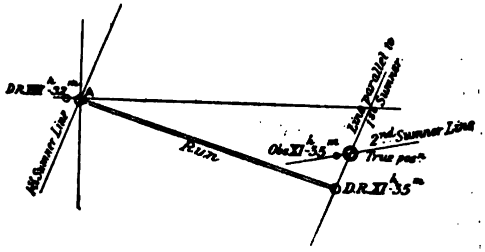
Find the lat. and long. of second position of the ship.

On working the first observation as a chronometer we obtain a position A, lat. 49° 22′ N., long. 8° 39¾′ W., which is placed on the chart (see fig. 176), and the first Sumner line drawn through that position at right angles to N. 112° E., which is the sun's true bearing.

The run is then laid off and the D. R. position at 11^h 35^m A.M. found, lat. 49° 11½′ N., long. 7° 46¾′ W., through which position a line is drawn parallel to the first Sumner line.

The second observation is then worked as an ex-meridian, using the hour angle 22^m 20^s found in the usual way, resulting in a position lat. 49° 16′ N., long. 7° 46¾′ W. A second Sumner line is drawn through this at right angles to the sun's true bearing, N. 170° E,[1] and the intersection of this with the transferred first Sumner line gives the ship's position at 11^h 35^m, lat. 49° 16′ N., long. 7° 41′ W.

Fig. 176.



Sumner lines are usually laid down on the chart, or if the scale of that is small, on the skeleton chart supplied for the purpose; but the position may be *calculated* as in the following example :

Example 3

In lat. by D. R. 40° N. Declination 13° N.

---

[1] The Sumner line for an ex-meridian observation is conveniently laid down as follows:

From the position of the ship, in the latitude obtained by the ex-meridian observation and the assumed longitude, lay off half the hour angle in arc measured on the longitude scale, East (true) if the body is East of meridian, and West (true) if West of meridian.

From the extremity of this line draw a line at right angles to it in a direction away from the body and equal in length to the 'reduction to the meridian' measured on the latitude scale.

The extremity of this latter line joined to the position of the ship is the required Sumner line.

Ship apparent time (from calculating a sun chronometer) $19^h\ 56^m\ 11^s$, longitude 18° 30′ W.

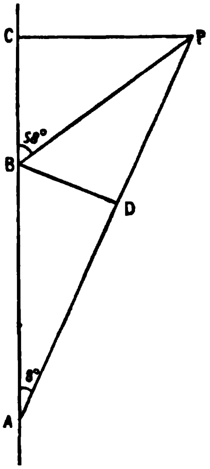
Azimuth taken from the azimuth tables N. 98° 45′ E., so that the direction of the tangent, or Sumner line, will therefore be N. 8° 45′ E.

After an interval by chronometer of $3^h\ 0^m\ 10^s$ another observation is taken, worked with longitude 18° 30′ W., supposing there is no 'run,' and a latitude of 40° 11′ N. obtained.

Turning to the azimuth tables with an hour angle of $22^h\ 56^m$, we find the azimuth to be N. 148° E., and the direction of the tangent is accordingly N. 58° E.

A P and B P are the two tangents whose intersection in P is the true position of the ship (fig. 177), P C is perpendicular to the meridian A B, and B D to A P.

FIG. 177.



$$A B = 11', \; B A D = 8°, \; B P D = 50°.$$
$$\therefore \text{ by traverse table } B D = 1{\cdot}5, \; B P = 2,$$
$$B C = 1{\cdot}1', \; P C = 1{\cdot}7'.$$

The latitude of P will therefore be 40° 12′ N. and the longitude 18° 28′ W.

If there had been any 'run' during the interval between the observations, the first position would have been corrected by traverse table, as on page 221, so as to bring it up to the time of making the second observation, and the second observation worked with the hour angle derived from the corrected longitude.

Remarks A great advantage of the *graphical* mode of procedure lies in the readiness with which the two Sumner or position lines can be transferred on the chart to a new position, and crossed with a third or even fourth line, or if in sight of land by a bearing of some object, thus affording additional certainty of the position.

It must, however, be clearly understood that, in whatever way this problem is solved, the resulting *longitude* entirely depends upon the *chronometer*.

With very little modification all the preceding investigations are as applicable to any other heavenly body as to the sun. The geographical position of the body being in latitude the same as its declination and in longitude the same as its hour angle from Greenwich expressed in arc, which equals its hour angle from the ship ± longitude of ship.

At twilight the position of a ship can be satisfactorily fixed by simultaneous observations of two stars. Care must be taken in this case that the stars differ considerably in bearing, so as to ensure a clear intersection between the two Sumner lines, and if one of the stars be near the meridian the Sumner line must be found by the ex-meridian method.

When simultaneous observations are made they are, of course, worked with the *same* latitude, viz. that by D.R., and the reason why if both are chronometer sights two different longitudes are found is that the D.R. latitude is erroneous.

In using the azimuth tables to find the direction of the Sumner lines the student will remember that with stars, planets, or moon, the tables are entered with the *hour angles* of those bodies.

Probably the most valuable line obtainable is from a star or planet at morning twilight, crossed by a subsequent line obtained from the sun. Venus is often most favourably situated for such an observation. The navigator should wait for his second observation till the sun has reached a convenient altitude, and it is advisable that the star should be of opposite declination to the sun, so as to secure a good angle between the two lines; a result which will also be attained by selecting a *high* star near the meridian.

In finding a Sumner line from a star, if the declination exceeds 23° or the altitude 60°, the azimuth cannot be found from the azimuth tables, and must be either taken from Weir's azimuth diagram, or observed by compass, or calculated.

The value of *one* Sumner line found by either of the preceding methods lies in the fact that since, if the G. M. T. is correct, the ship must be *somewhere* on this line, its direction gives the bearing of any point of land which may lie upon it when produced in either direction.

Thus suppose in thick weather when near the land a Sumner line (say N. 10° E.) is found to cut the coast line at 20′ west of a harbour it is desired to enter; if course is altered as convenient so as to make a diff. long. of 20′, on steering N. 10° E. the ship will then be heading for the harbour.

An error in the chronometer time (and therefore in G. M. T.) does not affect the *direction* of this line, but moves it parallel to itself east or west by an amount equal to this error. An error in altitude moves the line parallel to itself, either from or towards the heavenly body, by an amount equal to the error.

Since for small zenith distances (10° to 12°) the form of the curve of equal altitude closely approximates *in low latitudes* to that of a circle, this curve may sometimes be laid down on the chart instead of the tangent.

This is effected by taking as centre a point whose lat. is the same as the declination of the sun, and whose longitude equals the Greenwich apparent time, the radius being equal in miles to the true zenith distance in minutes of arc.

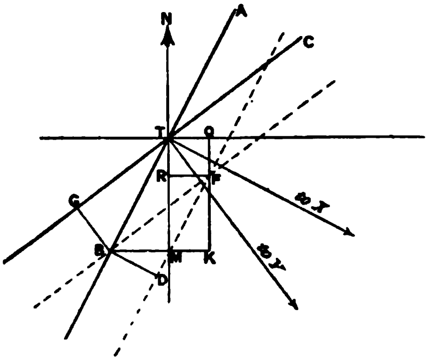
After an interval admitting of a due change of bearing, another arc of a circle of position is similarly drawn, the first circle being shifted on the chart to allow for the ship's run.

One of the intersections of these arcs is the position at time of second observation.

Note.[1]—To find the errors of the deduced latitude and longitude (δl and δg), corresponding to errors of δa_1 and δa_2 of the observed altitudes—

In the accompanying figure, let A T B, C T G represent two correct lines of position obtained from two altitudes of the same or different heavenly bodies X and Y, which intersect at T, the true place of the observer; also let F D and F B represent two faulty lines of position caused by errors of δa_1 and δa_2 in the altitudes, which would place the observer at an erroneous point F.

FIG. 178.



Through F draw F R and Q K perpendicular to the meridian and parallel of latitude at T.

Through B draw B D, B G, and B K perpendiculars to A B, C G, and Q K.

Then $ABK = ATQ = MTX = A_1$ (suppose)

$FBK = CTQ = MTY = A_2$ „

and $TBF = GTB = XTY = A_1 - A_2$.

also $BFD = GTB = A_1 - A_2$.

---

[1] By Mr. G. Williams, Naval Instructor, R.N. Fig. 178 is supposed to be drawn on a globe or plane chart.

$$
\begin{aligned}
\text{Now } \delta l &= \mathrm{TR} = \mathrm{TM} - \mathrm{RM} \\
&= \mathrm{TB} \,.\, \sin \mathrm{TBM} - \mathrm{FB} \,.\, \sin \mathrm{FBK} \\
&= \frac{\mathrm{BG}}{\sin \mathrm{GTB}} \,.\, \sin \mathrm{TBM} - \frac{\mathrm{BD}}{\sin \mathrm{BFD}} \,.\, \sin \mathrm{FBK} \\
&= \frac{\delta a_2 \,.\, \sin \mathrm{A}_1}{\sin \overline{\mathrm{A}_1 - \mathrm{A}_2}} - \frac{\delta a_1 \,.\, \sin \mathrm{A}_2}{\sin \overline{\mathrm{A}_1 - \mathrm{A}_2}} \\
\delta l &= \frac{\delta a_2 \,.\, \sin \mathrm{A}_1 - \delta a_1 \,.\, \sin \mathrm{A}_2}{\sin \overline{\mathrm{A}_1 - \mathrm{A}_2}} \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
\text{Again } \mathrm{TQ} &= \mathrm{BK} - \mathrm{BM} \\
&= \mathrm{FB} \,.\, \cos \mathrm{FBK} - \mathrm{TB} \,.\, \cos \mathrm{TBM} \\
&= \frac{\mathrm{BD}}{\sin \mathrm{BFD}} \,.\, \cos \mathrm{FBK} - \frac{\mathrm{BG}}{\sin \mathrm{GTB}} \,.\, \cos \mathrm{TBM} \\
\delta g \,.\, \cos \text{lat.} &= \frac{\delta a_1 \,.\, \cos \mathrm{A}_2 - \delta a_2 \,.\, \cos \mathrm{A}_1}{\sin \overline{\mathrm{A}_1 - \mathrm{A}_2}} \\
\delta g &= \frac{\delta a_1 \,.\, \cos \mathrm{A}_2 - \delta a_2 \,.\, \cos \mathrm{A}_1}{\sin \overline{\mathrm{A}_1 - \mathrm{A}_2} \,.\, \cos \text{lat.}} \qquad (2)
\end{aligned}
$$

From an examination of the formulæ (1) and (2) it will be seen that if δa_1 and δa_2 are equal or nearly equal (as the probabilities are the personal and instrumental errors will be of the same quantity and quality), then we have

Firstly.—$\mathrm{A}_1 - \mathrm{A}_2$ should be near 90° as is possible.

Secondly.—If the navigator be more anxious about his latitude than his longitude, then from (1) we see that the two observations should be taken on opposite sides of the prime vertical.

Thirdly.—If the observer be uneasy about his longitude, then from (2) we find that the two observations should be taken on opposite sides of the meridian.

The last two considerations would be of great importance if expecting soon to make the land.

MARCQ SAINT-HILAIRE'S METHOD.

The following method of dealing with curves of position has been proposed by M. Marcq Saint-Hilaire, and is illustrated by fig. 179.[1]

Let A represent the position of a ship by dead reckoning when an observation of a heavenly body in *any* position is taken, the bearing also being either observed or taken from the azimuth tables. With the hour angle deduced from the D. R. longitude the true altitude is *calculated* (as on page 215), and the difference between this and the *observed* true altitude is laid off on the line of bearing. If the *observed* altitude is less than that *calculated*, the position must be further away from the geographical position of the body than was expected, and so the difference is laid off *away from* the heavenly body, and *vice versâ*; through the point B, thus found, C D, a tangent to the line of bearing, is drawn.

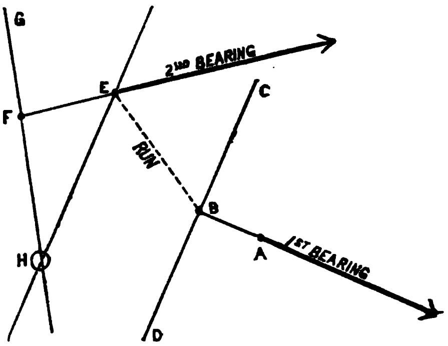
---

[1] The reader should here refer to the explanation of this method given in *Ex-Meridian Altitude Tables*, by Staff-Comm. Brent, R.N., and Messrs. Walter and Williams, Naval Instructors, R.N.

The run of the ship B E is laid off from B, and at E another altitude of the same or another heavenly body is observed and also calculated, the hour angle being found from the long. of E; the difference between these is dealt with as in the first case, and the intersection of the corresponding tangent G H, with a line through E parallel to the first tangent, gives H the position at time of second observation.

An advantage of this method is that no distinction of cases is involved, and whether far from or near to the meridian the observations can be always worked in the same way, or, if preferred, that nearer the prime vertical worked as a Sumner, and that nearer the meridian by this method; but, as in Sumner's method, the lines of position should intersect at a good angle (not less than 25° nor more than 155°).

Fig. 179.



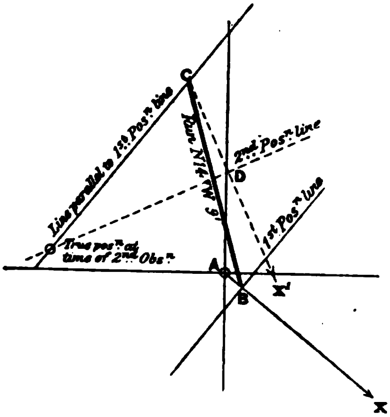
Example.—At time of first A.M. observation of the sun, the position by dead reckoning was lat. 37° 50′ N., long. 9° 15′ W.; the G. M. T. being $21^{h}\ 24'\ 9''$, by applying the long. in time and equation of time ($-13^{m}\ 41{\cdot}5^{s}$) in the usual way the hour angle is found to be $20^{h}\ 33^{m}\ 27{\cdot}5^{s}$, with which declination, 17° 20′ 43″ S., the zenith distance is computed (see page 215) and found to be 73° 26′ 17″. The observed zenith distance (true) being 73° 25′ 17″ and the sun's true bearing S. 51° E.

From the D. R. position A (fig. 180) lay off the sun's true bearing A X, and since the *observed* zenith distance was less than that computed, the observed must be *nearer* the sun than was supposed, so a distance of 1′ is laid off on A X towards X, and through this position B, the first position line, is drawn at right angles to A X.

The run N. 14° W. 9′ is then laid off from B, and at its extremity C a line is drawn parallel to the first position line and the zenith distance computed with that position lat. 37° 59′ N., long. 9° 18′ W.

Working with decl. 17° 19′ 19″ S., eq. of time 13^m 42^s and G. M. T. 23^h 24^m 30^s, we find the computed zenith distance 58° 54′ 20″ and the sun's true bearing S. 24° E., while the observed zenith distance was 58° 50′ 20″, hence the observer must again be nearer the sun than was supposed, so the difference of the two zenith distances C D is laid off on C X′ towards the sun, and the second position line drawn through D.

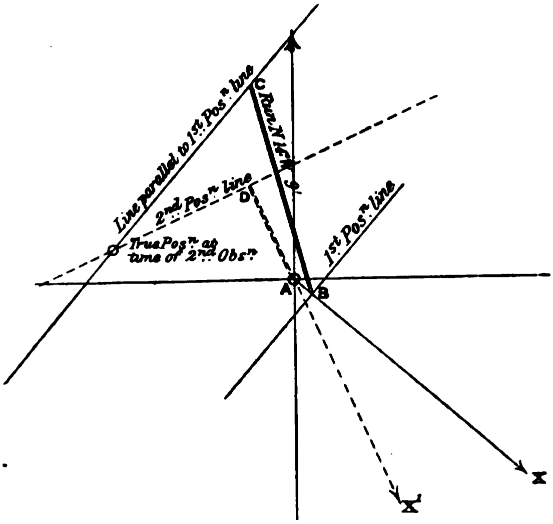
Fig. 180.


C
Line parallel to 1st Posn. line
Run N 14 W 9′
2nd Posn. line
D
1st Posn. line
True posn. at time of 2nd Obsn.
A
B
X′
X

The intersection of the second position line with the transferred first line will be the position of the ship at time of second observation, viz. lat. 57° 31′ N., long. 9′ 24 W.

The calculation may be shortened by using the position of A

Fig. 181.


C
Line parallel to 1st Posn. line
Run N 14 W 9′
2nd Posn. line
D
1st Posn. line
True Posn. at time of 2nd Obsn.
A
B
X
X′

to work *both* sights with and laying off the differences between observed and calculated zenith distances from the point A as illustrated in fig. 181.

Double Chronometer Rule.[1]

Theory

In considering in Chap. XIII., page 367, the effect of an error in latitude $l_{\prime}$ on an hour angle deduced from it, we saw that, except when near the meridian,

$$(1) \qquad h_{\prime} \text{ (in time)} = l_{\prime} \, \frac{\text{cot azimuth}}{\text{cos lat}} \cdot \frac{1}{15},$$

the azimuth being measured through 180° from the elevated pole.

Suppose, then, that two altitudes of a heavenly body both *west* of meridian have been observed at the chronometer times C' and C'', and that the first altitude worked with the dead reckoning latitude $L_{\prime}$ gave the S. M. T. $= t$, and that the second altitude worked with the D. R. latitude $L_{\prime\prime}$ (found by applying the run in latitude to $L_{\prime}$) gave the S. M. T. $= t''$.

If, then, the correction for run in longitude be applied to t, and t' be the result, we have t' and t'' the two results of the observations reduced to the instant when the *second* altitude was observed, and if the estimated latitudes were *correct*, $t'' - t'$ should equal $C'' - C'$, both being expressed in *mean* time.

Now the effect of an error in latitude $l_{\prime}$ on the first observation will be in this case to alter the S. M. T. reduced to the time of the second observation to $t' + l_{\prime} \dfrac{\text{cot az.}}{\cos L_{\prime}} \cdot \dfrac{1}{15}$, and that of t'' for the same error in latitude to $t'' + l_{\prime} \dfrac{\text{cot az.}}{\cos L_{\prime\prime}} \cdot \dfrac{1}{15}$, the azimuths being those calculated, observed, or taken from the azimuth tables for the positions at the times of making the observations, which azimuths we shall call $A_{\prime}$ and $A_{\prime\prime}$.

Hence, we shall find

$$\left(t'' + l_{\prime} \frac{\cot A_{\prime\prime}}{\cos L_{\prime\prime}} \cdot \frac{1}{15}\right) - \left(t' + l_{\prime} \frac{\cot A_{\prime}}{\cos L_{\prime}} \cdot \frac{1}{15}\right) = C'' - C',$$

from which we obtain

$$(2) \qquad l_{\prime} \text{ (in arc)} = \frac{(C'' - C') - (t'' = t')}{\dfrac{\cot A_{\prime\prime}}{\cos L_{\prime\prime}} - \dfrac{\cot A_{\prime}}{\cos L_{\prime}}}$$

This expression for the value of $l_{\prime}$ may be simplified by supposing $L_{\prime} = L_{\prime\prime}$, or by taking the mean value, when we finally obtain, for the general expression, after some reductions,

$$(3) \qquad l_{\prime} = \frac{\{\, (C'' = C') - (t'' - t') \,\} \cos L_{\prime\prime} \sin A_{\prime} \sin A_{\prime\prime}}{\sin (A_{\prime} - A_{\prime\prime})}.$$

---

[1] Proposed in 1847 by M. Pagel of the French Navy; a translation of his work will be found in the *Mercantile Marine Magazine* for 1865.

The value of l, or the correction to be applied to the latitude at time of making the second observation being thus found, the correction of t' or S. M. T. at the same instant is found by substitution in equation (1).

As originally explained by M. Pagel, such a number of various cases and conditions arise as to practically render the problem useless, but by the following more graphic method of illustration (where the heavenly body observed is supposed to be the sun) we are led to the same results in a more practical way.[1]

Let P G (figs. 182, 183) represent the meridian of Greenwich.

z the *true* zenith of the observer at time of second observation.

x the sun's place at the first observation.

y the sun's place at the second observation.

$a\,z\,c$ and $b\,z\,d$ small portions of circles of equal zenith distance which may, except with great altitudes, be considered to be straight lines.

FIG. 182.

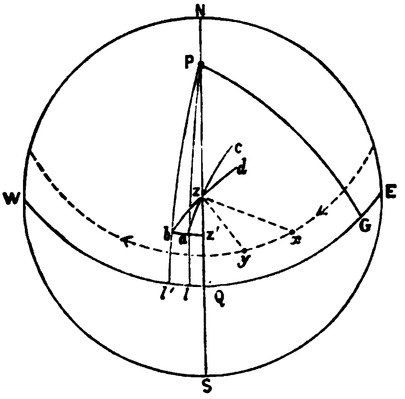
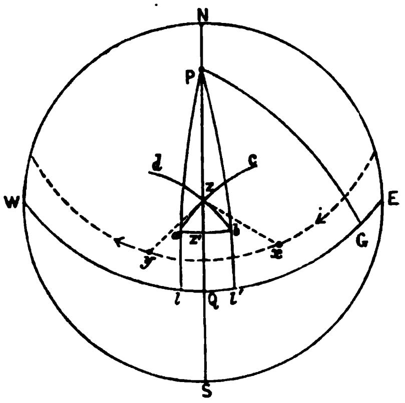


FIG. 183.



$z\,q$ the true lat. at time of second observation, and z' Q the lat. by dead reckoning at the same instant.

a and b the positions where the circles of equal zenith distance intersect the parallel through z'.

Then if the first chronometer is worked with the lat. D.R. at that time and corrected for run, the first observation places the ship at a when the second observation is taken, the second places her at b, both a and b being, of course, on the same parallel of lat. as z'.

The longitudes corresponding to these positions are Gl and Gl'. Then G l—G l=$l\,l'$, the diff. long.

In the triangles $z\,b\,z'$, $z\,a\,z'$, which being very small may be considered as plane triangles, ab is known (=departure corresponding to ll') and zaz' (=to the first true bearing $z'zx$, since they are both complements to z_1za) and zbz' (=the second true bearing $z'zy$, since both are complements of $z'zb$); hence we can find zz' the correction for the latitude $z'\text{Q}$; and thence $z'a$, $z'b$, and $\text{Q}l$, $\text{Q}l'$ the corrections for the two longitudes.

---

[1] By Mr. A. C. Johnson, Naval instructor R.N., late of H.M.S. *Britannia*.

Let the true bearings of the sun at the times of first and second observations be represented by $\text{A}_{,}$ and $\text{A}_{,,}$.

$$\text{Then } \text{Z}\,\text{Z}_1 = \text{Z}a \,.\, \sin \text{Z}a\text{Z}_1 = \text{Z}a \,.\, \sin \text{A}_{,}$$

$$\text{but } \text{Z}a = ba\,\frac{\sin \text{Z}ba}{\sin b\text{Z}a} = ba\,.\,\frac{\sin \text{A}_{,,}}{\sin(\text{A}_{,} \pm \text{A}_{,,})}$$

The upper sign applying to fig. 183, and the lower to fig. 182.

$$\therefore \text{Z}\,\text{Z}_1 = ba\,\frac{\sin \text{A}_{,}\,.\,\sin \text{A}_{,,}}{\sin(\text{A}_{,} \pm \text{A}_{,,})}.$$

Now ba or the departure$=ll_{,}$ or diff. long. × cos lat. or al,

$$\therefore \text{Z}\,\text{Z}_1 \text{ or corr. in lat.} = \frac{\text{diff. long.} \times \sin \text{A}_{,} \times \sin \text{A}_{,,} \times \cos \textit{lat.}}{\sin(\text{A}_{,} \pm \text{A}_{,,})}.$$

$$\text{Also } a\text{Z}_1 = \text{Z}\,\text{Z}_1 \,.\, \cot \text{Z}a\text{Z}_1 = \text{Z}\,\text{Z}_1 \,.\, \cot \text{A}_{,},$$

$$\therefore a\text{Z}_1 \,.\, \sec al \text{ or lat.} = \text{Z}\,\text{Z}_1 \,.\, \cot \text{A}_{,} \,.\, \sec \text{lat.},$$

$$\text{and } b\text{Z}_1 \,.\, \sec \text{lat.} = \text{Z}\,\text{Z}_1 \,.\, \cot \text{A}_{,,} \,.\, \sec \text{lat.}$$

But $a\text{Z}_1$. sec lat. and $b\text{Z}_1$. sec lat. are equal to $\text{Q}l$ and $\text{Q}l_{,}$, which are the corrections (in arc) to be applied to the longitudes of a and b.

Hence we arrive at the same results as in the preceding investigation.

Practical rule

(1) Two chronometer observations on either or both sides of the meridian are to be taken at an interval of an hour and a half or two hours, or at a less interval, provided that the change in the sun's bearing is not less than a point and a half or two points.

(2) The first observation is worked with the lat. D.R., and this latitude and the first longitude obtained are corrected for the run of the ship in the interval. The second observation is worked with this corrected latitude, and the longitudes are marked (1) and (2).[1]

(3) The true bearing of the sun at each observation is either observed or taken by inspection from the azimuth tables for the D.R. lat.

(4) For the correction in lat. add together log. diff. between

---

[1] If two stars or a star and the sun are employed the most *eastern* observation is marked (1) and the most *western* (2).

(1) and (2), [L sin first true bearing, L sin second true bearing, L cosec diff. or sum of bearings, according as the observations are made on the *same* or *opposite* sides of the meridian] and L cos lat. at time of second observation; the sum, rejecting 40, will be log. correction of lat. in miles.[1]

(5) For the correction in long. add to the L cot of each true bearing the log. correction in lat. and the L sec lat. at place of second observation; on subtracting 20 from each sum the remainder is the log. correction of long. in minutes of arc, which on being applied to (1) and (2) should give the same result.

To obviate the practical difficulty attending the *sign* of the corrections for latitude and longitude the following tables may be referred to.

TABLE (A).

To Name the Correction for the D. R. Latitude.

The observer's Latitude with reference to the Sun's Declination.	Observations on the SAME Side of Mer.		Observations on OPPOSITE Sides of Mer.	
	Longitudes.	Corr. for D. R. Lat.	Longitudes.	Corr. for D. R. Lat.
NORTHWARD	(1) East of (2)	N.	(1) East of (2)	S.
	(1) West of (2)	S.	(1) West of (2)	N.
SOUTHWARD	(1) East of (2)	S.	(1) East of (2)	N.
	(1) West of (2)	N.	(1) West of (2)	S.

TABLE (B).

To Name the Correction for the Approximate Longitude.

A.M., or SUN EAST of Meridian.			P.M., or SUN WEST of Meridian.		
True Bearing reckoned from	When Corr. in Lat. is	Corr. in Long. is	True Bearing reckoned from	When Corr. in Lat. is	Corr. in Long. is
N.	N.	W.	N.	N.	E.
N.	S.	E.	N.	S.	W.
S.	N.	E.	S.	N.	W.
S.	S.	W.	S.	S.	E.

These results may be remembered by combining the letters N.N.W. and S.S.W. for A.M. sights, and N.N.E. and S.S.E. for P.M., observing that when the second letter is reversed the third is reversed in like manner.

NOTE.—The bearings are always to be considered as less than 90°, and if when corrected for variation and deviation they exceed 90° they must be subtracted from 180° and referred to the opposite point of the compass; thus N. 100° W. would become S. 80° W., and so on.

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[1] Tables giving the value of the quantities enclosed in a bracket have been published by Mr. A. C. Johnson, R.N., late of H.M.S. *Britannia*; they also form Table 25 of Inman's Tables.

These tables may also be conveniently embodied in the following rules:

Observations in Same Quadrant.	*Observations in Different Quadrants.*
Allow *both* corrections for longitude to the *eastward* or both to the *westward*;	Correct the *easterly* longitude towards the *west*, and the *westerly* towards the *east*,

in such a manner as to make the two longitudes agree. If they do not agree this shows that the corrections have been wrongly applied, and herein we have a valuable safeguard against error.

Example On April 25, at $7^h\ 30^m$ A.M., in lat. D. R. 49° 20′ N., long. D. R. 5° 40′ W., the following observation of the sun was taken:

Obs. alt. ☉ 24° 49′ 20″; time by chron. $8^h\ 1^m\ 50^s$; true bearing S. 82° E.

The ship then ran by compass N.E. by E.½E. (Dev. 21° E. Var. 22° W.) 12 miles, and at $9^h\ 40^m$ A.M. the observed altitude $\underline{\odot}$ was 43° 52′ 10″; time by chronometer $10^h\ 10^m\ 22^s$; true bearing S. 51° E. Find the position of the ship at time of second observation.

On April 20 (noon) the chronometer was slow on Greenwich mean time $11^h\ 51^m\ 22^s$; daily rate $5{\cdot}6^s$ losing. Index correction +2′ 10″; dip for 20 feet. Corrected declination at time of first observation 13° 21′ 30″ N., and at second observation 13° 23′ 30″ N., the corresponding equations of time being $2^m\ 11^s$ and $2^m\ 12^s$ subtractive from S. A. T.

From the first observation we find S. A. T. $19^h\ 31^m\ 58^s$, and resulting longitude 5° 53′ W., which corrected for run becomes 5° 37′ W. (1) at time of second sight.

From the second observation we find S. A. T. $21^h\ 40^m\ 58^s$, and resulting longitude 5° 46′ W. (2), the difference of longitude being 9′.

Then corr. in lat. $= 9' \, . \sin 82° \, . \sin 51° \operatorname{cosec} (82° - 51°) \cos 49° 26'$, or latitude at second observation

$= 8{\cdot}74'$, which, on referring to table on page 399, is seen to be N. or additive; therefore the required latitude is 49° 35′ N.

Corr. in long. (1) $= 8{\cdot}74' \, . \cot 82° \, . \sec 49° 35' = 1{\cdot}8'$.

This correction on referring to above table is seen to be E., and therefore the required longitude is 5° 35′ W., which result may be checked by similarly correcting longitude (2), which should give the *same* result.

The preceding example, worked by means of Mr. Johnson's table (Table 25 of Inman), would be arranged as follows:

Bearings.	Table 24.	Longitude.
S. 82° E.	(*a*) 0·22	(1) 5° 37′ W.
S. 51° E.	(*b*) 1·26	(2) 5° 46′ W.
	diff. 1·04	9

$$\text{Correction of second lat.} = \frac{9}{1{\cdot}04} = 8{\cdot}6'.$$

$$\text{,,} \quad \text{longitude (1)} = 8{\cdot}6 \times 0{\cdot}22 = \ 1{\cdot}9'.$$

$$\text{,,} \quad \text{,,} \quad (2) = 8{\cdot}6 \times 1{\cdot}26 = 10{\cdot}8.$$

The correction for second lat. is seen to be N. on reference to Table A, p. 399, hence the true lat. at time of second observation =49° 35′ N.

The correction for both longitudes (1) and (2) is seen to be E. on reference to Table B, p. 399, hence when corrected *both* agree in giving the true long. at time of second observation = 5° 35′ W.

Note.—The corrections (*a*) and (*b*) taken from Table 35 of Inman should be subtracted from one another if the bearings are in the *same* quadrant, and added to one another if in *different* quadrants.[1]

In considering the limits of application of this problem we must observe— **Remark**

(1) That equation (1), page 396, upon which the investigation and also the values of $p_{,,}$ and $p_{,}$ depend, is only strictly true when the heavenly body observed is on the prime vertical, and hence neither observation should be taken when near the meridian, especially in high latitudes.

(2) The azimuths employed, if not observed, have been either calculated or taken from the azimuth tables for incorrect latitudes —viz. those by D. R., which *theoretically* would necessitate a recalculation, using the true latitude of the second place of observation.

(3) From the investigation on page 398, we see that when $A_{,}$ and $A_{,,}$ differ by 90° the circumstances of observation are favourable, except when limited by our first remark.

The reader will observe that this method is applicable to other heavenly bodies than the sun, and that by working the observations to the nearest minute and using only five places of decimals in the logarithms—a degree of accuracy sufficient for all practical purposes at sea—a considerable saving of time and trouble may be effected.

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[1] Except when they are in opposite quadrants, in which case take the difference. This, however, will rarely happen, and is not worth taking into consideration.

EXAMPLES.[1]

1. On June 1, in lat. by D. R. 49° 5′ N., long. by D. R. 8° 30′ W., the alt. of the sun was 35° 52′ at 8 A.M. S. M. T., the corresponding G. M. T. being $20^h\ 31^m\ 34^s$, decl. and eq. of time 22° 10′ N. and $2^m\ 22^s$ subtr. from App. time.

The vessel then ran N.W. (true) 25 miles, and at $10^h\ 10^m$ A.M. when the G. M. T. was $22^h\ 44^m\ 10^s$ the altitude of the sun was 55° 5′, and decl. and eq. of time 22° 11′ N. and $2^m\ 21^s$.

Find the lat. and long. of second place of observation.

Ans. Lat. 49° 33′ N.; long. 9° 9′ W.

2. On March 14, in lat. 49° 40′ N., long. 7° 35′ W. (by D. R.) at $7^h\ 30^m$ A.M., when the G. M. T. was $20^h\ 10^m\ 2^s$ the alt. of the sun was 12° 44′, decl. and eq. of time 2° 15′ S. and $9^m\ 10^s$ add. to App. time.

The vessel then ran S.W. true 30 miles, and at $9^h\ 40^m$ A.M., when the G. M. T. was $22^h\ 21^m\ 40^s$, the alt. of the sun was 30° 20′ and decl. and eq. of time 2° 17′ S. and $9^m\ 8^s$.

Find the lat. and long. of second place of observation.

Ans. Lat. 49° 14′ N.; long. 8° 1′ W.

3. On December 24, in lat. 48° 30′ N., long. 8° 55′ W. (by D. R.), at dawn at $6^h\ 30^m$ A.M. the alt. of Jupiter (W. of mer.) was 42° 38′, the G. M. T. being $19^h\ 5^m\ 25^s$, R. A. M. S. $18^h\ 17^m\ 12^s$, R. A. planet $10^h\ 32^m\ 4^s$, decl. of planet 10° 22′ N.

The vessel then ran E. by N. (true) 36 miles, when at $9^h\ 30^m$ A.M., the G. M. T. being $22^h\ 2^m\ 42^s$, the sun's alt. was 10° 40′, the decl. and eq. of time being 23° 13′ S. and 7^s add. to App. time.

Find the lat. and long. of second place of observation.

Ans. Lat. 48° 41′ N.; long. 8° 12′ W.

4. On May 1, at $3^h\ 45^m$ A.M., when the horizon could be plainly seen in lat. 50° 10′ N., long. 7° 35′ W. (by D. R.), the alt. of α Aquilæ E. of mer. was 45° 12′, G. M. T. $16^h\ 15^m\ 40^s$, and R. A. and decl. of star $19^h\ 45^m\ 7^s$, 8° 34′ N., the R. A. M. S. being $2^h\ 40^m\ 10^s$.

The vessel then ran E. true 24 miles, and at 6 A.M. the altitude of the sun was 11° 35′, G. M. T. $18^h\ 25^m\ 3^s$, decl. and eq. of time 15° 18′ N., $3^m\ 13^s$ subtr. from App. time.

Find the lat. and long. of second place of observation.

Ans. Lat. 50° 6′ N.; long. 7° 13′ W.

5. On May 25, in lat. 49° 20′ N., long. 6° 45′ W. (by D. R.),

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[1] These examples are intended to be worked by *all* the methods given in this chapter, the graphical solutions being placed on Admiralty Chart No. 1.

The altitudes given are *true* altitudes of the sun's *centre*; the declination and equation of time, &c., are *corrected* for Greenwich data.

at 8 A.M. the alt. of the sun was 35° 6′; G. M. T. 20^h 23^m 2^s; decl. 21° 2′ N.; eq. of time 3^m 16^s subtr. from App. time.

The ship ran E. (true) 30 miles, and at about 11^h 30^m A.M. (the elapsed time by chronometer being 3^h 31^m 20^s) the following observation was made near the meridian:

Alt. of sun 61° 2′; decl. 21° 1′ N.; eq. of time 3^m 15^s.

Find the lat. and long. of second position of ship.

Ans. Lat. 49° 24′ N.; long. 5° 56′ W.

6. On April 21, in lat. 49° 22′ N., long. 8° 40′ W. (by D. R.), at 8^h 32^m A.M. the alt. of the sun was 33° 32′; G. M. T. 21^h 6^m 8^s; decl. 12° 2′ N.; eq. of time 1^m 28^s subtr. from App. time.

The ship then ran S. 80° E. (true) 35 miles, and at about 11^h 35^m A.M. (the elapsed time by chronometer being 3^h 1^m 10^s) the following observation was made near the meridian:

Alt. of sun 52° 31′; decl. 12° 4′ N.; eq. of time 1^m 29^s.

Find the lat. and long. of second position of the ship.

Ans. Lat. 49° 9′ N.; long. 7° 55′ W.

7. On January 1, in lat. 49° 20′ N., long. 7° 25′ W. (by D.R.), at 10^h 12^m A.M. the alt. of the sun was 13° 44′; G. M. T. 22^h 45^m 20^s; decl. 23° 2′ S.; eq. of time 3^m 58^s add. to App. time.

Course was then shaped N. 65° E. (true), and when the vessel had run 42 miles the Bishop lighthouse was sighted bearing N. 28° W. (true).

Find the lat. and long. of the ship's position at that instant.

Ans. Lat. 49° 40′ N.; long. 6° 18′ W.

THE TIDES.

A complete discussion of the phenomena of the tides would be beyond the scope of the present work, but the great practical utility of a general knowledge of the subject in the art of navigation (especially round our own coasts) would seem to justify the following elementary remarks.

General description

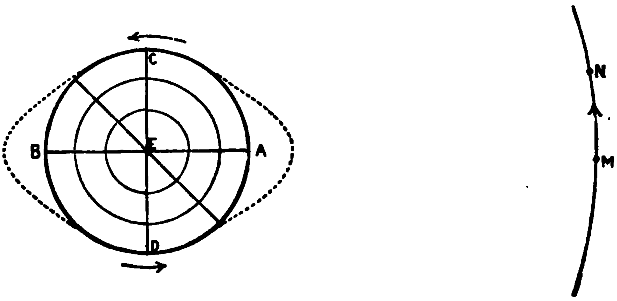
The phenomena of the tides usually observed round the coasts of Great Britain and Ireland are those of a regular rise and fall of the water separated by intervals of somewhat more than six hours, the tide rising twice and falling twice in each day, or rather in about 24 hours and 48 minutes. The times when the water is at its highest come later and later at each successive tide by an average interval of about half an hour, and at the end of a fortnight high water occurs at about the same time of day or night as at the beginning.

It is also a matter of observation that the height to which the water rises differs not only at different places, but from day to day at the *same* place, twice in each month rising to a maximum height above the mean level. The recurrence of these phenomena at about the times of new and full moon early suggested a connection between them, and we may regard the tides as an effect of the attraction of the moon on the water of the globe, modified by a similar but lesser attraction of the sun.

Daily changes

In fig. 184, if E represent the earth, seen from a point above the north pole and supposed to be covered with water, and M the moon, the effect of her attraction will be to draw the water toward A, supposing it capable of moving freely on the surface; and since, on the opposite side of the earth B, the tendency of the attraction of the moon is to draw the earth away from the water, a similar and nearly equal accumulation will occur there; hence, as the earth turns round in the direction of the arrows, every place on arriving near A and B will experience high water, and on passing C and D low water.[1]

FIG. 184.



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[1] If the earth were *at rest* the high waters would be experienced *at* A and B, but in consequence of rotation and the inertia of the water, they occur some time *after*

The accumulations of water at A and B are termed tidal waves, that at A the superior, and that at B the inferior wave; these tidal waves must be regarded as an effect of the attraction of the moon on each successive portion of water brought under it by the rotation of the earth.

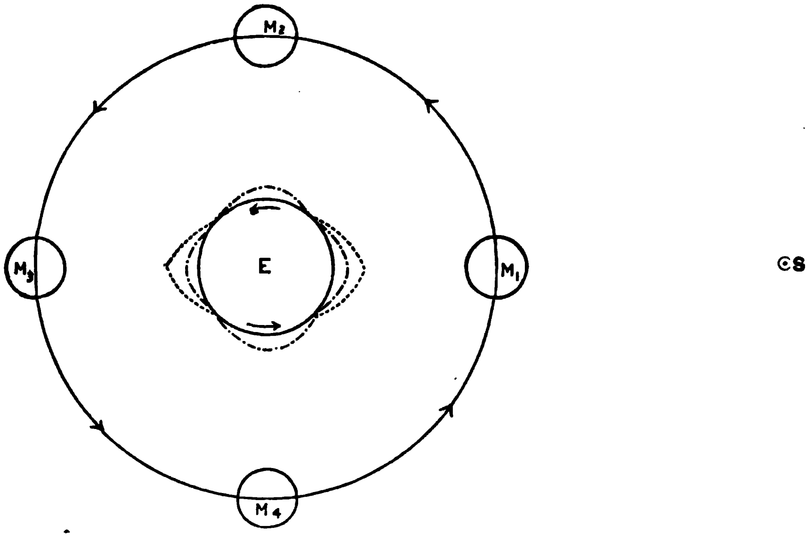
While the earth is making a complete rotation the moon has moved in the direction M N, and the same daily phenomena will recur at an interval later by the amount of that motion—that is, on an average, 48 minutes.

Similar but smaller daily effects are produced by the sun's attraction, the observed result being their *united* effect further and very greatly modified by the obstructions offered by the land.

Fortnightly changes

The fortnightly changes in the height of the tide are illustrated by fig. 185, where E represents the earth, S the sun, and M_1, M_2, M_3, M_4 four positions of the moon in its monthly orbit, the whole seen from the northern side, when the directions of the motions of rotation and revolution will be as shown by the arrows.

Fig. 185.



Springs

At M_1, M_3 (new and full moon), since the sun and moon are acting together[1]

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passing through those positions: these intervals are termed *lunitidal* intervals—that is, intervals between upper or lower passages of the moon across the meridian of a place and the following high waters.

[1] At M_3 the moon's *nearest* wave coincides with the sun's *farthest* wave. The tidal effect of the sun is less than that of the moon, since the cause of the tides is the *difference* of the attraction of either body exerted at the surface and at the centre of the earth. That *difference* in the case of the moon is such that the force of attraction on the nearest part of the earth is $\frac{1}{30}$th greater than on the centre, while in the case of the sun it is only $\frac{1}{1200}$th part greater; the *total* force of attraction of the sun being, however, much greater than that of the moon, the real proportion between the tidal effects of the sun and moon is about 1 to 2¼, and if the earth were covered with water spring tides would bear to neap tides the ratio of $(2\frac{1}{4}+1)$ to $(2\frac{1}{4}-1)$, or 13 to 5. Under existing circumstances the neap tide may be considered *on an average* to rise

and producing their greatest effect, the tides are then equal and at their highest being termed *Spring* tides.

Neaps At M_2, M_4 (the first and last quarters) the sun and moon are acting at right angles to one another, and the smaller effect due to the sun tends to neutralise the more powerful effect of the moon, the result being that the water is more evenly distributed over the globe and nowhere rises so high: these tides are termed *Neap* tides. In some localities at the first and last quarters *four* high waters are actually experienced, but more frequently a modifying effect on the ordinary two high waters occurs. The disturbing effect of the sun *between* springs and neaps is shown by the tide not being formed so nearly *under* the moon as at springs, but nearer to or farther from the sun; this effect, termed *priming and lagging*, causes the intervals between successive high waters to be sometimes considerably greater or less than the average. Thus for about three days after spring tides the interval between the corresponding tides of successive days is *less* than the average, and the tides are said to prime; while for about three days after neaps the opposite effect is produced, and the tides are said to lag.

Priming and lagging

Moon's age The time which elapses between the moon occupying the positions M_1, M_2, M_3, M_4 differs at different seasons of the year, owing to neither of the orbits of the earth and moon being circular, and also to the point of the moon's nearest approach to the earth (perigee) differing each month. Since the average complete revolution of the moon with regard to the sun is effected in $29^d\ 12^h\ 44^m$, these intervals may roughly be considered as $7\frac{1}{2}$ days each, and are usually estimated by the moon's age, which is supposed to begin with the time of new moon.[1]

Hence, in general terms—

Summary At new moon the sun and moon are said to be in *conjunction*, and the moon's age is 0 days; it is on the meridian of every place at nearly the same time as the sun, and spring tides are caused.

At the first quarter the moon's age is about $7\frac{1}{2}$ days; it is on the meridian of every place at 6 hours after the sun, and neap tides are caused.

At full moon the moon is said to be in *opposition*, and her age is about 15 days; it is on the meridian at 12 hours after the sun, and spring tides are again generated.

At the last quarter the moon's age is about $22\frac{1}{2}$ days; it is on the meridian at 18 hours after (or 6 hours before) the sun, and neap tides are again caused.

The tide produced by the sun is greatest at the equinoxes (March 20 and September 22), when it has no declination, and would also exceed the average when the earth is nearest to it (January 1), but for the fact that it does not then exert its attraction over the equator. This tide recurs every 12 hours.

The tide produced by the moon is greatest twice a month when it has no

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three-quarters the height of springs, the height in both cases being estimated above the level of low water of ordinary spring tides; the proportion between springs and neaps is, however, very different in different places.

[1] New moon occurs each time at *noon* on some *one* meridian—that is, the sun and moon cross that meridian at the same time. At places east or west of that meridian the instant of the sun and moon being in line may occur at *any* hour of that day (local time), depending upon the longitude, but they will nevertheless cross the meridian at *nearly* the same time. The same consideration applies of course to the time of full moon in regard to its occurrence at midnight.

declination, and especially so when at such times it is nearest to the earth. This tide recurs on an average every 12 hrs. 24 mins.

The heights of successive spring and neap tides differ from each other, since the new and full moons sometimes occur when the moon is nearest to the earth, and sometimes when it is farthest away; they are also greater when the new or full moon passes the meridian of a place near the zenith, and less when it crosses at some distance north or south of the place; while the attractive influence of the sun is greatest in January, when the earth is nearest to it, and least in July when it is farthest away.[1]

The most powerful cause of difference is, however, probably due to the rotation of the earth, which causes bodies on the surface to have a tendency everywhere (except at the poles) to recede from the axis. This tendency (termed centrifugal force) is most powerful at the equator, since the velocity of rotation is there at its maximum, and at the new and full moons which occur about the equinoxes (March 20 and September 22), when both sun and moon are nearly vertically over the equator, this centrifugal tendency acts directly *with* their attractions, and unusually high tides, known as equinoctial springs, occur.

Mean Sea Level

Observations show that, notwithstanding the great differences in the height of tide at different places, and even at the *same* place at different times of the month, the mean level of the sea is practically everywhere the same.[2] A good determination of the mean sea-level by the simple operation of taking means may be made, in less than two days, with even a moderate number of observations *properly distributed so as to subdivide both solar and lunar days into not less than three equal parts.* Suppose, for example, we choose 8-hour intervals, both solar and lunar. Take a lunar day at $24^h\ 48^m$ solar time, which is near enough, and is convenient for division; and choosing any convenient hour for commencement, let the height of the water be observed at the following times, reckoned from the commencement:

h m	h m	h m
0 0	8 0	16 0
8 16	16 16	24 16
16 32	24 32	32 32

The observations may be regarded as forming three groups of three each, the members of each group being separated by 8 hours solar or lunar, while one group is separated from the next by 8 hours lunar or solar. In the mean of the 9 results the lunar and solar semidiurnal and diurnal inequalities are all four eliminated.

Nine is the smallest number of observations which can form a complete series. If the solar day be divided into *m* and the lunar into *n* equal parts, where *m* and *n* must both be greater than 2, there will be *mn* observations in the series; and if either *m* or *n* be a multiple of 3, or of a larger number, the whole series may be divided into two or more series having no observation in common, and each complete in itself. The accuracy of the method can thus be tested, by comparing the means obtained from the separate sub-series of which the whole is made up.

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[1] The greatest and least distances of the earth from the sun are in the proportion of about 30 to 29; the greatest and least distances of the moon from the earth are in about the proportion of 8 to 7.

[2] To test this important point a series of levels in 1830 were carried across from Wick rocks, on the Bristol Channel, to Axmouth, on the English Channel, and though at low water the level of the sea at these places differed 12 feet, the *mean* level coincided almost exactly.

Should the ship's stay not permit of the employment of the above method, a very fair determination may be made in less than a day, by taking the mean of n observations taken at intervals of the nth part of a lunar day, n being greater than 2. Thus if $n = 3$ these observations require a total interval of time amounting to only $16^h\ 32^m$. The theoretical error of this method is very small, and the result thus obtained is decidedly to be preferred to the mere mean of the heights at high and low water.

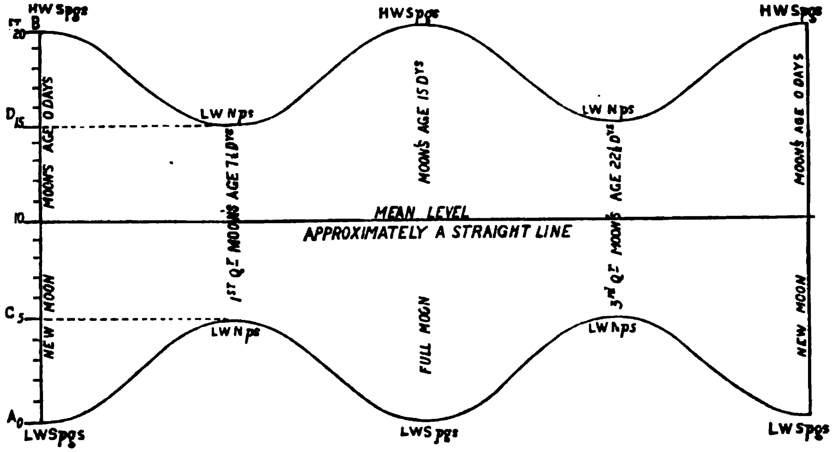
The mean level thus determined is subject to meteorological influences, and it would be desirable, should there be an opportunity, to redetermine it at the same place at a different time of the year. Should a regular series of observations for a fortnight be instituted, it would be superfluous to make an independent determination of the mean sea-level by either of the above methods at the same time.

The following diagram illustrates the movement of the water throughout a lunar month, and also the terms employed on Admiralty charts with reference to the heights of spring and neap tides.

FIG. 186.

In this figure the level of the sea at high water throughout a month is shown by the upper curve, and that at low water by the lower curve.

A B = rise of springs. A D = rise of neaps. = C D range of neaps.'



Atlantic tides

The theoretical considerations of the tidal waves on the supposition of the earth being covered with water are practically very greatly modified by the fact of so much of the surface being occupied by land;[1] thus in the Atlantic Ocean we find, since the times of high water become later and later on proceeding from the southern to the northern part,[2] that the tidal wave (compared to the fluttering of a flag) proceeds from south to north instead of from east to west, travelling in deep water at some hundreds of miles an hour, and only rising a few inches above the mean level, but rising many feet in the shoaler water nearer the land, and only travelling at the rate of a few miles an hour.

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[1] The proportion of land to water on the earth is about 1 to 4.

[2] This was illustrated by Dr. Whewell in 1833 by connecting by curved lines upon a chart of the Atlantic Ocean those places where the Establishment of the Port reduced to Greenwich time was the same. Such lines are termed *co-tidal lines*.

Vulgar establishment

The interval between the time of the moon's meridian passage on the day of new or full moon and that of the high water which follows, expressed in mean time, is defined as the Establishment of the Port (H. W. F. & C.), or Vulgar Establishment; this is practically equivalent to considering the H. W. F. & C. given on charts as the P.M. time of high water on the day of new moon or the A.M. time of high water on the day of full moon.

If the moon were the *sole* cause of the tides high water would follow the meridian passage of the moon on *every* day by about the same interval, and therefore would occur about 48 minutes later every day.[1]

The disturbing influence on the tide caused by the sun, however, prevents this regularity, and this disturbance affects different localties so variously that no *general* law can be established. This difference appears in the different ratio of the rise of spring tides to the rise of neap tides (the semi-mensual inequality of heights). In general the rise of spring tide above mean water is about double that of neap tide, which gives the solar tide *one-third* of the lunar tide. But in some cases the spring tide exceeds the neap tide only by one-third, which gives the solar tide only *one-seventh* of the lunar tide.

Also the difference of the greatest and least lunitidal intervals—that is, the semi-mensual inequality of the times—shows the difference of the solar tidal force at different places. The difference of the greatest and least intervals is $1^h\ 28^m$ at London and Liverpool, but at Plymouth it is $1^h\ 36^m$, and at Portsmouth $1^h\ 21^m$. On the coast of North America it is generally less than $1^h\ 20^m$, while at some places on the coasts of France and Ireland it is above 2^h.

We may take $1^h\ 28^m$ as the mean value of this difference, which agrees with the supposition that the solar tide is about one-third the lunar tide. Since, then, the daily difference between the times of moon's meridian passage and that of high water depends upon the *relative* positions of the moon, sun, and earth, no accurate laws can be framed for any place until a complete series of observations have been made at that place, embracing a period covering *every* change in these relative positions; this period is 18 years 11 days, and for the few localities where such a long series of tidal observations (13,000) have been made, tables have been calculated[2] which enable predictions to be made of times and heights of tides at any future period.

Tide tables

Tide calculations

Those times and heights are made applicable (approximately) to all important harbours on the coasts of Great Britain, Ireland, and the north coast of France by adding or subtracting certain tidal constants (given in the Tide Tables) to the time and height given on the day in question for the nearest of these standard ports.

For other places on the globe, apart from information derived on the spot, perhaps the best method of ascertaining the time of high water on any day is the following:

Example:—To find the time of high water *approximately*.

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[1] That is, since *two* high waters occur in the lunar day, these would be separated by an average interval of 12 hrs. 24 mins. It should be understood that the tidal wave does not occasion a transference of the *particles* of the water, but only a transfer of *form*; this may be illustrated by comparing the tidal wave with wind waves, when a similar effect is produced, though the origin of these waves is of course very different.

[2] These are published in the Admiralty Time Tables; they were originally compiled by Sir J. Lubbock in 1839; and relate to 24 standard ports, 23 being in the United Kingdom, and one (Brest) in France.

To the time of moon's meridian passage (corrected for longitude) add the establishment of the port or H. W. F. & C. given on the chart.

If the sum be less than 12 hours, it is the time of high water P.M.; if it exceeds 12 hours, it is the time of high water next morning; and to obtain the time for P.M. the previous day, subtract $12^h\ 24^m$.

If the sum exceed 24 hours, it is the time of high water P.M. the next day; for the time P.M. on the proposed day, subtract 24 h. 48 m.

When the age of tide is known the results can be greatly improved by correction from one of the following tables:

(1) *Tide a day and a quarter old.*—Minutes to be added to or subtracted from the establishments, according to the hour of the moon's transit on the half-day in question.

Hour of the Moon's Transit after Sun	h. 0	h. 1	h. 2	h. 3	h. 4	h. 5	h. 6	h. 7	h. 8	h. 9	h. 10	h. 11
Correction of the Vulgar Establishment to find the Lunitidal Interval	m. 0	m. −16	m. −32	m. −47	m. −57	m. −60	m. −47	m. −16	m. +15	m. +28	m. +25	m. +15

For example, if the establishment be $2^h\ 27^m$, at what hour will the high water come after a moon's transit which takes place at 4^h A.M.? The minutes to be added to $2^h\ 27^m$ for 4^h transit are, by the table -57^m, that is 57^m to be subtracted; therefore the high water will be at $1^h\ 30^m$ after the moon's transit, that is at $5^h\ 30^m$.

(2) *Tide two days and a half old*:

Hour of Moon's Transit	h. 0	h. 1	h. 2	h. 3	h. 4	h. 5	h. 6	h. 7	h. 8	h. 9	h. 10	h. 11
Correction of the Establishment	m. 0	m. −15	m. −31	m. −47	m. −62	m. −72	m. −75	m. −62	m. −31	m. 0	m. +13	m. +10

This table to be used in the same way as the other.

Hence we see that the age of the tide most affects the lunitidal interval when the time of moon's transit is between 7 and 8 hours.

At both springs and neaps the tide rises or falls most rapidly at or near half-tide; to find the height of tides at any intermediate hour between high and low water, Table (B) of the tide tables is employed, or Inman's Tables, No. 10, but these are only applicable when the times of rising and falling are equal.

The *Corrected Establishment* is the mean time of high water on the afternoon of the day of that new moon which crosses the meridian of a place *with* the sun—that is, at 12 o'clock.

Although this occurs but rarely at any one place, yet by means of a graphical representation it may be readily determined from a series of observations of the Vulgar Establishment,[1] and it is that which is usually given on charts as H. W. F. & C.

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[1] A pamphlet illustrating the mode of doing this is published by the Hydrographic Office and placed in the chart boxes; the *Mean* Establishment is the mean of the lunitidal intervals for a fortnight or semilunation, and is always less than the Vulgar Establishment.

Age of tide

The greatest effect of the sun and moon in causing the rise of tide must evidently occur at the instant of new or full moon, and the interval between those dates and the time of occurrence of the *highest* tide subsequently experienced at any place is termed the *age of the tide*, or sometimes the *retard*.

Thus at London the highest tide does not occur until about 2½ days or five transits after new or full moon, and the age of the tide is said to be 2½ days; a similar effect is produced on the neap tide.

The highest tides take place on the west coast of Ireland and on the south coast of England, three transits after the new and full moon, unless diverted by gales of wind or other extraordinary causes. Along the east coast of England, they take place four transits after the new and full moon. In the river Thames they occur five transits after the same epoch. These differences arise from the fact that the same tidal wave which produces high water on the west coast of Ireland takes half a day in its progress thence to the east coast of England, and a whole day before it arrives in the river Thames.

Tidal streams

A knowledge of the direction in which the tidal stream will set a vessel is generally of more practical inportance in navigation than that of the time of high water and height of tide, which are chiefly of use when entering or leaving a harbour.

There are few places, except the heads of bays and inlets, where the tidal streams at any distance from the shore turn with the rising and falling of the water,[1] and in channels open at either end the stream or set of the tidal waters turns at *half-tide* by the shore: this is known as *tide and half-tide*.

Near the sides of a channel of any width, and whose sides are shallow, the direction of the tidal stream is rotatory. On the left hand, looking up the channel with the flood stream, the direction of the rotation is with the hands of a watch; and on the right-hand side, in the contrary direction, in the following manner. At low water the stream will be running down the channel; at half tide it will be flowing towards the shore; at high water it will be running up channel parallel to the shore; and at half ebb it will be ebbing directly away from the shore.[2]

In the upper parts of estuaries or tidal rivers, where shallow water prevails the duration of the flood stream is commonly shorter than that of the ebb.

The higher up the estuary the greater will this difference become. This is apparently due to the retardation of the advancing tide caused by friction over the shoals, and when the range of tide is great the water becomes heaped in the lower part of the estuary, finally rushing up the higher part in a wave which, in extreme instances, has a more or less vertical face, and is called a '*bore*.' In such a case the tide rises perhaps half its height in a few minutes, and the whole duration of flood stream will be confined to two or three hours, or even less, the remainder of the twelve hours being occupied by the downward or ebb stream.

Such local peculiarities are described in the sailing directions relating to the place in question, and our description will be limited to those localities round our own coasts where a knowledge of the set of the tidal streams is of most importance.

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[1] At London Bridge the stream runs *up* until the surface of the water has fallen about 2 feet.

[2] A good diagram showing the level of the water at high, low, and mean tide at the First Narrows in Magellan Strait, where tide and half-tide is well illustrated, will be found in *Wrinkles on Navigation*.

Throughout the English Channel and North Sea the direction in which a vessel will be set by the tidal stream at any hour can be referred to high water at Dover, and to facilitate this reference the locality is supposed to be divided into 21 compartments, whose limits are given in the Tide Tables.

On finding from the known position of the ship in which of these compartments she is situated, and also from the Tide Tables the number of hours before or after high water at Dover on the day in question, the probable direction and rate of the tidal stream can be taken from the Tide Table by inspection.

Throughout the Irish Channel the tidal streams set towards Liverpool while the water is rising there, and away from Liverpool when the water is falling there: this is quite irrespective of this various *times* at which it is high water on shore throughout this long channel; in fact, it is high water at *both* entrances at the same time that it is low water in the middle. The direction in which a vessel will be set by this tidal stream is therefore always found by ascertaining from the Tide Table the time of high water at Liverpool on the day in question, the rate being given in a tabular form.

Diurnal inequalities

A few of the more peculiar features of tides in localities where they are not so regular as round our own coasts will now be mentioned.

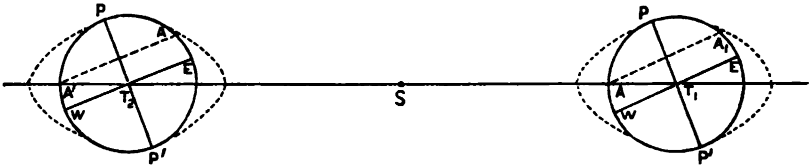
In most parts of the world observations show that consecutive high and low waters do not rise or fall to approximately the same level; this difference is termed the *diurnal inequality of height*, and may affect high waters only, low waters only, or both[1]; frequently, if the highest tides occur by day at one season of the year or period of the moon, the night tides will be highest at the opposite season or period.

In many places the regularity of the *times* of recurrence of high and low water are affected at different seasons in opposite ways; this is termed the *diurnal inequality in time.*

The causes of diurnal inequality are usually explained as follows:

If T_1, T_2, (fig. 187) represent the two positions of the earth with regard to S the sun, at midsummer and midwinter respectively, and A A_1 in each figure represents a position of the earth in lat. 23° N., shown both at noon and at midnight:

Fig. 187.



It is evident that at T_1 (midsummer) the *day* tide, caused by the sun, will rise higher than the *night* tide, since, though these tidal waves (shown by a dotted curve) are *equal*, yet by day A passes under the crest, while at night the crest is 46° to the southward of A_1, to which point A has been carried by the rotation of the earth.

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[1] At Singapore this diurnal inequality sometimes amounts to 6 feet, and chiefly affects the level of *low* water; at the Falkland Islands it is about 2 feet, and equally affects both high and low water; round our own coasts it does not amount to more than a few inches.

From similar considerations we see that at T_2 (midwinter) the *night* high water will be higher than that by *day*.

If, then, the sun *alone* were attracting the water, the effect on the diurnal inequality would be reversed every six months; but, as similar and much greater effects are produced by the moon in her *monthly* revolution round the earth, their united effect is much more complicated.

In some localities at neaps the two lunar and two solar tides appear *separately*, occasioning four tides in one day, while at other places they obliterate one another and a tide is only apparent at springs; Courtown, in Ireland, is an instance of the former, and Tahiti, in the Pacific Ocean, an instance of the latter occurrence.

Effect of barometric pressure

An increase or decrease of the pressure of the atmosphere over any locality (indicated by the rising or falling of the barometer) affects the height of tide, a *high* barometer occasioning a *low* tide, and *vice versâ*.

In the open ocean a rise of 1 inch would occasion a depression of about 1 foot, but in localities somewhat surrounded by land the effect is *less*, being about 7 inches at London and 11 inches at Liverpool.

Exceptionally high tides occur at all places at the times of new and full moon near the equinoxes (March 20 and September 22), and also when new or full moon occurs in January at the time when the moon is nearest the earth (perigee).[1]

The dates of such high tides are indicated by an asterisk in the Admiralty Tide Tables, and the effects are further increased at such times by a low barometer and wind blowing from such a direction as to cause an accumulation of water at the place considered.[2]

It may be interesting to note that the highest known tides occur in the Bay of Fundy, New Brunswick, where springs sometimes rise 70 feet; round our own shores the maximum occurs at Chepstow, Bristol Channel, where the spring rise is 60 feet; at London Bridge it is 21 feet.

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[1] The distance of the moon from the earth varies from 252,950 to 221,590 miles.

[2] A gale of wind has been known to alter the level of the river Thames at London by as much as 5 feet. This cause leads to the *time* of high water being *delayed* but the *height* increased; the opposite effect is, of course, produced by wind blowing *out* of a river or estuary.

PRACTICAL EXAMPLE ON NAVIGATION.

THE following example (the calculation of which forms a part of the instruction in practical navigation given to the senior classes of students at the Royal Naval College) is intended to be both worked through and laid down on the charts;[1] if the latter are not available the calculation may conveniently commence on page 415, May 2.

Illustrations will be found in this of most of the practical problems dealt with in former pages.

VOYAGE OF H.M.S. 'GREENWICH' FROM PORTSMOUTH TO MADEIRA, 1884.

At Portsmouth, on April 18, box chronometers A, B, C, and deck watch (D), were drawn from the Royal Observatory.

On April 20 time ball dropped at $5^h\ 11^m\ 3^s$, as shown by D; the comparisons with A, B, and C were as follows:

D	$5^h\ 21^m\ 20{\cdot}5^s$	D	$5^h\ 21^m\ 13^s$	D	$5^h\ 22^m\ 59^s$
A	5 13 00	B	11 9 00	C	3 13 00

On April 21 time ball dropped at $5^h\ 11^m\ 6{\cdot}5^s$ by D; the comparisons were—

D	$5^h\ 16^m\ 2^s$	D	$5^h\ 16^m\ 21{\cdot}5^s$	D	$5^h\ 17^m\ 58{\cdot}5$
A	5 8 00	B	11 4 00	C	3 8 00

On April 22 time ball dropped at $5^h\ 11^m\ 11^s$ by D; the following were the comparisons:

D	$5^h\ 16^m\ 21{\cdot}5^s$	D	$5^h\ 16^m\ 31{\cdot}5^s$	D	$5^h\ 17^m\ 58{\cdot}5^s$
A	5 8 00	B	11 4 00	C	3 8 00

The time ball was taken daily until 1 P.M. on April 25, when the errors and rates of the chronometers were determined.

Sailed for Plymouth on April 26, passing out of the Needles Channel at 2 P.M. (having previously calculated time of high water in that channel); took departure with Needles lighthouse bearing E. by N., and St. Catherine's lighthouse S.E½S. (head W.S.W.), and shaped course to pass 5 miles south of Shambles light vessel. At 3^h 30^m P.M. Priory church at Christchurch bore N.E.¾E., and Anvil Point lighthouse N.W.¾W. (head W. by N.½N.). At 6^h 30^m P.M. Shambles light bore N.N.W.½W., when, intending to double the angle on the bow, the patent log was put over; at 7^h 25^m P.M. the light bore N. by E.½E., and the patent log showed 5 miles; continued on the same course until 8^h 30^m P.M., when, with Portland High Light bearing N.E.¾N., and Shambles light E. by N.¾N., course was altered to pass 5 miles south of the Start.

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[1] The charts required bear the following numbers:—1; 2060*a*; 2450; and 2620 (for deviation see Table on page 21). The charts in the Officers' Atlas may also be used.

At 2 A.M., April 27, Start light bore N.W.¼N. (head N.W. by W.¾W.)., when, intending to double the angle on the bow, the patent log was put over; at 3 A.M. Start light bore N. by W.¾W., and the patent log showed 7·1 miles; at 3^h 45^m A.M., with Start light bearing N.N.E.¼E., and patent log showing 12·1 miles, course was altered for the Eddystone. At 6 A.M. the Eddystone bore N.W.½N., and Bolt Tail E.¾N. (head N.W.½N.), when the vessel was steered for Plymouth Sound.

During the above passage the set of the stream was calculated when off the Shambles and off the Start, also the time of high water at Plymouth.

During the stay at Plymouth the time ball was daily taken, and at noon on May 1 chronometer A was considered to be 4^h 3^m 22^s fast on G. M. T., daily rate $3{\cdot}6^s$ gaining.

Leaving Plymouth at 6 A.M. on May 2, took departure from Eddystone lighthouse, bearing N.E.¼E., distance 2 miles (head S.W.), and shaped course W.½N., with a fair wind.

At 9 A.M. observed a time azimuth. Time by A 1^h 24^m 1^s, comp. bearing of sun S. 21° E., head W.½N., and comp. bearing of sun with head S.W. by W.¼W., S. 29° E.

At noon, having run 48 miles, observed meridian alt. sun' L. L. 55° 41′ 45″, index error + 2′ 10″, dip for 20 ft.

N.B.—This index error and dip was employed throughout the voyage: the lower limb of sun was always observed, and the *observed* altitudes registered.

May 3.—At 2 A.M. the run from the previous noon was 115 miles, when, being in 80 fathoms of water, course was altered to S.W. by W.¼W.; and at 8^h 30^m A.M., having run 45 miles, took

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May 2.—Sun's dec. 15° 34′ 56″ N. Var. in 1 hour, 44·30″ incr. Var. 21° W.

sights for sun chronometer. Time by A, $1^h\ 5^m\ 55^s$; altitude, 36° 52′ 15″.

At noon, the run from sights being 22 miles, observed the meridian alt. of sun 58° 9′ 50″; and, after a further run of 62 miles on the same course, at 10 P.M. the wind drew ahead from the southward, with cloudy weather, and the vessel was put on the port tack.

May 4, at 6 A.M., having made 45 miles W. by N.½N., with half a point leeway, the vessel came up to W. by S. (leeway ½ point), and at noon had made 35 miles on that course, when the wind drawing ahead, ship was tacked to the eastward.

At 2 P.M., having made E.½S. 10 miles after noon (leeway ½ point), took the first sights for double altitude (Sumner) and deviation of compass, putting over the patent log. Time by A, $6^h\ 44^m\ 41^s$; altitude of sun, 51° 9′ 5″; comp. bearing, S. 54° W., head E.½S.

At $4^h\ 30^m$ P.M., when patent log showed 12·8 miles, course E.½S. (leeway ½ point), observed second sights for double altitude. Time by A, $9^h\ 10^m\ 46^s$; altitude, 27° 1′ 30″; comp. bearing S. 89° W., head E.½S.

From $4^h\ 30^m$ P.M. the courses made good were E. 12 miles, E.½N. 11 miles, and E.½S. 14 miles (leeway ½ point on each course), when at 1 A.M., May 5, ship was tacked to the westward.

From 1 A.M. until 8 A.M. the courses were as follows: N.W. by W.½W. 9 miles; W.N.W. 11 miles; and N.W. by W.¾W. 14 miles (leeway ¼ point on each course); sights were then taken for sun chronometer. Time by A, $12^h\ 39^m\ 55^s$; altitude, 32° 14′ 50″; comp. bearing S. 33° E. (head N.W. by W.¾W.).

Between time of sights and $11^h\ 45^m$ N.W. by W.¾W. 21 miles was made good, with leeway ¼ point, when an altitude of the sun

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May 3.—Sun's dec., 15° 52′ 32″ N. Var. in 1 hour, 43·65″ incr. Eq. of time, $3^m\ 18{\cdot}45^s$ sub. from A.T. Var. in 1 hour, $0{\cdot}255^s$ incr.

May 4.—Sun's dec., 16° 9′ 52″ N. Var. in I hour, 42·99″ incr. Eq. of time, $3^m\ 24{\cdot}31^s$ sub. from A.T. Var. in 1 hour, $0{\cdot}232^s$ incr. Var. 23° W.

May 5.—Sun's dec., 16° 26′ 55″ N. Var. in 1 hour, 42·31″ incr. Eq. of time, $3^m\ 29{\cdot}60^s$ sub. from A.T. Var. in 1 hour, $0{\cdot}210^s$ incr. Dec. of γ Ursæ Majoris, 54° 20′ 22″ N. Dec. of β Leonis, 15′ 13′ 13″ N. Dec. of α Cancri, 12° 18′ 21″ N. R.A. α Cancri, $8^h\ 52^m\ 8^s$. Sid. time at preceding G. M. noon, $2^h\ 55^m\ 2{\cdot}65^s$. Var., 23° W.

near meridian was observed. Time by A, $4^h\ 26^m\ 43^s$; altitude, 60° 11′ 30″.

The wind then became fair, and course was shaped S.W. by W.; at sunset (7 P.M.) an amplitude was observed, the sun setting by compass N. 32° W. (head S.W. by W.), and the run from $11^h\ 45^m$ A.M. to that time being 41 miles.

At $8^h\ 50^m$ P.M., having run 52 miles from $11^h\ 45^m$ A.M., the meridian altitudes of a north and of a south star were observed at the same time and also a star chronometer.

Mer. alt. γ Ursæ Majoris ...... 80° 44′ 20″.
„ β Leonis .... 60 15 50.

Time by A, $1^h\ 34^m\ 4^s$; altitude α Cancri, 41° 10′ 50″.

From this time until $11^h\ 30^m$ A.M., May 7, the run was 98 miles, when an altitude of the sun near the meridian was observed. Time by A, $4^h\ 30^m\ 56^s$; altitude, 62° 55′ 50″.

The run from these sights to noon was 4 miles.

At sunset (7 P.M.), having run 49 miles from noon, the sun was observed to set by compass N. 34° W.

At $9^h\ 40^m$ P.M., on a further run of 21 miles from sunset, the meridian altitude of the moon's lower limb was observed to be 41° 23′ 10″.

May 7, at 2 A.M., when 100 miles had been run from the previous noon, the altitude of Polaris was observed 41° 11′ 30″, and the meridian altitude of η Ophiuchi 32° 39′ at the same time; the wind then fell light, with cloudy weather, and at noon the vessel had made good 152 miles from the previous noon.

The wind then drew ahead from the southward, and the courses until 4 A.M., May 8, were as follows: W.½N. 30 miles, S.W. by W. 20 miles, and W.N.W. 32 miles (leeway ¼ point on each course), when the vessel was tacked to the eastward, and the

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May 6.—Sun's dec., 16° 43′ 42″ N. Var. in 1 hour, 41·62″ incr. Eq. of time, $3^m\ 34{\cdot}37^s$ add to M. time. Var. in 1 hour, $0{\cdot}21^s$ incr. Moon's dec., at 10^h 5° 23′ 25″ S. Var. in 10^m 93·2″ incr. Moon's semi. (corr.) 15′ 5″. Moon's hor. par. (corr.), 54′ 42″. Var., 24° W. Sid. time at G. M. noon, $2^h\ 58^m\ 59{\cdot}21^s$. Dec. of η Ophiuchi, 15° 34′ 48″ S.

May 7.—Sid. time at G. M. noon, $3^h\ 2^m\ 56^s$. Dec. of ζ Aquilæ, 13° 41′ 30″ N. Dec. γ Herculis, 19° 25′ 35″ N. R. A. γ Herculis, $16^h\ 16^m\ 48^s$.

May 8.—Sun's dec. 17° 16′ 26″ N. Var. in 1 hour, 40·2″ incr. Var. 22° W.

position at the same time determined from the following observations: Altitude of Polaris, 40° 13′ 30″; mer. alt. ζ Aquilæ, 63° 26′ 20″. Time by A, 9^{h} 5^{m} 36^{s}; altitude γ Herculis, 47° 57′ 10″.

From this point until noon the vessel made good S.E. by S. 40 miles (leeway ½ point), and the meridian altitude of the sun was observed 67° 15′ 40″, after which the wind came fair and course was shaped S.W.¾S.

At 4 P.M., having run 32 miles, the vessel was swung for ascertaining the deviation by bearings of the sun observed on eight points.

Time by A.	Ship's Head.	Bearing of Sun.
9^{h} 3^{m} 51^{s}	North	N. 71° W.
10 40	N.E.	West
15 42	East	S. 88° 10′ W.
21 0	S.E.	N. 86 40 W.
24 5	South	N. 73 30 W.
27 59	S.W.	N. 60 20 W.
33 52	West	N. 48 0 W.
39 40	N.W.	N. 46 10 W.
43 45	North	N. 64 30 W.

Course was then continued S.W.¾S., and at noon, May 9, the run from the previous noon was 195 miles.

From noon, May 9, to 4 A.M., May 10, the run on same course was 71 miles, when the following star observation was made with the view of combining it with a subsequent observation of the sun, to form a double altitude (Sumner). Time by A, 9^{h} 4^{m} 54^{s}; altitude μ Sagittarii, 31° 25′ 10″.

The patent log was then put over, and at 8 A.M., when it showed 91 miles, the following observation of the sun was obtained: Time by A, 1^{h} 7^{m} 13^{s}; altitude, 35° 1′ 50″.

Course was then shaped S.W. by S. for Porto Santo, and on running 41 miles from sights a time azimuth was taken: Time by A, 10^{h} 33^{m} 34^{s}; comp. bearing of sun, N. 51° 20′ W. (head S.W. by S.).

Porto Santo was sighted ahead at 4 A.M., May 11, after which courses were shaped as requisite for Funchal Bay.

May 9.—Sid. time at G. M. noon, 3^{h} 10^{m} 49^{s}. R. A. μ Sagittarii, 18^{h} 6^{m} 50^{s}. Dec. μ Sagittarii, 21° 5′ 16″ S.

May 10.—Sun's dec., 17° 48′ 1″ N. Var. in 1 hour, 38·74″ incr. Eq. of time, 3^{m} 47·77^{s} sub. from A.T. Var. in 1 hour, 0·92^{s} incr. Var., 22° W.

EXAMINATION FOR RANK OF LIEUTENANT, R.N.

PART I.

ROYAL NAVAL COLLEGE, NOVEMBER 1898.

NAVIGATION.—PAPER I.

Time allowed, 3 hours.

October 31st, 1898, landed at Blackpool, latitude 53° 48′ 30″ N., longitude 3° 2′ 45″ W., and about 7ʰ 40ᵐ P.M., mean time, took the following observations to rate chronometer:

Mean of Chronometer Times.	Mean of Obs. Alt. α Lyræ (W. of Mer.) in Artificial Horizon.
8ʰ 12ᵐ 0ˢ	98° 24′ 10″

October 20th, at G.M. Noon, the chronometer was 0ʰ 20ᵐ 25ˢ fast on G.M.T.

Proceeded to sea at 9ʰ P.M., shaping course N. 10° W.

Deviation, 3° E. Speed, 12′. Blackpool Light bearing N. 68° E.

9ʰ 15ᵐ P.M. Blackpool Light, S. 39° E.

11ʰ 30ᵐ P.M. Obs. Alt. Polaris, 55° 34′ 10″; Selker Light, N. 30½° E.

Altered course N. 46° W. Deviation, 1° 40′ E. Speed, 12′.

2ʰ A.M. Obs. Mer. Alt. Moon's U. L., 59° 49′ 40″.

„ Obs. Alt. Mars' Centre (E. of Mer.), 36° 42′ 0″.

Chronometer showed 2ʰ 20ᵐ 22ˢ.

Altered course N. 83° W. Deviation, 2° 40′ W. Speed, 12′.

6ʰ A.M. St. John's Point Light, N. 51° W.

6ʰ 15ᵐ A.M. The angular bearing on the bow was doubled.

Lay down the various courses, distances, and bearings. Obtain the position at 9ʰ 15ᵐ P.M.; and the D. R. and observed positions at 11ʰ 30ᵐ P.M., 2ʰ A.M., and 6ʰ 15ᵐ A.M.

All courses and bearings are by compass, unless otherwise stated.

The index error for all observations was 3′ 10″ —, and the height of eye was 17 feet.

NAVIGATION.—PAPER II.

Time allowed, 3 hours.

1. Define, with diagrams, the terms: Difference of Longitude, Rhumb Line, Leeway, Polar Angle, Right Ascension, Celestial Longitude, Sidereal Day.

Distinguish between True Course Steered and True Course Made Good; between a Knot and a Nautical Mile; and between the True Bearing of a distant object and its Mercatorial Bearing.

2. Prove the formula

diff. long. = dist. × sine course × sec. mid. lat.

A certain place on the earth's surface is revolving 360 miles an hour faster than a place whose latitude is 25° further south. Find its latitude.

3. Find the daily motion of the mean sun in the celestial equator, and the arc described by a meridian of the earth in a mean solar day.

The sun's apparent R.A. at mean noon was $14^h\ 25^m\ 46{\cdot}78^s$, and the equation of time was $16^m\ 18{\cdot}68^s$ (+ M.T.). Find the mean time of the next Transit of the First Point of Aries.

4. Explain the method of calculating the time of the moon's meridian passage in any given longitude, explaining what you mean by the 'moon's proper motion to the eastward.'

5. June 28th, 1898, in longitude 16° 45′ E., the true mer. alt. of the sun's centre below the pole was 9° 58′ 20″. Required the latitude.

6. Explain the construction of a vernier, and prove that the degree of accuracy = $\frac{l}{n}$, where l = length of a division on the scale of the instrument, n = the number of divisions on the vernier, $p.n. - 1$ = the number of divisions on the scale of the instrument.

Why is it that 61 or 121 divisions on the arc of a sextant are never used in constructing the vernier?

7. Calculate the value of the auxiliary angle A, having given, apparent altitude of the moon's centre 46° 28′; apparent altitude of the sun's centre 55° 21′; moon's horizontal parallax 55′ 47″.

8. The sun's declination is 21° N.; find the latitude of the place where morning twilight commences at 6 o'clock.

NAVIGATION AND NAUTICAL ASTRONOMY. PART 2.

Time allowed, 2 *hours*.

1. A port bears from a ship S. 70° W., distant 40 miles. A current runs N. 45° E., 1¼ knots. The ship can sail 8 knots within 6 points of the wind (S.S.W.), and reaches the port in two tacks. Find, without drawing to scale or using any table except the Traverse Table, the distance made good on each tack.

2. May 15th, 1898, in latitude 32° 38′ N., longitude 16° 55′ W., the sun had equal altitudes (*approximately* 55°) at the following times by chronometer:

$8^h\ 47^m\ 56^s$ A.M. $1^h\ 40^m\ 42^s$ P.M.

Required the error of the chronometer on G.M.T.

3. Given Meridian Passage of Venus, October 30th, $2^h\ 15^m\ 30^s$.

.... 31st, $2^h\ 13^m\ 18^s$.

Transit of First Point of Aries, October 30th, $9^h\ 23^m\ 10^s$.

Find the R. A. of Venus at G. M. Noon, October 31st.

4. Investigate the formula for finding the latitude by altitude of the Pole-star.

5. Show that if c be the co-latitude of the place, and ω the obliquity of the ecliptic, the greatest and least values of the inclination of the ecliptic to the horizon are $c + \omega$, and $c - \omega$.

DEFINITIONS IN NAUTICAL ASTRONOMY.

The celestial concave is the surface of an infinitely large sphere, of which the earth is the centre, and on which the motions of the heavenly bodies appear to take place. **Celestial concave**

The true place of a body is the point in which the line joining the centre of the earth to the centre of the body meets the celestial concave. **Place of a body**

The apparent place of a body is the place in which its centre is seen on the celestial concave by an observer. **Apparent place**

The axis of the heavens is coincident with the earth's axis produced. **Axis of the heavens**

It is about this diameter that the celestial concave appears to revolve from east to west.

The poles of the heavens are the extremities of the axis. **Pole of the heavens**

The equinoctial or celestial equator is the great circle in which the plane of the terrestrial equator cuts the celestial concave. **Equinoctial**

Its axis and poles are the axis and poles of the heavens.

The celestial meridian is the great circle in which the plane of the terrestrial meridian cuts the celestial concave. **Celestial meridian**

The earth's orbit is a plane and nearly circular curve described about the sun. **Earth's orbit**

The ecliptic is the great circle in which the plane of the earth's orbit cuts the celestial concave. **Ecliptic**

It is the great circle on which the sun appears to move during the year.

The obliquity of the ecliptic is the angle between the ecliptic and the equinoctial. **Obliquity of the ecliptic**

Its value may be assumed to be 23° 27′.

The ecliptic cuts the equinoctial in two points: that in which the sun passes from the south to the north of the equinoctial is called *the first point of Aries.* **First point of Aries**

This point is called the vernal equinox, while the other point of intersection is called the autumnal equinox or *first point of Libra.*

The solstitial points are the two points of the ecliptic which are at the greatest distance from the equinoctial. **Solstitial points**

These points are midway between the equinoxes.

When the sun is near the solstitial points its declination does not appreciably change for several days. Hence the name Solstice.

The horizon at any place is the great circle in which the celestial concave is met by a plane touching the earth. **The horizon**

Sometimes the horizon is thought of as determined by a plane through the centre of the earth parallel to the above plane, and it is then called the rational *or theoretical horizon, although it is absolutely identical with the horizon defined above.*

The zenith The zenith is the point overhead in which the perpendicular to the plane of the horizon at any place meets the celestial concave.

Reduced zenith The reduced zenith is the point overhead in which a radius of the earth passing through the place of the observer meets the celestial concave.

The true and reduced zenith are nearly coincident, the earth being almost a sphere.

Nadir The nadir is the point in the celestial concave diametrically opposite to the zenith.

The zenith and nadir are the poles of the horizon.

Circles of altitude Circles of altitude or vertical circles are great circles which pass through the zenith.

Parallels of altitude Parallels of altitude are small circles whose planes are parallel to the plane of the horizon.

The celestial meridian as defined on p. 126 *is the great circle which passes through the zenith and the poles of the heavens.*

North and south points The north and south points are the points in which the celestial meridian cuts the horizon.

Prime vertical The prime vertical is the circle of altitude whose plane is perpendicular to the plane of the celestial meridian.

East and west points The east and west points are the points in which the prime vertical cuts the horizon.

Altitude The altitude of a heavenly body is the arc of a vertical circle between the body and the horizon.

Zenith distance The zenith distance of a heavenly body is the arc of a vertical circle between the zenith and the body.

Angle of elevation The angle of elevation of a body above the observer is the angle between a line drawn to it and the horizontal plane through the observer's eye.

Angle of depression The angle of depression of a body below the observer is the angle between a line drawn to it and the horizontal plane through the observer's eye.

Azimuth The azimuth or true bearing of a heavenly body is the angle between the celestial meridian and the circle of altitude passing through the body.

It is measured by an arc of the horizon from the north or south point, either eastward or westward, and may be of any value, from 0° *to* 180°.

Amplitude The amplitude of a heavenly body is the angle between the prime vertical and the circle of altitude passing through the body when on the horizon.

The amplitude of a rising body is measured by an arc of the horizon from the east point, and may be of any value, from 0° *to* 90° *north or south.*

The amplitude of a setting body is measured from the west point, and may be of any value from 0° *to* 90° *north or south.*

Twilight parallel The twilight parallel is a parallel of altitude 18° below the horizon.

Visible horizon The visible or sea horizon is the small circle of the celestial concave which bounds the view of the observer, being the apparent meeting of the sea and sky.

The dip is the angle of depression of the sea horizon.

Note.—*The correction for dip is –.*

Refraction

The correction for refraction is the angle through which a ray of light from a body is deflected in passing through the earth's atmosphere.

The correction for refraction is – ; it vanishes at the zenith and is a maximum at the horizon.

Parallax

The parallax is the apparent angular shifting of an object arising from a change in the point of view.

The correction for parallax in altitude is the angle subtended at the body by a radius of the earth passing through the observer.

The correction for parallax is + ; it is a maximum in the horizon and vanishes at the zenith.

Observed altitude

The observed altitude of a heavenly body is the altitude of the body above the sea horizon as taken with a sextant.

Apparent altitude

The apparent altitude of a heavenly body is the observed altitude when corrected for index error, dip, and semidiameter, and is the angle of elevation of the line drawn from the observer to the apparent place of the body.

True altitude

The true altitude is the apparent altitude corrected for refraction and parallax, and is the angle of elevation of the line drawn from the centre of the earth to the true place of the body.

Transit

The transit of a heavenly body is its passage over the celestial meridian.

Circles of declination

Circles of declination are great circles which pass through the poles of the heavens.

Circles of declination are sometimes called hour circles.

Parallels of declination

Parallels of declination are small circles whose planes are parallel to the plane of the equinoctial.

Right ascension

The right ascension of a heavenly body is the arc of the equinoctial between the first point of Aries and the circle of declination which passes through the body.

Right ascension is measured in time eastward—that is, in the direction of the sun's motion—and may be of any magnitude from 0 h. to 24 h.

Declination

The declination of a heavenly body is the arc of the circle of declination between the equinoctial and the body.

Declination is measured from the equinoctial, and may be of any magnitude, from 0° to 90° north or south.

Polar distance

The polar distance of a heavenly body is the arc of a circle of declination between the elevated pole and the body.

Polar angle

A polar angle is the angle between any two circles of declination.

Hour angle

The hour angle of a heavenly body is the angle between the celestial meridian and the hour circle passing through the place of the body.

Hour angle is measured westward from the celestial meridian, and may be of any magnitude, from 0 h. to 24 h.

Circles of celestial latitude

Circles of celestial latitude are great circles passing through the poles of the ecliptic.

Parallels of celestial latitude

Parallels of celestial latitude are small circles whose planes are parallel to the plane of the ecliptic.

Celestial longitude Celestial longitude is the arc of the ecliptic between the first point of Aries and the circle of celestial latitude passing through the place of the body.

Celestial longitude is measured eastward, and may be of any magnitude, from 0° *to* 360°.

Celestial latitude Celestial latitude is the arc of a circle of latitude between the ecliptic and the place of the body.

Celestial latitude is measured from the ecliptic, and may be of any magnitude, from 0° *to* 90° *north or south.*

Mean sun The mean sun is an imaginary body which is supposed to describe the equinoctial with uniform velocity in the same time that the real sun describes the ecliptic with variable velocity.

Apparent noon Apparent noon is the instant when the centre of the real sun is on the celestial meridian.

Mean noon Mean noon is the instant when the mean sun is on the celestial meridian.

Apparent time Apparent time is the hour angle of the real sun.

Mean time Mean time is the hour angle of the mean sun.

Equation of time The equation of time is the difference between apparent and mean time.

Sidereal time Sidereal time is the hour angle of the first point of Aries.

Apparent solar day An apparent solar day is the interval of time between two successive transits of the sun over the same celestial meridian.

Mean solar day A mean solar day is the interval of time between two successive transits of the mean sun over the same celestial meridian.

Sidereal day A sidereal day is the interval of time between two successive transits of the first point of Aries over the same celestial meridian.

Solar year A solar year is the interval of time between two successive passages of the sun through the first point of Aries.

Mean solar year A mean solar year is the mean of several solar years.

Sidereal year A sidereal year is the interval of time between two successive passages of the sun through the same point of the celestial concave.

Civil time Civil time is reckoned in two intervals of twelve hours named A.M. and P.M., as they come before or after the noon of the day which begins at midnight.

Astronomical time Astronomical time is reckoned in one period of twenty-four hours beginning at noon.

Greenwich date The Greenwich date is the astronomical time at Greenwich of the instant when an observation is taken.

INDEX

- Abbreviations on charts, 43
- Absolute altitude, 340
- Acceleration, 166, 192
- Adjustments of sextant, 137
- Age of moon, 406
  - tide, 411
- Airy's great circle method, 104
- Almanac, Nautical, 178
- Alphabet, Greek, 118
- Altitude circles, 127
  - absolute, 340
  - apparent, 155
  - corrections of, 145
  - double, 254
  - observed, 155
  - near meridian above pole, 241, 267
  - near meridian below pole, 253
    - errors in, 362
  - off meridian, calculated, 215
  - on meridian, calculated, 214
    - prime vertical, 280
  - rate of change of, 369
    - near meridian, 371
    - near prime vertical, 370
  - true, 156
- Altitude azimuth, 322
  - errors in, 371
- Amplitudes, 317
  - errors of, 321
- Aphelion, 159
- Apogee, 151
- Arc converted to time, 177
  - semi-diurnal, 332
- Aries, first point of, 123, 379
  - time of transit of, 175, 187
- Artificial horizon, 336
- Ascension, oblique, 125
  - right, 124
    - of moon, 187
      - planets, 188
      - stars, 125
      - sun, 186
- Astronomical day, 176
  - seasons, 380
  - time, 176
- Atlantic tides, 408
- Augmentation of moon's semi-diameter, 153
- Autumnal equinox, 123
- Auxiliary angle A, 294
- Axis of earth defined, 2
  - heavens defined, 121
- Azimuth, 127
  - altitude, 322
    - errors in, 371
  - diagram, Weir's, 322
  - tables, 329
  - time, 325
    - errors in, 373

- Balance, chronometer, 275
- Balls, time, 335
- Barometer, effect on tides, 413
  - vernier, 144
- Bearing and angle, 53
  - compass, 113
  - cross, 52
  - four-point, 59
  - magnetic, 53
  - true, 113
    - of a terrestrial object, 330
- Bearings, change of, 315
  - cross, 52
  - on charts, 44
  - plate, 60
- Binnacle, 12
- Bissextile, 170
- Blackburne's tables, 330, 368
- Box chronometer, 276
- Boxing the compass, 10

- Calendar, 170
  - Gregorian correction of, 171
- Cardinal points, 10
- Centering error, 145
- Change of altitude at any time, 272, 371
  - near meridian, 371
    - prime vertical, 370
- Charts, 30
  - abbreviations, 43
  - catalogue of, 44
  - correction of, 42
  - Gnomonic, 39
  - Mercator, 30, 39
  - number of, 43
  - Plane, 30

Charts, remarks on, 42
  star, 118
  wind and current, 83
Chord method (Sumner's), 385
Chronometer, construction of, 274
  history of, 274
  journal, 277
  performance of, 276
  rate of, 354
  star, 288
  sun, 283
  tests of, 276
Circle of illumination, 382
  declination, 124
  equal altitude, 384
  hour, 124
  six o'clock hour, 124
  vertical, 127
Civil day, 176
  time, 176
Clearing lunar distance, 293
Collimation error, 138
Colure, equinoctial, 194
Combined altitudes, 272
Comparison of chronometers, 277
Compass construction, 8
  deviation, 120
  error, total, 16
  journal, 100
  standard, 8
  steering, 8
  total error, 16
Composite sailing, 105
Compression of earth, 377
Concave, celestial, 121
Constellations, 117
Convergency, 52
Conversion of R. A. and declination into lat. and long. and *vice versâ*, 132
Course, compass, 12
  made good, 15
  magnetic, 18
  steered, 15
  through water, 82
  true, 7
Cross bearings, 52
Current, 83
  chart, 83
  sailing, 81
Curve of equal altitude, 384

Danger angle, 60
Date, Greenwich, 179
Day defined, 157
  apparent solar, 168
  length of, 332
  longest, 333
  lunar, 157
  mean solar, 160
  shortest, 333
  sidereal, 158, 168
  solar apparent, 158
  mean, 160
Day's work, 83

Dead reckoning, 1
Declination defined, 124
  circles, 124
  parallels, 124
  of moon, to correct, 187
  planet, to correct, 188
  star, 125
  suns to correct, 183
Definitions in nautical astronomy, 421
  navigation, 110
Departure, 69
  course, 65
Deviation of compass, 20
  finding, 319, 324, 327
  table, 21
Diagram, astronomical, 127, 129, 130
  azimuth, Weir's, 322
Dip, apparent, 147
  of sea horizon, 146
  shore horizon, 148
  tabular, 147
  true, 147
Distance, 8
  scale, 41
Diurnal inequality of tides, 412
Double altitudes, direct method, 254
  errors of, 373
  of stars, 263
  sun, 257
  chronometer rule, 396
Doubling angle on bow, 59
Douwes' method of double alts., 269
Dumb card, 60

Earth, velocity in orbit, 159
Easting, 3
Ecliptic, 123
  obliquity of, 110
Equal altitudes on both sides of mer., 343, 347, 351
Equal alts. on same side of mer., 353
  longitude by, 312
  circles of, 384
  curves of, 384
  of stars, 353
  parallel of, 383
Equation of time, 161
  fig. of parts, 163
  to correct, 185
  equal altitudes, 345
Equator defined, 2
  celestial, 123
Equi-angular spiral, 7
Equinoctial, 123
Equinoxes, 159
  precession of, 124, 379
Error of chronometer, 339, 343, 353
Establishment of port, 409

Fixed stars, 117
  to find, 118
Fixing positions, 52
Formulæ of sailings, 89
Four-point bearing, 59

GEOCENTRIC position, 131, 156
Geographical position of sun and heavenly bodies, 382
Gnomonic projection, 39
Godfray's great circle method, 103
Great circle defined, 2
    methods of drawing, 103, 104
    sailing, 101
Greek alphabet, 118
Greenwich date approx., 179
    exact, 181
Gregorian correction, 171
Ground log, 15

HARRISON'S watch, 274
Heights on charts, 43
Heliocentric position, 131
Herne's method for long., 312
High-water line, 43
    time of, 409
Horary tables, 202
Horizon, 125
    apparent or visible, 125
    artificial, 336
    errors in position of, 360
    rational, 125
    sensible, 125
    true distance of, 147
    visible or apparent, 125
        distance of, 147
Hour angle defined, 201
    calculated, 203
    errors in, 365
    notes on, 209
    of moon, 201
        star, 206
        sun, 205
  circles, 124
    six o'clock, 124

ILLUMINATION, circle of, 382
Index error sextant, 140
Inequality diurnal, 412
Irradiation, 144

JOHNSON'S method, 396
Journal compass, 100
    chronometer, 277

KEPLER'S laws, 116
Kew certificates, 145
Knot, length of, 13

LAGGING, 406
Latitude celestial, 131, 378
    defined, 3, 227
    difference of, 3
    equals alt. of pole, 227
    scale, 41
    terrestrial, 3
Latitude by alt. near mer., 241, 250, 267
    change of alt., 273
    combined alts., 272
    double alts., 254
    Douwes' method, 269
    mer. alt. above pole, 230
        below pole, 239
        moon, 235
        planet, 238
        sun, 233
        star, 237
    Pole star, 254
    three alts., 270
    Sumner's method, 381
Leeway, 26
Lemonnier on long., 312
Level, mean sea, 407
Libra, first point of, 123
Lights on charts, 43
Lines of position, 382
Log book, 83
  Dutchman's, 12
  glass, 13
    error of, 13
  ground, 15
  line, 13
    error of, 13
  patent, 14
  ship, 12
Logarithms, proportional, 297
Longitude, celestial, 131
    defined, 2, 279
    difference of, 3, 279
    scale, 41
    by chron. moon, 290
        star, 288
        sun, 283
      equal alts., 312
      lunar, 290
Loxodromic curve, 7
Lubber line, 12
Lunar, by star, 301, 304
    sun, 300
    planet, 301
  distance stars, 291
  refinements omitted at sea, 311
  notes on, 308

MAGNETIC course, 18
    meridian, 18
Marcq Saint-Hilaire's method, 393
Mean sea level, 407
  sun, daily motion of, 165
  time, 160
Mercator chart, 30
    sailing, 77
Meridian altitude above pole, 230
    below pole, 239
    of moon, 235
      star or planet, 237
      sun, 233
    not maximum alt., 233, 236
    errors in, 361

Meridian defined, 2
    magnetic, 20
    prime, 110
    reduction to, 248
    secondary, 358
Meridional diff. lat., 32
    parts, 32, 107
Middle latitude sailing, 73
Mile, nautical and geographical, length of, 3
Moon's altitude, correction of, 197
    decl., to correct, 187
    equatorial hor. par., 190
    hor. par., to correct, 189
    mer. passage, approx., 216
        exact, 219
    motions, 221
    passing meridian, 209
    right ascension, 188
    semi-diameter, 151
        to correct, 189

Nadir, 126
Napier's curve, 24
    diagram, 99
Natural scale, 41
Nautical Almanac, 178
    astronomy defined, 1
        definitions, 421
    mile, length of, 3
Navigation defined, 1
    definitions, 110
Neap tides, 406
New style, 171
Night observations, 361
Northing, 3

Oblique ascension, 125
Obliquity of ecliptic, 123
Observations at night, 361
Observatory, Royal, 309
Old style, 171

Pagel's method, 397
Parallax, 151
    diurnal, 156, 378
    horizontal, 152
    in altitude, 152
    of moon, 189
        sextant, 141
        star or planet, 153, 188
        sun, 153
Parallel of equal altitude, 383
    latitude, 3
    sailing, 66
Perigee, 151
Perihelion, 158
Plane chart, 30
    sailing, 69
Planets, elements of, 117
    mer. alt., 237
    movement, laws of, 116
Points of compass, names of, 10
    equinoctial, 123
Points, solstitial, 124
Polar distance, 124
Pole star for lat., 254
    bearing of, 329
    corrections explained, 252
    not at pole, 119
Poles, 121
    defined, 2
Position, fixing of in various ways, 52
    lines of, 382
Port, establishment of, 409
Precession of equinoxes, 124, 379
Prime vertical, 127
    altitude on, 280
    time of crossing, 280
Priming and lagging, 406
Projections, 30
    stereographic, 128
Proper motions of stars, 380
Proportional logarithms, 297
Protractor, 50
    rectangular, 50
    semi-circular, 50

Quadrantal points, 10

Range of tide, 408
Rate of chronometer, 354
    harbour, 358
    sea, 358
Reduced latitude, 378
Reduction to meridian, 248
Reflection, laws of, 133
Refraction, 148
    correction of, 150
Retardation, 166, 194
Revolution of earth, 116
    planets, 116
Rhumb line, 7
Right ascension, 124, 186
    of moon, to correct, 188
        planet, to correct, 188
        star, 125
        sun, to correct, 186
Rising, time of, 333
Rotation of planets, 116
    sun, 116
Royal Observatory, 309
Run, corr. for, in alt., 221, 381
    time, 224

Sailings, 65
    current, 81
    great circle, 101
    Mercator, 77
    middle latitude, 73
    parallel, 66
    plane, 69
    traverse, 73
Scale, natural, 41
    of latitude, 41
        longitude, 41
Sea rate of chron., 357
Seasons illustrated, 122, 380
Second differences in lunars, 2

Semi-diameters, moon, 151
        planet, 117
        sun, 151
Semi-diurnal arcs, 332
Sensible horizon, 125
Setting, time of, 333
Sextant, 133
    adjustments, 137
    centering error, 145
    collimation, 138
    double, 136
    index error, 140
    names of parts, 133
    notes on, 144
    side error, 137
    stand, 339
Shadwell's star-tables, 262
Sidereal time, at mean noon, 187
        converted into mean solar, and *vice-versâ*, 167
        mean and apparent, 164
Solar system, 115
        elements of, 117
Solstices, 159
Solstitial points, 124
Soundings, 88
    machine, 88
    on charts, 43
Southing, 3
Spheroidal form of earth, 377
Spiral, equi-angular, 7
    spring, 274
Spring tides, 406
Standard chronometer, 279
    compass, 8
Stars, altitude, corr. of, 200
    catalogue, 118
    chronometer, 288
    clock, 158
    fixed, 117
    lunar distance, 291
    mer. passage, time of, approx., 212
            exact, 213
    number of, 117
    passing meridian, 209
    proper motion of, 380
    tables, Shadwell's, 262
    to find, 119
Station pointer, 51
Steering compass, 8
Stereographic projection, 128
Straight line method (compasses), 24
Stream, tidal, 411
Sumner lines, 385
    method, 381
Sun, altitude, corr. of, 195
    chronometer, 280
    crosses line, 184
    declination, corr. of, 184
    dial, 165
    geographical position of, 382
    parallax, 153
    right ascension, corr. of, 186
    semi-diameter, 151
Swinging ship, 98

TABLES of azimuth, 185
    deviation, 21
    ex mer. 249
    hour angles, 202
    latitude corr., 286
    solar system, 117
    tides, 409
    variation, 16
Terminator, 382
Tidal streams, 411
    waves, 405
Tides, 404
    age of, 411
    tables, 409
Time and longitude converted, 185
    apparent solar, 164
    astronomical, 176
    azimuth, 325
        errors in, 373
        tables, 185
    balls, 335
    civil, 176
    conversion of, 191
    converted to arc, 177
    defined, 157
    equivalents, 167
    graphically shown, 163
    mean solar, 164
    problems on, 171
    sidereal, 164
    standards of, 157
Total error of compass, 161
Towson, ex-meridian tables, 249
    great circle method, 103, 329
Traverse sailing, 73
    table, 71
Tropical year, 168
True bearing of a terrestrial object, 330
Twilight, 334

VARIATION, 16
Velocity of planets, 116
Vernal equinox, 123
Vernier of barometer, 144
    sextant, 142
Vertex of great circle, 102
Vertical circles, 127
    prime, 127
    angle of, 378

WALKER's taffrail log, 14
Weir's azimuth diagram, 322
Westing, 3

YEAR anomalistic, 169
    defined, 158
    Julian, 170
    mean solar, 168
    sidereal, 168
    tropical, 168

ZENITH, 125
Zodiac, 123

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NAVIGATION

AND

NAUTICAL ASTRONOMY

Astronomical Observatory
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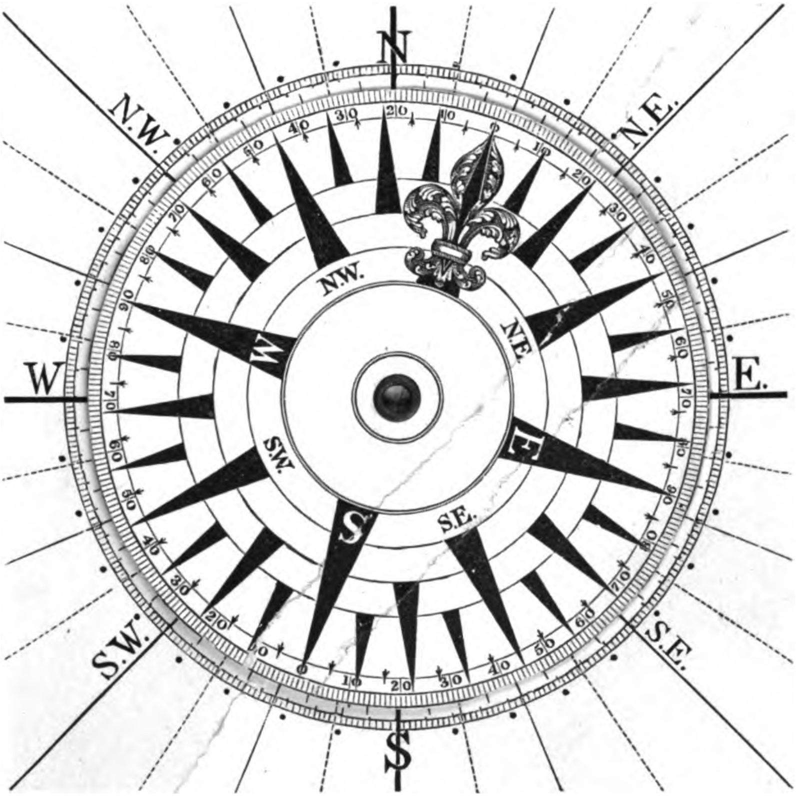
1899

PLATE I.

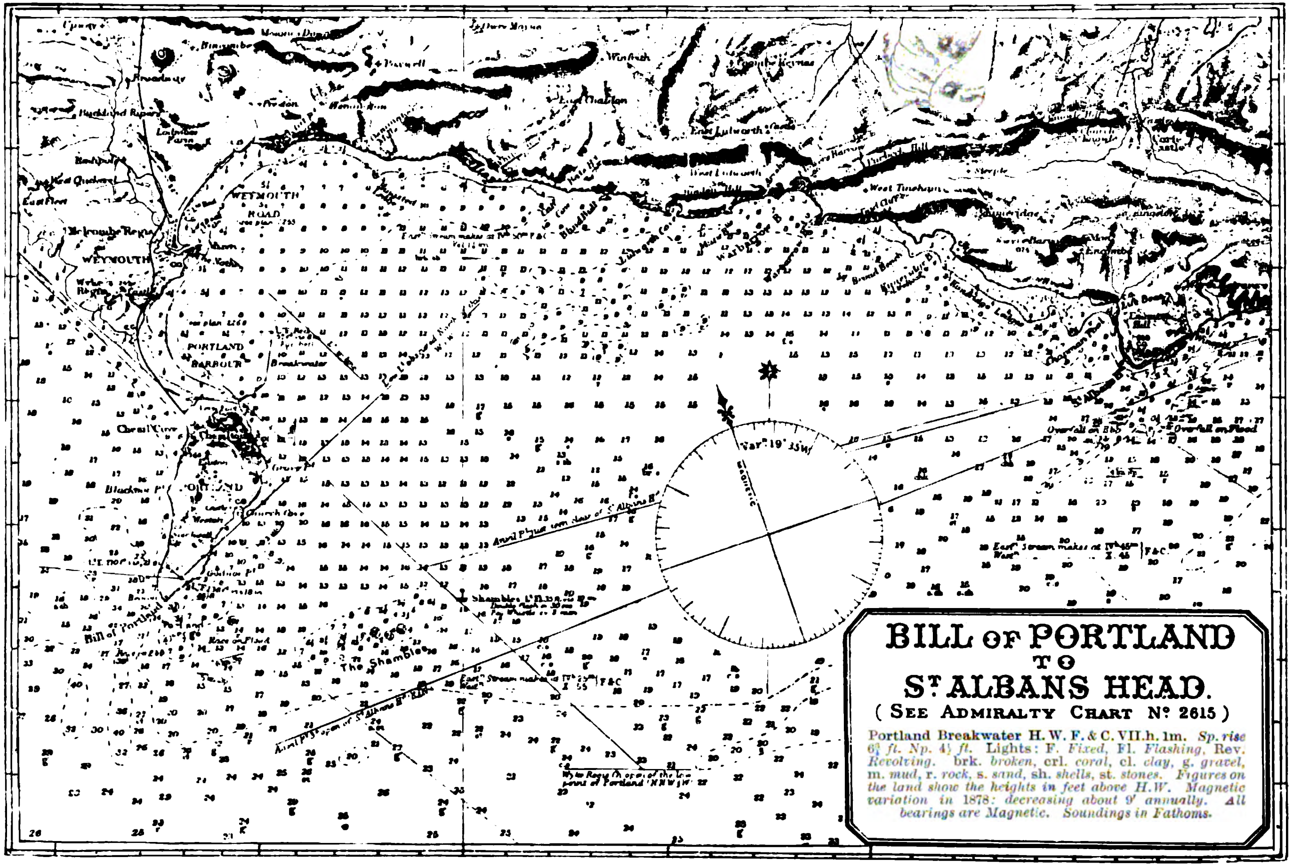


If using these cards for correcting for *Variation* alone, the fixed card should be considered *true*, and the north point of the inner card moved east or west of true north by an amount equal to the Variation.

If correcting for *Deviation* alone, consider the fixed card as Magnetic, and separate the north points by an amount equal to the deviation.

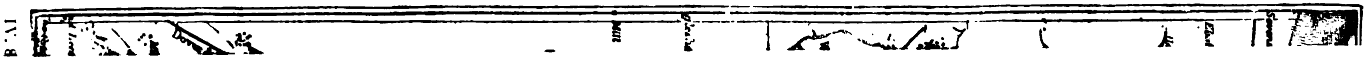
If correcting for both Variation and Deviation, consider the outer card *true*, and separate the north points by an amount equal to the sum of the Variation and Deviation if of like names, or otherwise by their difference.

PLATE 2.



BILL OF PORTLAND
TO
ST. ALBANS HEAD.
(SEE ADMIRALTY CHART No. 2615)

Portland Breakwater H. W. F. & C. VII.h. 1m. *Sp. rise 6¾ ft. Np. 4½ ft.* Lights: F. *Fixed*, Fl. *Flashing*, Rev. *Revolving*. brk. *broken*, crl. *coral*, cl. *clay*, g. *gravel*, m. *mud*, r. *rock*, s. *sand*, sh. *shells*, st. *stones*. *Figures on the land show the heights in feet above H.W. Magnetic variation in 1878: decreasing about 9′ annually. All bearings are Magnetic. Soundings in Fathoms.*



g to the vertical
ndard Compass.
ction of courses.

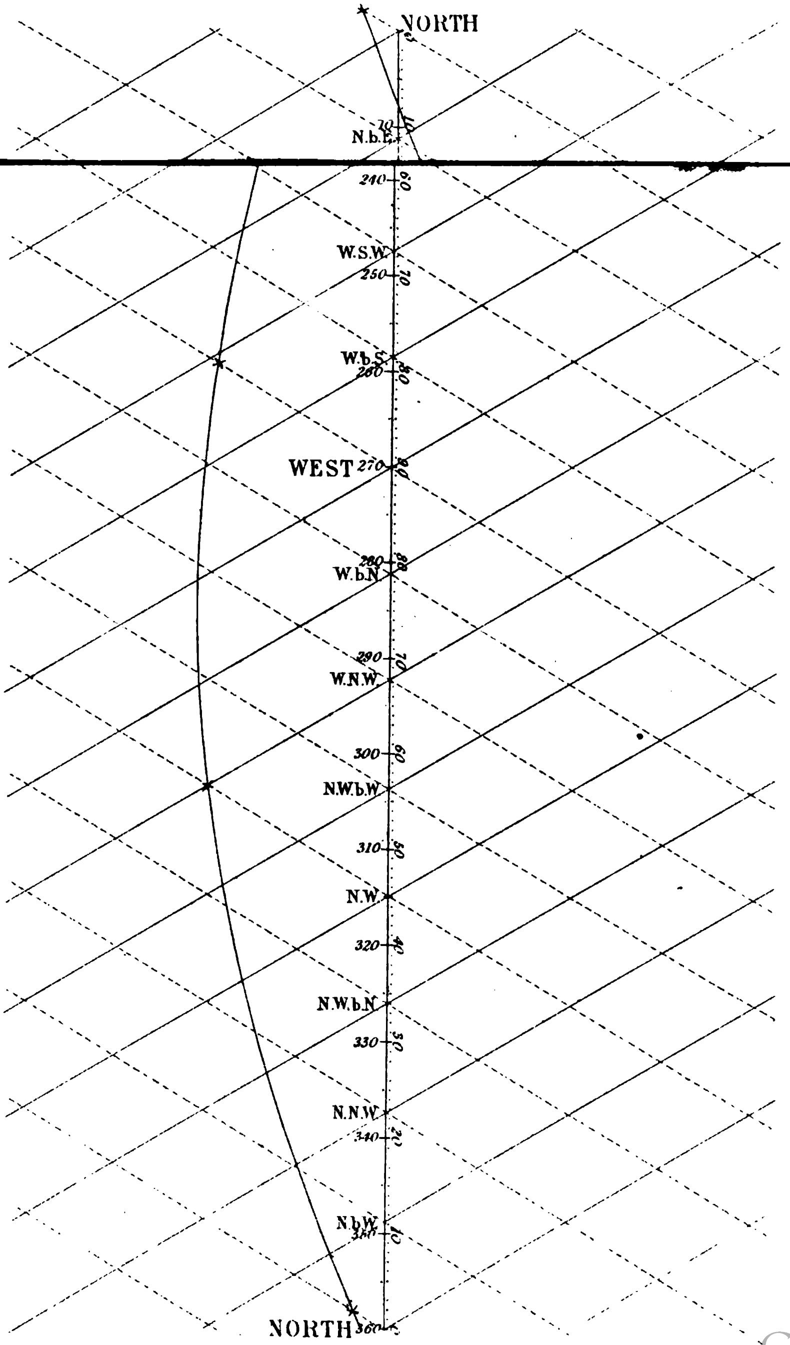
Plate 4.

NAPIER'S DIAGRAM.

(CONSTRUCTED FROM THE DEVIATION TABLE USED IN THIS WORK.

North point of compass
drawn to the
West.

North point of compass
drawn to the
East.



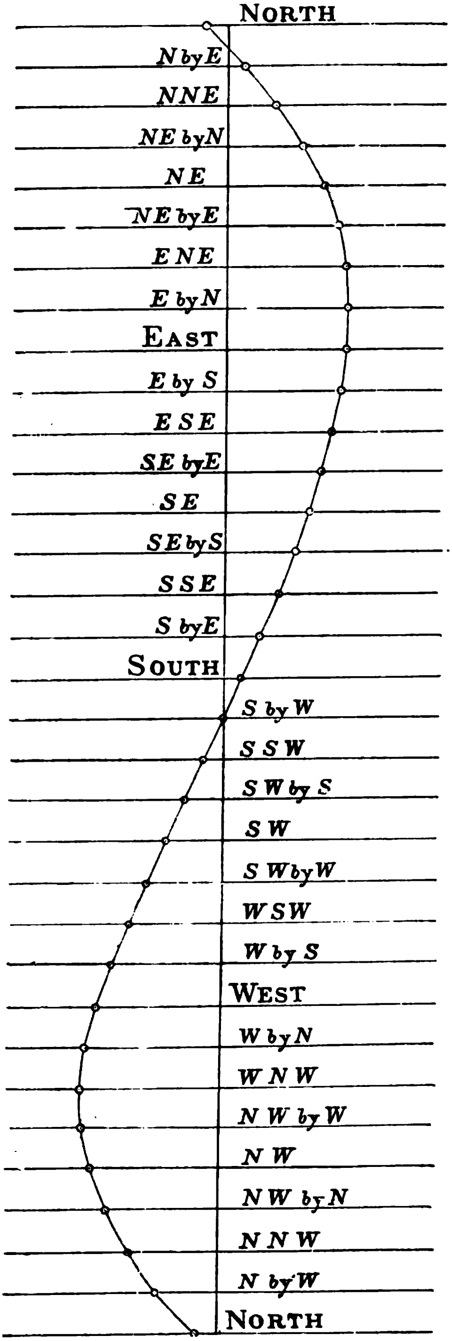
then move in a
course required.
to the vertical

d then move in a
course required.
g to the vertical
ndard Compass.
ction of courses.

PLATE 5.

DEVIATION CURVE.

DRAWN FROM THE DEVIATION TABLE USED IN THIS WORK.



NOTE.—This curve is useful for testing the accuracy of observations for deviation, but not for correcting courses. Easterly deviations are laid off to the right of the central line, and westerly deviations to the left; these may be laid down from *any* scale of equal parts.

PLATE 6.

ARCHIBALD SMITH'S (STRAIGHT LINE) COURSE TABLE,

CONSTRUCTED FROM THE DEVIATION TABLE USED THROUGHOUT THIS WORK,

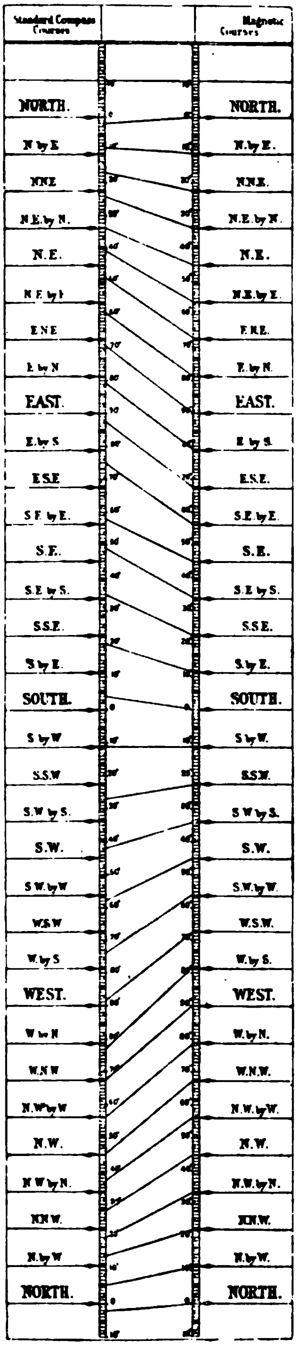
For the Use of Officers of the Watches
when Tactical Manœuvres are being performed.

EXPLANATION.

I. How to fill up the Table.

This Table may be filled up two ways, viz.—Either by drawing the lines from all the points, half-points and quarter-points in the 'Magnetic Course' Column, as in the Example, or from all the points, &c., in the 'Standard Compass Course' Column.

The first will be the most useful to the Officers of the Watches in Tactical Manœuvres, and has been adopted in the Example.



EXPLANATION.

II. How to use the Table.

1ST CASE.—If the Fleet is steering a given course by Signal (which always means a Magnetic Course), and a Signal is made to alter course a given number of points to Starboard or Port, run the finger up or down on the Magnetic Course Column the required number of points, and then across on the Straight line to the Standard Compass Course Column, and the Standard Compass Course to be Steered will be found.

2ND CASE.—If the Fleet is not steering a Magnetic Course given by Signal, then run the finger across from the 'Standard Compass Course' which is being steered on the Straight line to the opposite Column, and proceed as in Case 1.

PLATE 6.

ARCHIBALD SMITH'S (STRAIGHT LINE) COURSE TABLE,

Constructed from the Deviation Table used throughout this Work,

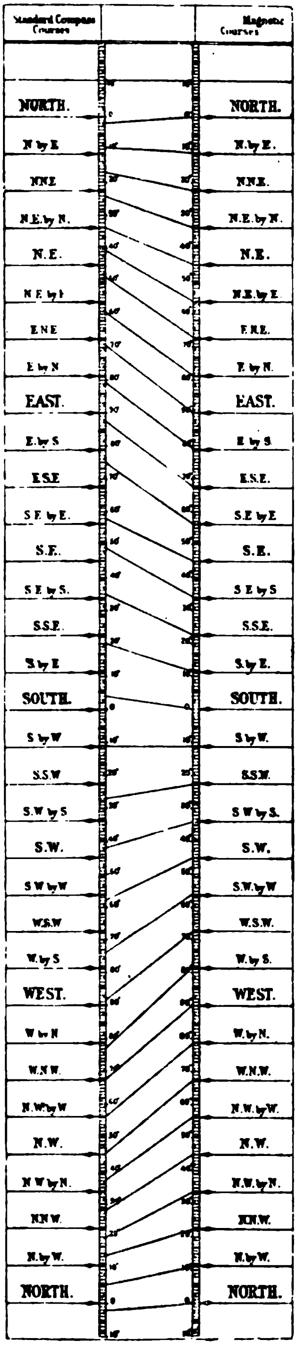
For the Use of Officers of the Watches
when Tactical Manœuvres are being performed.

EXPLANATION.

I. How to fill up the Table.

This Table may be filled up two ways, viz.—Either by drawing the lines from all the points, half-points and quarter-points in the 'Magnetic Course' Column, as in the Example, or from all the points, &c., in the 'Standard Compass Course' Column.

The first will be the most useful to the Officers of the Watches in Tactical Manœuvres, and has been adopted in the Example.



EXPLANATION.

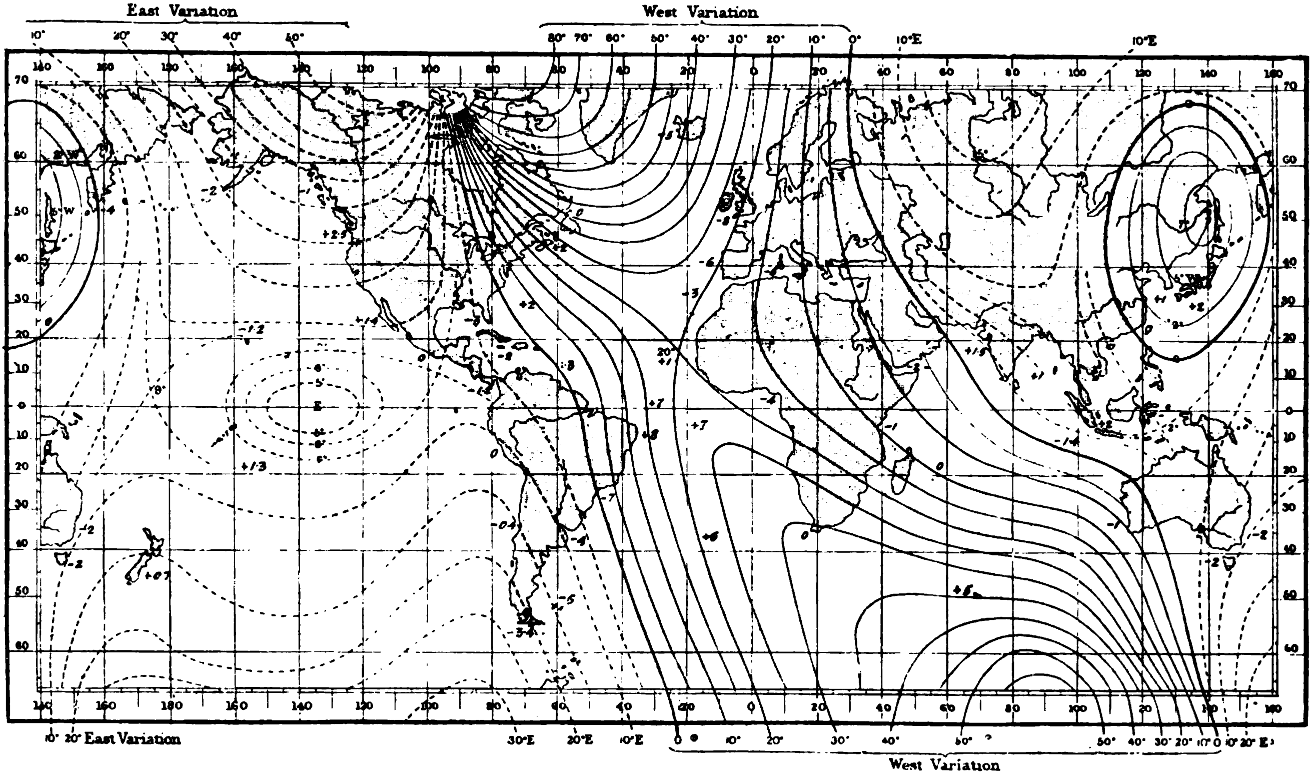
II. How to use the Table.

1st Case.—If the Fleet is steering a given course by Signal (which always means a Magnetic Course), and a Signal is made to alter course a given number of points to Starboard or Port, run the finger up or down on the Magnetic Course Column the required number of points, and then across on the Straight line to the Standard Compass Course Column, and the Standard Compass Course to be Steered will be found.

2nd Case.—If the Fleet is not steering a Magnetic Course given by Signal, then run the finger across from the 'Standard Compass Course' which is being steered on the Straight line to the opposite Column, and proceed as in Case 1.

PLATE 7.

Lines of Equal Magnetic VARIATION 1882

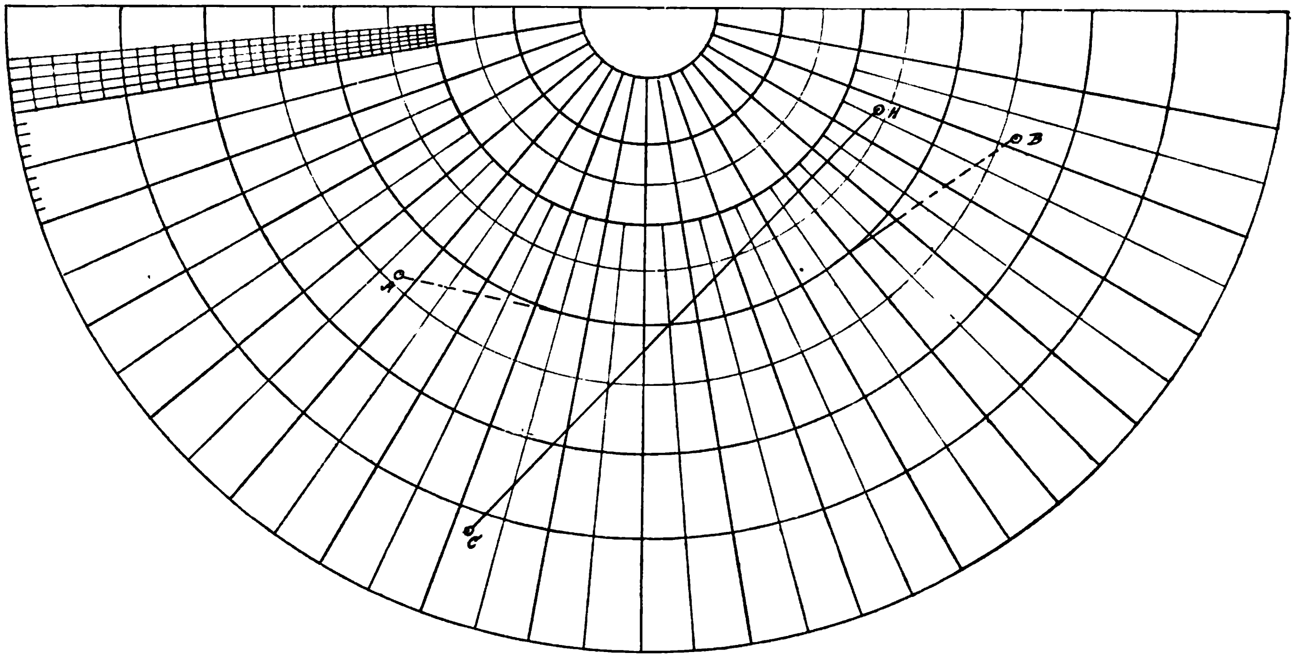


NOTE.—The figures on this chart preceded by the sign + or – show the approximate annual changes of variation in minutes of arc, + indicating an annual increase and – an annual decrease.

This information is given on one of the upper diagrams on the Variation Chart.

PLATE 8.

GODFRAY'S GREAT CIRCLE DIAGRAM.



This diagram is available for any places being both in North, or both in South latitude, since *any* meridian may be assumed as that of Greenwich.

H C represents the great circle between Cape Horn and the Cape of Good Hope.

A B represents a composite track between A and B not passing south of lat. 50°.

The lat. and long. of intermediate positions are transferred to a Mercator chart.

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